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A simple and fundamental design rule for resisting delamination in bimaterial structures

Thomas D. Moore [a,*], John L. Jarvis [b]

[a] *Packaging Development in the Assembly Department, Analog Devices B.V., Limerick, Ireland*
[b] *Department of Aeronautical and Mechanical Engineering, University of Limerick, Ireland*

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Abstract

The determination of the peeling stress in the interface between the two layers of a bimaterial structure has long been of particular interest in the microelectronic industry. This is because peeling of deposited layers on silicon wafers, and delamination of encapsulated integrated circuits continue to be of great concern. Until now beam theory has not delivered a simple method of determining the direction of the peeling stress at the free edge of the structure, although a recent paper has proposed a formulation which approximates to the reality.

Past studies have focused on determining the stresses in the interface. In this paper a different approach is taken. A fundamental formulation is derived for the moment that arises in the interface as a result of the naturally occurring reversal in sign of the peeling stress. It is shown that this peeling moment M_p completely generates the additional bending needed to ensure that the radius of curvature of each layer at the interface is identical, even when they have differing flexural rigidities.

The direction and magnitude of the peeling moment M_p is determined in this paper, and its value is confirmed by finite element analysis for seven test cases representing a wide range of bimaterial structures. The expression for M_p is simple and easily computed, and it provides a fundamental design rule for resisting delamination in bimaterial structures.

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1. Introduction

Several instances of bimaterial assemblies occur in the manufacture of integrated circuits (ICs). The vast majority of IC assemblies consist of a silicon chip bonded at elevated temperature to a substrate of a greatly different material. This constitutes a bimaterial assembly, with the silicon chip as the upper layer and the die-attach paddle or substrate as the lower layer.

Generally the assembly is encapsulated with an epoxy moulding compound at high temperature, and in many instances nowadays this encapsulated product may also be approximated as a bimaterial assembly. For example a plastic ball grid array (BGA) comprises the substrate as the lower layer and the encapsulant as the upper layer.

Finally the individual layers deposited or grown at elevated temperature in the fabrication of the IC wafer may be treated as a bimaterial assembly of that layer onto the receiving structure.

In each case the delamination of the bimaterial assembly is a potential hazard to the reliability of the IC. The objective of this work is to provide a greater understanding of the root cause and nature of the peeling force which arises from the thermomechanical stress in the bimaterial. From this understanding a simple criterion is developed by which the direction of the peeling stress can be determined, and which enables

* Corresponding author. Tel.: +353-61-495139; fax: +353-61-303111.

E-mail addresses: tom-d.moore@analog.com (T.D. Moore), john.jarvis@ul.ie (J.L. Jarvis).

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a comparison of the relative magnitudes of the peeling stress under various layer geometries and material properties.

2. Prior work

The foundation for the application of beam theory in the study of bimaterial beams was laid by Timoshenko [1] in 1925. This enabled the axial stresses and the radius of curvature of the structure to be calculated under thermomechanical loading. Timoshenko stated that the interfacial forces were concentrated near the free ends, and consisted of shearing and peeling forces. In his Section IV Timoshenko extended the bimaterial beam theory to apply to the bimaterial plate. He also examined the behaviour of the beam in the specific instance where the thicknesses and moduli of elasticity of the two layers were identical, and showed that in this circumstance the peeling force was zero.

Timoshenko's extension of the theory to the bimaterial plate was applied to the analysis of 3-D IC structures by Glascock and Webster [2]. They used the theory to develop charts for determination of bow (deflection) and axial stress in the silicon, when attached to various metal substrate materials.

Suhir [3] extended Timoshenko's theory by including a provision for the effect of the local shear stress on the axial displacement of the beam. An additional component of axial displacement at any point on the beam was introduced, proportional to the value of the shear stress at that point. This approach allowed expressions to be formulated for the shear and peeling stresses in the interface. In the initial formulation the shear stress did not close to zero at the free end; and a separate normal force at the free end was needed to equilibrate the peeling stresses. In a later formulation [4], Suhir delivered zero shear at the free end; the formulation proved to be quite complex to implement in practice.

A valuable benchmarking exercise was made by Eischen et al. [5] for the comparison of various approaches to determining the stress in electronic bimaterial structures. They compared solutions from Hess (an extension of [1] with an eigenfunction solution truncated to 30 terms with 101 points in the least squares fit), Suhir (as in [3]) and finite element analysis (FEA) (300 8-node elements containing 981 nodes). The FEA solution and the Hess solution concurred, while the Suhir solution showed some discrepancies from these.

