Efficiency of spectrometers

B. Carli and V. Natale

A general formula for the calculation of the signal to noise ratio (SNR) of a spectroscopic measurement is derived, and its factors are discussed. Particular attention is paid to identify which losses are necessarily present in the measurement and which can be avoided or reduced. The formula can also be used for the comparison of the different spectroscopic methods.

I. Introduction

Two recent papers[1,2] have suggested a revised value of the multiplex advantage in Fourier spectroscopy (FS): $(N/8)^{1/2}$ instead of the conventional value $(N)^{1/2}$, where N is the number of spectral elements. This revised value raised the objection that the factor $8^{1/2}$ was already well known under other names and could therefore be incorrectly used twice.[3]

No general criterion exists for the calculation of the efficiency of a spectrometer and for the comparison of the different methods. As a consequence different definitions are often used for the calculation of the multiplex advantage in FS, and a scattered range of values is quoted in the literature.

The purpose of this paper is to describe the different spectroscopic techniques within a single logic frame and to deduce a tentative classification of efficiency factors. The attention of this paper is focused mainly on measurements in the IR spectral region where the different spectroscopic techniques have comparable merits, but most of our results have general value.

In Sec. II some basic considerations on detection and coding of a signal are made. In Sec. III we recall some properties of an algorithm, which led to the first evaluation of the multiplex advantage in FS as equal to $(N/8)^{1/2}$ and gives a single mathematical tool to describe and analyze the different spectroscopic methods.[4] Using the results of Secs. II and III a comprehensive expression for the calculation of the SNR in the spectral measurement is given in Sec. IV.

II. Coding Problem

The measurement of a signal requires the knowledge of a reference level corresponding to zero signal. In an ideal case, this information can be derived from an *a priori* calibration of the detecting system. In the measurement of IR radiation power it is in practice always given by a real measurement taken during the measurement time. This imples that, if a time t is spent for the measurement, the signal is acquired for a time $t/2$, and an equal time $t/2$ is used to measure the zero level. The loss in SNR relative to the ideal case is equal to $\frac{1}{2}$ for a detector noise limited measurement.

Usually the signal and the zero level are compared by means of a suitable alternator, and the difference between the two levels is measured by demodulation plus integration of the modulated output from the detector. As calculated by Treffers,[2] the SNR of this measurement is equal to

$$\frac{S}{N} = \frac{P\sqrt{2}}{(\text{N.E.P.})} \frac{\int_{-\infty}^{+\infty} M(t)I(t)D(t)dt}{\left[\int_{-\infty}^{+\infty} D^2(t)I^2(t)dt\right]^{1/2}}, \tag{1}$$

where P is the power which reaches the measuring system, N.E.P. is the detector noise equivalent power, $M(t)$ is the function which modulates P, $D(t)$ is the demodulating function, and $I(t)$ is the integration function. Equation (1) is valid only in the hypothesis of detector noise limited measurements. We will refer to the modulation and demodulation process used for the signal measurement as signal coding. Equation (1) can also be written as

$$\frac{S}{N} = \frac{P(2\tau)^{1/2}}{(\text{N.E.P.})} \cdot \alpha, \tag{2}$$

where τ is the integration time, and α is the signal coding efficiency which depends on $M(t)$, $I(t)$, and $D(t)$. The values of α for different modulation and demodulation functions are listed in Table I.

The authors are with CNR Instituto di Ricerca sulle Onde Elettromagnetiche, 50127 Firenze, Italy.

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Table I. Signal Coding Efficiency (α)

		M_1	M_2	M_3
D_1	I_1	1	—	—
	I_2	$\sqrt{2}$	—	—
D_2	I_1	—	$1/\sqrt{8}$	$2/\pi$
	I_2	—	1/2	1/2
D_3	I_1	—	$1/\pi$	$2/\pi$
	I_2	—	$\sqrt{2}/\pi$	$1/\sqrt{2}$
D_4	I_1	—	$1/\sqrt{8}$	$\sqrt{2}/\pi$
	I_2	—	1/2	$2/\pi$

Modulating function:
$M_1 = 1$ no modulation;
$M_2 = 1/2(1 + \cos\omega t)$ cosine modulation;
$M_3 = 1/2 + 2/\pi \sum_{k=0}^{\infty} 1/(2k+1) \cos k\omega t$ square wave modulation;
where $f = \omega/2\pi$ is the modulation frequency.