Most recently, Ru [6] introduced an amendment of Suhir's theory, in which the contribution to the axial displacement from shear was in two parts; one part was proportional to the shear at that point, and the other was proportional to the second derivative of the shear. This resulted in formulations for shear and peeling stress along the interface in the beam which gave results of the same nature and often of the same order of magnitude as the FEA and Hess results reported in [5].

Klingbeil and Bueth [7,8] took a fracture mechanics approach to the analysis of bimaterial structures, and presented rules for design of debond–resistant interfaces. The key recommendation was that to prevent initiation of delamination the peeling stress at the edge should be compressive, and to prevent propagation of the delamination crack, the signs of the edge stress intensity factor K^* and the interface stress intensity factor K_1^* should both be positive. These factors are functions of the ratio of the thicknesses, and Dundar's α, i.e. the relative stiffness of the two materials.

In addition these authors [7] drew attention to the many factors which influence delamination and peeling in practice. These may individually or collectively give rise to delamination in a structure which is designed in accordance with the recommendations above.

The consequences of having a tensile value for the peeling stress are well illustrated in an experiment by Mirman [9]. In this study it was shown that a bimaterial consisting of a Kovar layer and a plastic layer delaminated under thermomechanical stress due to cooling by 30 °C, and resisted delamination when heated by 80 °C. It was shown by FEA that under cooling the peeling stress at the edge was tensile, while under heating it was compressive. The shear stress in the interface changed in the same proportion as the peeling stress; it was concluded that the principle cause of initiation of delamination was the peeling stress rather than the shear stress.

3. Bending moments in a thermostat beam

In the first section of his paper [1] Timoshenko examined the deflection of the beam under uniform heating. He made the usual beam theory assumptions, including the assumption that the thickness of each layer was very much smaller than the radius of curvature taken up by the bent beam after a uniform temperature change. A shear force develops at the interface between the two layers as a result of the difference in the coefficients of thermal expansion (TCEs) of the layers. This gives rise to tensile and compressive axial forces, P_1 and P_2 in the two layers. The radius of curvature ρ of the bent beam was expressed as

$$\frac{1}{\rho} = \frac{(\alpha_2 - \alpha_1)(t - t_0)}{\frac{h_1 + h_2}{2} + \frac{2(E_1 I_1 + E_2 I_2)}{h_1 + h_2}\left[\frac{1}{E_1 h_1} + \frac{1}{E_2 h_2}\right]} \tag{1}$$

where h_1 and h_2 are the beam thicknesses, α_1 and α_2 are the TCEs, E_1 and E_2 are the moduli of elasticity, I_1 and I_2 are the moments of inertia of the upper and lower

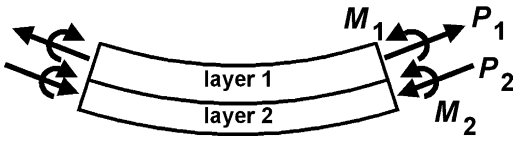


Fig. 1. An arbitrary element of the bimaterial beam deformed by the bending moments generated within it (from Fig. 1 of Ref. [1]).

layer respectively, and t and t_0 are the final and initial temperatures of the beam of unit width.

An arbitrary element of the bimaterial beam remote from the ends is shown in Fig. 1.

Timoshenko showed that the axial forces P_1 and P_2 are in equilibrium and may be labelled simply as P. Its size can be determined as follows:

$$P = \frac{2(E_1 I_1 + E_2 I_2)}{(h_1 + h_2)\rho} \tag{2}$$

He also examined the distribution of stresses in an arbitrary cross-section remote from the ends, and stated that for each material the axial stresses in the material can be reduced to a central force P as used above, and a couple M_1 or M_2 as shown in Fig. 1. He then considered the stresses near the ends of the beam. He began by making the following simplification. When the layer thicknesses are identical (i.e. $h_1 = h_2$) then their moments of inertia are also identical viz. $I_1 = I_2$; also the lever arms for each layer are identical, and thus $M_1 = M_2$. If at the same time their Young's Moduli are identical, viz. $E_1 = E_2$ then this gives rise to an identical radius of curvature at the interface for each layer.

In the general situation, $E_1 I_1$ is not equal to $E_2 I_2$ and the moments M_1 and M_2 acting alone will not cause the radii of curvature at the interface for each layer to be identical. Further explanation is needed to account for the identical interfacial curvature of each layer.

The explanation lies in the generation of additional forces and moments between the layers. Timoshenko identified the presence of a peeling stress which varied along the interface, which together with the shearing stress near the end could be reduced to two equal and opposite couples of such magnitude as to produce (with the couples previously considered) equal curvatures $1/\rho$ at the interface.

4. Moments generated by the peeling stress distribution

In this section a means of determining the additional couples needed to adjust the bending of both layers to the common radius of curvature ρ will be developed. It has been shown by various authors [1,10] that in a long and slender beam the peeling stress is zero at all locations except in a region near the free ends of the beam. It was further shown [10] that the shear stress is also zero except in the same region. In addition it has been shown that in this local region a change of sign occurs in the peeling stress [5,6]; these papers have also shown that the maximum value of the peeling stress occurs at the free edge.

It is this reversing peeling stress which gives rise to the additional moment necessary to produce identical interfacial curvatures.

The peeling stress arises from the action of one layer on the other normal to the interface. An example of the direction and sign of the peeling stress is shown in Fig. 2.

In this example the action is positive (tensile) at the free edge. Inboard from the free edge the action becomes negative (compressive). The integration of the peeling stress over the zone in which it is tensile gives an equivalent tensile force acting through a point located in the tensile zone. Similarly the integration of the peeling stress over the zone in which it is compressive gives an equivalent compressive force acting through a point located in the compressive zone.

These equivalent forces are necessarily equal and opposite to maintain equilibrium of forces in the normal direction. This combination of forces in opposite directions with a lever arm between them gives rise to a moment labeled M_p in Fig. 2.

The direction of the moment is also shown by the curved arrows in Fig. 2; it promotes sagging. In accordance with convention, moments which promote

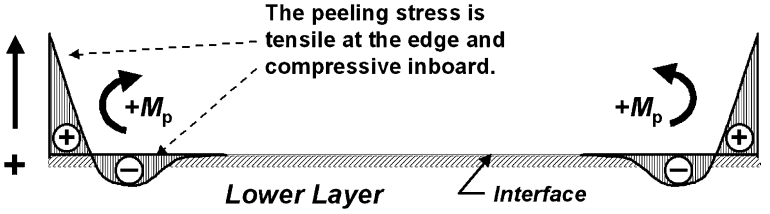


Fig. 2. Tensile and compressive action of the upper layer on the lower layer of a bimaterial beam due to a uniform change in temperature.

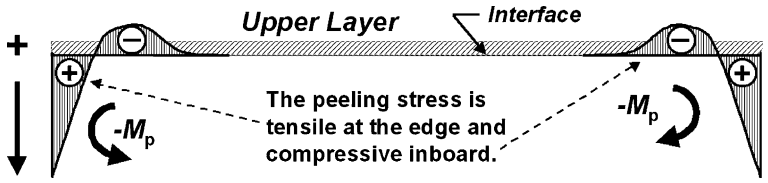


Fig. 3. Tensile and compressive action of the lower layer on the upper layer of the same bimaterial beam as in Fig. 2.

sagging are shown positive, and moments which promote hogging are shown negative.

The equal and opposite moment arising from the corresponding action of the lower layer on the upper layer is shown in Fig. 3. This moment promotes hogging of the layer and is therefore shown negative.

If the peeling stress at the free end were compressive, (becoming tensile inboard) the direction of the moment would be opposite to that shown by the curved arrows in Figs. 2 and 3.

It is evident from Figs. 2 and 3 that each instance of the peeling stress generates a couple M_p close to the point of occurrence. The couples are equal and opposite, being the action of one layer on the other. Thus considering Figs. 1–3, identical interfacial curvatures are produced when

$$\frac{M_1 - M_p}{E_1 I_1} = \frac{1}{\rho} = \frac{M_2 + M_p}{E_2 I_2} \tag{3}$$