Demodulating function:
$D_1 = 1$ no demodulation;
$D_2 = \cos\omega t$ cosine demodulation;
$D_3 = 4/\pi \sum_{k=1}^{\infty} 1/(2k+1) \cos k\omega t$ square wave demodulation;
$D_4 = 4/\pi \cos\omega t$ equivalent demodulation when high frequencies filter plus square wave demodulation are used.

Integration function:
$I_1 = 1/\tau$ for $0 \le t < \tau$;
$I_1 = 0$ for $t < 0$ and $t \ge \tau$;
$I_2 = (1/\tau) \exp(t/\tau)$ for $t \le 0$;
$I_2 = 0$ for $t > 0$.

From Table I we can see that the use of the signal coding, essential for the signal measurements, leads to a loss in SNR relative to the ideal case of a measurement without coding. This loss can be partially regained in a few cases. A possibility is a more efficient use of the incoming power by shining the radiation alternatively upon two detectors (double output modulation) instead of wasting radiation for half of the time (single output modulation). The double output modulation allows a gain of $\sqrt{2}$ in SNR relative to the single output modulation. Furthermore, the losses due to signal coding can be avoided altogether when other codings (e.g., spectral and/or spatial coding) are present in the measurement (see Sec. III).

The possible recovery of some of the signal coding losses will be taken into account in Sec. IV using a factor G_1, equal to $\sqrt{2}$ for double output modulation and equal to 1 for single output modulation.

III. Spectral Analysis

We can acquire information on a spectral distribution $S(\nu)$ by filtering the incoming radiation with a set of frequency dependent filters with transparency $f_i(\nu)$ and measuring the integral transmitted power:

$$P_i = \int_{\nu_{\min}}^{\nu_{\max}} E(\nu) \cdot S(\nu) \cdot f_i(\nu) d\nu. \quad (3)$$

$E(\nu)$ is the instrumental efficiency (see Appendix), and $f_i(\nu)$ are the filtering functions (FFs) typical of each spectroscopic method. The FF is a function which approximates a Dirac delta for monochromators (grating, prism, and Fabry-Perot spectrometers) a cosine function plus offset for FS, a Hadamard function plus offset for Hadamard spectroscopy (HS), a sawtooth modulation for the mock interferometer, or any other function, which can be optically obtained (high or low pass filter, bandpass filter, etc.).

The retrieval of the spectral distribution from the experimental measurements requires an *ad hoc* computing method for each spectroscopic technique. This divides spectroscopy in a few almost independent fields [Dirac spectroscopy (DS), FS, HS], each with its formalism. In order to simplify the comparison of the different spectroscopic methods we will refer to an algorithm which can be used for the retrieval of the spectrum from measurements made with any FF.[4] This algorithm leads to a formula which gives the spectral distribution as a function of the measurements and of the FFs. An important property of this formula is that it reduces itself to the *ad hoc* computing method when the FFs have the classical form.

The existence of an analytical relationship between the retrieved spectrum and the measurements ensures us that the instrumental function can always be calculated and makes it possible to obtain the relationship between the noise level in the experimental measurement $\langle \sigma_I^2 \rangle$ and the noise level in the retrieved spectral distribution $\langle \sigma_R^2 \rangle$ (Ref. 4):

$$\langle \sigma_R^2 \rangle = \frac{1}{\beta^2(\nu)} \langle \sigma_I^2 \rangle, \quad (4)$$

where $\beta(\nu)$ is a parameter which can be calculated from the FFs.

A. Instrumental Function

Remembering that the instrumental function $F(\nu,\nu_0)$ is by definition the retrieved spectral distribution when the input spectrum is monochromatic (at frequency ν_0) and of unit intensity, we deduce that the relationship between the retrieved spectrum $S_R(\nu)$ and the input spectrum $S_I(\nu)$ can always be expressed as

$$S_R(\nu) = \int E(\nu_0) \cdot S_I(\nu_0) \cdot F(\nu,\nu_0) d\nu_0. \quad (5)$$

In particular when $F(\nu,\nu_0) = F(\nu - \nu_0)$, Eq. (5) becomes a convolution:

$$S_R(\nu) = [E(\nu) \cdot S_I(\nu)] * F(\nu). \quad (6)$$

B. Parameter β

The parameter β measures the noise amplification or attenuation due to the retrieval calculations. The values of β for different spectroscopic methods, when ideal FFs are used, are listed in Table II.