Re-ordering yields an expression for the couple M_p that arises from the peeling stress. This is

$$M_p = \frac{M_1 E_2 I_2 - M_2 E_1 I_1}{E_1 I_1 + E_2 I_2} \tag{4}$$

The moment M_1 arises from the offset of the axial force P from the interface; the magnitude of the offset is $h_1/2$. Thus $Ph_1/2$ may be substituted instead of M_1, and similarly for M_2. With this substitution and using the expression in Eq. (2) in place of P the following expression evolves:

$$M_p = \frac{h_1 E_2 I_2 - h_2 E_1 I_1}{(h_1 + h_2)\rho} \tag{5}$$

Finally substituting $h_1^3/12$ for I_1 and $h_2^3/12$ for I_2 (recalling that the beam is unit width) we get

$$M_p = \frac{h_1 h_2}{12(h_1 + h_2)\rho}(h_2^2 E_2 - h_1^2 E_1) \tag{6}$$

The sign of M_p is determined solely by the sign of $(h_2^2 E_2 - h_1^2 E_1)$ and the sign of ρ. The sign of ρ is determined by the difference in TCEs of the materials and the sign of the temperature change. The term $(h_2^2 E_2 - h_1^2 E_1)$ is the key as to how the layer thicknesses and elasticities influence the likelihood of delamination. As this is a square function of h_1 and h_2, relatively small changes in thickness may cause a significant change in the propensity of a bimaterial structure to delaminate.

The maximum peeling stress at the free edge as shown in Figs. 2 and 3 may be either positive (tensile) or negative (compressive) depending on the sign of M_p; the figures illustrate a condition in which the maximum peeling stress is tensile. If the maximum peeling stress is tensile then it promotes delamination at the free edge, while if it is compressive it inhibits delamination (pinching instead of peeling). Thus the sign of M_p provides a fundamental design rule for resisting delamination in bimaterial beams.

5. Other occurrences of $(h_2^2 E_2 - h_1^2 E_1)$

The magnitude of the expression $(h_2^2 E_2 - h_1^2 E_1)$ diminishes to zero as $h_2^2 E_2$ approaches $h_1^2 E_1$ in value. The expression changes sign at the equality, and the maximum normal stress switches from pinching to peeling, or vice versa. At the equality, when $h_2^2 E_2 = h_1^2 E_1$ the moment imposed on each layer by the shear at the interface is exactly sufficient to cause identical radii of curvature at the interface. An additional moment is not needed and as a consequence the peeling stress does not arise. Timoshenko's statement in the paragraph following Fig. 4 of his paper [1] specifies a subset of this equality.

A related expression arises several times in the literature on bending of bimaterial beams. In both [3,4] the variable μ is a multiplier which determines the sign and magnitude of the peeling stress (and in these formulations the shear stress also). The expression for μ is $(h_2 D_1 - h_1 D_2)/[2(D_1 + D_2)]$ where D is the flexural rigidity; this can be simplified by considering $v_1 \approx v_2$ to get the following (note that μ equals zero when $h_2^2 E_2 = h_1^2 E_1$):

$$\mu = \frac{1}{2}\frac{E_1 h_1^2 - E_2 h_2^2}{E_1 h_1^2 + E_2 h_2^2} \tag{7}$$

The formulation for the peeling stress in [6] uses an expression identical to μ as a multiplier. It also appears in Eq. (2.12) of that paper. A discussion ensued therein which questioned the abnormal dependency of the peeling stress on the geometrical and material parameters; in fact, Timoshenko took advantage of this de-

pendency to generate an example free of peeling stress, as used in [1, Eq. (9)].

The present analysis shows that this phenomenon, viz. the diminishment of the expression $(h_2^2E_2 - h_1^2E_1)$ to zero as $h_2^2E_2$ approaches $h_1^2E_1$ is a natural property of the structure, and is accompanied by a reduction to zero of the peeling stress. This is also pointed out by Matthys and De Mey [11] following their Eq. (5).

6. Modelling as independent layers in FEA

The moment M_p may be determined from the geometric and material properties of the layers and the temperature change by using any one of Eqs. (4), (5) or (6). FEA is used to demonstrate that the peeling moment from these equations gives rise to identical bending in the two layers and also that this bending is identical to the bending in a bimaterial beam in which the two layers are continuously bonded. In this section the logic behind the finite element model is presented, and then the structure is described.

The peeling stress in the interface varies continuously along the region close to the free edge of the beam. It was shown in Section 4 that the peeling stress reverses sign generating a couple M_p. In the FEA implementation the peeling stress distribution was replaced by a pair of equal and opposite forces of moment M_p. The moment arm was taken as approximately 25% of the total beam thickness because by observation of FEA analyses of bonded IC assemblies this arm was typical of the distance between the centroids of the positive and negative peeling stress zones.

However, it is emphasized that for consideration of bending, the moment may be applied as any combination of normal forces, and that these forces may be subdivided so long as the moment takes effect from the free end and so long as the net moment computes to the value of M_p. In the bonded beam model the shear and peeling were distributed along the interface rather than being replaced by the statically equivalent point forces as in the independent (non-bonded) FEA model.

The axial force P arises from the mismatch in coefficients of thermal expansion of the bonded layers. The effect of one layer on the other is exerted through shear at the interface, close to the free edge of the beam. The magnitude of the total shear force is identical to the magnitude of the axial force in reaction to it, viz. P.

In this FEA implementation a compressive axial force of magnitude P was applied to the free edge at the interface of the layer of greater length, and a corresponding tensile axial force was applied to the free edge at the interface of the other layer. These force the interface fibres in each layer to take up the same axial length when superimposed on the effect of the temperature change.

The normal forces which comprise M_p outlined above together with the axial force P define the action of one layer on the other in FEA.

The model type used was a plane stress representation of a bimaterial beam. In the analysis each layer was treated as an independent entity, i.e. unbonded, and the actions of one layer on the other were replaced by point forces applied so as to generate moments.

The modeling was done in ANSYS 5.7, with 104 elements of type PLANE42 in each layer. Linear elastic properties were used. In each analysis a second bimaterial beam was included, identical to the first in every respect except that the two layers were bonded continuously along the interface. This was subject to the temperature change but not to the peeling moment. For simplicity this second beam is not shown in the figures.

A line of symmetry exists at the mid-point of the beam, cutting vertically through the two layers. Accordingly only one half of the beam was modeled. Each layer has its nodes on this line anchored in x (the axial direction) in order to provide a reaction to the axial force and a reaction to the moments. In each layer also, one of these nodes was anchored in y (the normal direction) in order to prevent relative movement of the layers at the plane of symmetry. For each layer, this latter node lay on the interface.

The layers were positioned with a separation of a few microns in order to facilitate detection of the onset of pinching. Each layer was subject to thermal expansion, and each was subject to an axial force representing the shear stress together with a pair of equal but opposite normal forces representing the peeling stress. The forces on one layer were matched by corresponding forces on the other layer, with the axial forces being of opposite sign.

An illustration is provided in Fig. 4, showing the constraints at the line of symmetry, and the forces near the free end. To allow a suitable scale the mid-section of the beam is omitted from Figs. 4 and 6.

In this illustration the axial forces are applied as compressive on the lower layer and tensile on the upper

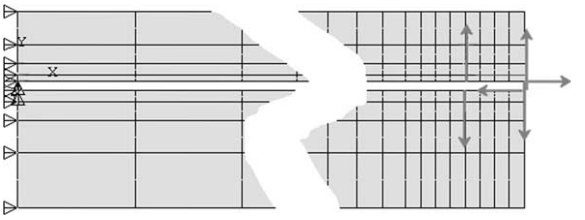


Fig. 4. The model structure showing the beam comprising two independent layers and the forces and constraints at each end of it.

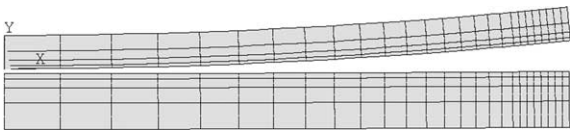


Fig. 5. Bending of two independent layers due to axial forces.

layer, and the peeling moment is applied as positive (sagging) on the lower layer and negative (hogging) on the upper layer.

The deformed independent layers are shown in Fig. 5 with the peeling moment M_p initially set to zero. Under thermomechanical stress each layer bends in the same direction, each being subject to a moment arising from the offset of the axial force from the neutral axis of the layer. These are the moments M_1 and M_2 of Eq. (2) above. In the general case each layer adopts a different radius of curvature; this is evident in Fig. 5.

In the general case peeling stresses arise. A moment of identical magnitude but opposite value was applied to the two layers to adjust their bending. This moment was varied iteratively until identical curvatures were generated on the lines representing the interface of the two layers.

A peeling effect is evident in Fig. 5. It was found that a positive moment was required to counteract a peeling effect, and a negative moment was required to counteract a pinching effect. From consideration of Figs. 2 and 3, it is also clear that a positive moment also indicates that the peeling stress at the free end is positive in sign.

The effect of adding the desired moment M_p is shown in Fig. 6. Note that this moment gave rise to a separation between the layers at the free end equal to the separation between the layers at the plane of symmetry. Thus the curvature at the interface is identical.