From Table II we can see that the loss of a factor $1/4$

Table II. Spectral Coding Efficiency (β)

Spectroscopic method	Spectral coding efficiency β
Dirac spectroscopy	1
Fourier spectroscopy	$(N/8)^{1/2}$
Hadamard spectroscopy	$(N/4)^{1/2}$
Mock interferometer (data retrieval with first harmonic approximation)	$(N/2\pi^2)^{1/2}$
Mock interferometer (data retrieval with real FFs)	$(N/12)^{1/2}$

for HS and of 1/8 for FS respect to the full multiplex advantage N is confirmed. This result is not surprising since a multiplex measurement is not a mere superposition of N channels on the same detector, but it is also a coding which makes possible the retrieval of the power in the N different channels. The notion that the losses, relative to the full multiplex advantage, present in multiplex spectroscopy are related to the spectral coding automatically suggests how these losses can be partially regained. The spectral coding in fact makes use of a modulation which is often also suitable for the signal detection. In some spectroscopic methods therefore the signal coding is superfluous, and its losses can be avoided. This is done in FS where a fast scanning instrument is frequently used and could be extended to HS.

Furthermore, if the instrument is not dissipative, the reflected power is

$$P_R = \int_{\nu_{\min}}^{\nu_{\max}} E(\nu)S(\nu)[1 - f_i(\nu)]d\nu. \tag{7}$$

This second output can be exploited in two ways:

(1) two detectors can be used, one at each output, with a gain in SNR of $\sqrt{2}$;

(2) the signal modulation can be arranged in order to measure the difference between the two complementary outputs with a gain in SNR equal to 2. This solution reduces the noise due to source fluctuations in multiplex spectroscopy; on the contrary it enhances this noise if used in nonmultiplex spectroscopy.

The possibility of recovering some of the spectral coding losses will be taken into account in Sec. IV using a factor G_2 equal to $\sqrt{2}$ for double output, equal to 2 for complementary output modulation, and equal to 1 otherwise.

IV. SNR in the Spectrum

From Eqs. (2–5) we deduce that the SNR of the spectral measurement is in general equal to

$$\left(\frac{S}{N}\right)_S = \frac{\int E(\nu_0)S_I(\nu_0)F(\nu,\nu_0)d\nu}{\text{N.E.P.}} G_1G_2\alpha\beta(2\tau)^{1/2}, \tag{8}$$

where G_1 and G_2 are the factors related to the possible optimization of the instrument discussed in Secs. II and III. In Eq. (8) τ may be substituted by TE_t/N, where T is the total time of the measurement, and E_t is a factor which we can call time efficiency and is equal, by definition, to τ/t_m with $t_m = T/N$ the sampling period.

Since $S_I(\nu_0) = I_I(\nu_0)A\Omega$, where $I_I(\nu_0)$ is the intensity of the signal and $A\Omega$ the throughput, and since $E(\nu)$ is in general a slowly varying function in comparison to $F(\nu, \nu_0)$, Eq. (8) may be written as

$$\left(\frac{S}{N}\right)_S = \frac{\int I_I(\nu_0)A\Omega F(\nu,\nu_0)d\nu}{\text{N.E.P.}} E(\nu)G_1G_2\alpha\beta\left(\frac{2TE_t}{N}\right)^{1/2}. \tag{9}$$

Equation (9) does not include the effect on SNR due to the apodization. This technique is widely used because the instrumental function often shows undesired features such as secondary lobes or long tails. In this case it may become desirable to decrease the spectral resolution in order to improve the contrast of the spectrum. This can be done by convolving the spectrum with a suitable function $C(\nu)$ normalized to 1. As a result, the SNR of the spectrum varies from

$$\left(\frac{S}{N}\right)_{\text{UNAPODIZED}} = K\frac{\int I_I(\nu_0)F(\nu,\nu_0)d\nu_0}{\text{N.E.P.}} \tag{10}$$

to

$$\left(\frac{S}{N}\right)_{\text{APODIZED}} = K\frac{\int I_I(\nu_0)F'(\nu,\nu_0)d\nu_0}{[\int(\text{N.E.P.})^2 \cdot C^2(\nu)d\nu]^{1/2}}, \tag{11}$$

where $F'(\nu,\nu_0) = F(\nu',\nu_0) * C(\nu - \nu')$. The SNR is multiplied by a factor

$$B(\nu) = \frac{\left(\frac{S}{N}\right)_{\text{APODIZED}}}{\left(\frac{S}{N}\right)_{\text{UNAPODIZED}}} = \frac{\int I_I(\nu_0)F'(\nu,\nu_0)d\nu_0}{\int I_I(\nu_0)F(\nu,\nu_0)d\nu_0 \cdot [\int C^2(\nu)d\nu]^{1/2}}. \tag{12}$$