It will also be noted in this detailed view that each layer has adopted a different length at the neutral axis, but that the lengths at the interface are identical. This arises directly from the use of Timoshenko's formulation for the axial force, because that is based on identical displacements of the interface fibres of each layer.

7. FEA results and comparison with theory

Seven cases of bimaterial structure were investigated as described below. For each case the FEA model was run with iterative estimates of M_p until the vertical displacement δ_U of the node at the free edge of the interface of the upper layer was identical to the vertical displacement δ_L of the corresponding node on the lower layer. The values of δ_U and M_p were noted.

In each analysis the vertical displacement of the node on the interface at the free edge δ_B of the bonded beam was also noted to serve as a validation of the model with the independent layers.

Then for each case M_p was computed from Eq. (5), and the vertical displacement of the free end δ was computed using the well known formula from [1], $\delta = L^2/8\rho$, where L is the full length of the beam.

The seven representative structures were as follows:

- An example similar to [1] in which the thicknesses and elastic moduli of the two layers were identical; this is case 1.
- An example in which the thicknesses differed, and the elastic moduli were identical; this is case 2.
- An example in which $h_2^2 E_2 = h_1^2 E_1$; this is case 3.
- One example from [3]; this is case 4.
- The two examples from [5], which were also used to investigate the model of [6]; these are cases 5 and 6.
- An example from a BGA assembly in current production [12]; this is case 7.

The input data for these cases are set out in Table 1. The table also contains the values of δ as computed and from the two beams in FEA (viz. δ_L and δ_B). A validation of the FEA modelling approach using independent layers is provided by comparison of the values for the vertical displacement δ. An examination of Table 1 shows that δ as calculated from the widely accepted formula is often identical to four significant digits to the results from FEA of the model with the independent layers. In no case is the difference as much as 1%.

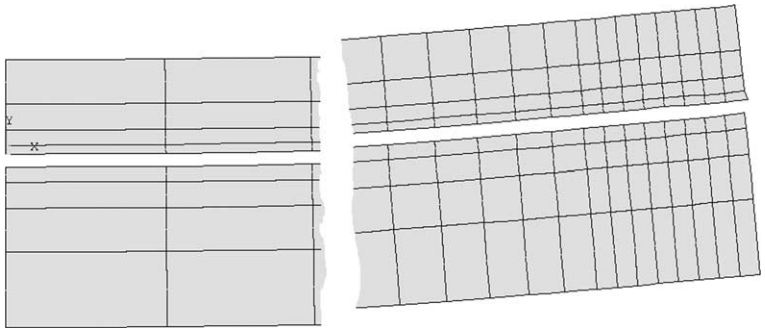


Fig. 6. Both ends of deformed model with peeling moment M_p added; the equal spacing shows identical curvature at the interface.

Table 1
Input data for seven cases of bimaterial beams, and corresponding results from first principles and from FEA

Variable	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7
h_1 (m)	0.0015	0.0015	0.0001	0.0005	0.0025	0.006096	0.000280
E_1 (Pa)	70.38e9	70.38e9	180e9	120e9	70.38e9	200e9	150e9
α_1 (°C^{-1})	23.6e−6	23.6e−6	2.8e−6	3.2e−6	23.6e−6	2.5e−6	2.9e−6
v_1[a]	0.345	0.345	0.300	0.42	0.345	0.3	0.28
h_2 (m)	0.0015	0.0025	0.0003	0.002	0.0025	0.000254	0.000550
E_2 (Pa)	70.38e9	70.38e9	20e9	70e9	325e9	69e9	20e9
α_2 (°C^{-1})	4.9e−6	4.9e−6	15e−6	23.6e−6	4.9e−6	23.6e−6	12.4e−6
v_2[a]	0.345	0.345	0.300	0.345	0.293	0.345	0.39
ΔT (°C)	240	240	−150	240	240	240	−240
$L/2$ (m)	0.0254	0.0254	0.0025	0.0254	0.0254	0.0762	0.010
Results data							
δ-[1] (m)	−7.24e−4	−5.09e−4	−2.14e−5	7.30e−4	−3.69e−4	2.042e−4	−1.811e−4
δ_L-lower (m)	−7.25e−4	−5.13e−4	−2.16e−5	7.34e−4	−3.69e−4	2.042e−4	−1.814e−4
δ_B-bonded (m)	−7.24e−4	−5.11e−4	−2.15e−5	7.31e−4	−3.68e−4	2.041e−4	−1.810e−4
M_p (N m)	0	−34.70	0	18.87	−189.6	−10.62	0.1808
M_p-FEA (N m)	0	−34.58	0	18.87	−188.0	−10.62	0.1823

[a] v_1 and v_2 are the Poisson's ratios of the respective materials.

Further validation is made by comparing the value of the vertical displacement δ_B of the bonded beam model with the vertical displacement δ_L of the model with independent layers. The difference does not exceed 0.5%.

Table 1 also shows a comparison between M_p as given by Eq. (5) and as found by the iterative solution in FEA. The results show that Eq. (5) gives values for M_p that are often identical to four significant digits to the values from FEA. In no case is the difference as much as 1%. This indicates a very close correspondence between the results from Eq. (5) and the FEA data for a very wide variety of bimaterial cases.

8. Conclusions

A new expression is derived for the peeling moment M_p in a bimaterial beam under thermomechanical load. The expression is fundamental, and involves only the dimensions and material properties of the layers of the beam. It is simple and easy to compute.

The peeling moment M_p is in itself sufficient to correct the bending of both layers of the beam when there is a difference in thickness or elasticity of the layers.

The magnitude and sign of the peeling moment M_p is easily determined using the expression derived in this paper.

The sign of M_p provides a fundamental design rule for resisting delamination in bimaterial beams.

From the sign of M_p the sign of the peeling stress at the free end of the beam is found with certainty. In addition, an indication of the relative magnitude of the maximum peeling stress may be determined for comparative analyses.

Some previous formulations from beam theory either consistently gave the incorrect sign for the maximum peeling stress, or contained a component in the formulation in which the sign was uncertain. In this paper the uncertainty is removed.

The factors which determine the sign of the peeling moment and of the maximum peeling stress are the radius of curvature ρ and $(h_2^2E_2 - h_1^2E_1)$; M_p is positive if the product of these factors is positive. When M_p is positive the maximum peeling stress is tensile and a risk of delamination exists.

The latter factor is sensitive to the square of the layer thicknesses, and more specifically to the square of the ratio of the thicknesses.

The factor is sensitive to the ratio of the elasticities of the materials.

The reduction of the expression $(h_2^2E_2 - h_1^2E_1)$ to zero as $h_2^2E_2$ approaches $h_1^2E_1$ in value is a natural property of the structure, and is accompanied by a corresponding reduction to zero of the peeling stress.

References

[1] Timoshenko S. Analysis of bi-metal thermostats. J Opt Soc Am 1925;11(September):233–55.
[2] Glascock H, Webster H. Structured copper: a pliable high conductance material for bonding to silicon power devices. IEEE Trans CHMT 1983;6(4):460–6.
[3] Suhir E. Stresses in bi-metal thermostats. ASME J Appl Mech 1986;53(September):657–60.
[4] Suhir E. Interfacial stresses in bi-metal thermostats. ASME J Appl Mech 1989;56(September):595–600.
[5] Eischen JW, Chung C, Kim JH. Realistic modeling of edge effect stresses in bimaterial elements. ASME J Electron Packaging 1990;112(3):16–23.

[6] Ru CQ. Interfacial thermal stresses in bimaterial beams: modified beam models revisited. ASME J Electron Packaging 2002;124(1):141–6.
[7] Klingbeil NW, Bueth JL. On the design of debond–resistant bimaterials—Part I: free-edge singularity approach. Eng Fract Mech 2000;66:93–110.
[8] Klingbeil NW, Bueth JL. On the design of debond–resistant bimaterials—Part II: a comparison of free-edge and interface crack approaches. Eng Fract Mech 2000;66: 110–28.
[9] Mirman IB. Effects of peeling stresses in a bimaterial assembly. ASME J Electron Packaging 1991;113(4):431–3.
[10] Suhir E. Predicted thermal stresses in a bimaterial assembly adhesively bonded at the ends. AIP J Appl Phys 2001; 89(1):120–9.
[11] Matthys L, De Mey G. An analysis of an engineering model for the thermal mismatch stresses at the interface of a uniformly heated two layer structure. IMAPS Int J Microcircuits & Electron Packaging 1996;19(2): 323–9.
[12] Moore TD, Jarvis JL. Failure analysis and stress simulation in small multichip BGAs. IEEE Trans Adv Packaging 2001;24(2):216–23.