If the spectrum is equal to a constant over the integration interval, remembering that F, C, and F' are functions normalized to 1, we obtain

$$B(\nu) = \frac{1}{[\int C^2(\nu)d\nu]^{1/2}} = a, \tag{13}$$

where $a > 1$ is a constant, typical of the function used for the convolution. In the case of FS the same operation is more simply done by apodizing the interferogram with a function $A(x) = \text{FT}[C(\nu)]$. The constant a as a function of $A(x)$ is equal to

$$a = \frac{2D}{\int_{-D}^{+D} A^2(x)dx}. \tag{14}$$

If $S_I(\nu_0) = \delta(\nu_0 - \nu^*)$

$$B(\nu) = \frac{F'(\nu,\nu^*)a}{F(\nu,\nu^*)}. \tag{15}$$

For $\nu = \nu^*$, $B(\nu)$ is always smaller than 1, which implies a SNR loss in line spectroscopy. The values of a for a flat spectrum and of $B(\nu^*)$ for a monochromatic spectrum, for different apodization functions, have been calculated by Harris.[5] With reference to Table I of that paper: $a = (\text{ENBW} \cdot \text{COHERENT GAIN})^{-1/2}$ and $B(\nu^*) = (\text{ENBW})^{-1/2}$. Including these considerations on apodization into Eq. (9) we obtain

$$\left(\frac{S}{N}\right)_S = \frac{\int I_I(\nu_0)F'(\nu,\nu_0)d\nu}{\text{N.E.P.}} E(\nu)A\Omega a\alpha G_1\beta G_2\left(\frac{2TE_t}{N}\right)^{1/2}, \tag{16}$$

where $F'(\nu,\nu_0)$ reduces itself to $F(\nu,\nu_0)$ when no apodization is used.

In order to calculate the SNR of a spectral distribution represented by a sequence of discrete points, we need to calculate, using the instrumental function, the resolution interval $\Delta\nu$ of the measurement and define an average intensity

$$I_m = \frac{\int I_I(\nu_0)F'(\nu,\nu_0)d\nu}{\Delta\nu}. \tag{17}$$

Introducing the relationship (17) into Eq. (16), we finally obtain

$$\left(\frac{S}{N}\right)_S = \frac{I_m\Delta\nu}{\text{N.E.P.}} E(\nu)A\Omega a\alpha G_1\beta G_2\left(\frac{2TE_t}{N}\right)^{1/2}. \tag{18}$$

V. Conclusions

An original classification of efficiency factors in spectroscopy and a general formula for the calculation of the SNR based on the concepts of signal coding and spectral coding have been considered. This classification underlines the possibility of a few configurations capable of optimizing the SNR:

(a) both signal coding and spectral coding inherently produce two outputs, and more than one detector can be used for simultaneous measurements [see Figs. 1(a) and 1(b)];

(b) the signal coding can operate between the two complementary outputs of the spectral coding [see Fig. 1(c)];

(c) the spectral coding can be exploited also as signal coding [see Fig. 1(d)]; this possibility suggests fast scanning HS in parallel to fast scanning FS. These instrumental layouts, aiming to increase the performances of a spectrometer, have often been applied to specific instruments[6–10] but, to our knowledge, have never been identified as a general property of a spectroscopic measurement.

The losses of a factor 1/4 in HS and of a factor 1/8 in FS with respect to the full multiplex advantage N are confirmed.

Appendix: Efficiency Factors

All the power losses and/or instrumental errors present in a spectrometer can be described as an attenuation and/or deformation of the FFs, and the resulting effect in SNR can always be calculated through the factor $\beta(\nu)$. In fact an attenuation of the FFs causes an equivalent amplification in the retrieval: this leads to the right intensity level in the spectrum and is detected by the factor $\beta(\nu)$ as a noise amplication. This is a general property, which ensures that any instrumental factor can be accounted for.

In practice those losses which are not directly related to the optical filtering process and do not vary with the FFs can be more simply extracted from the FFs and collected in an instrumental efficiency factor $E(\nu)$ as in Eq. (3). Losses of this type are:

(1) input losses: losses due to the use of a fraction of the input beam (e.g., polarizing instruments, use of beam splitter for more contemporaneous measurements);

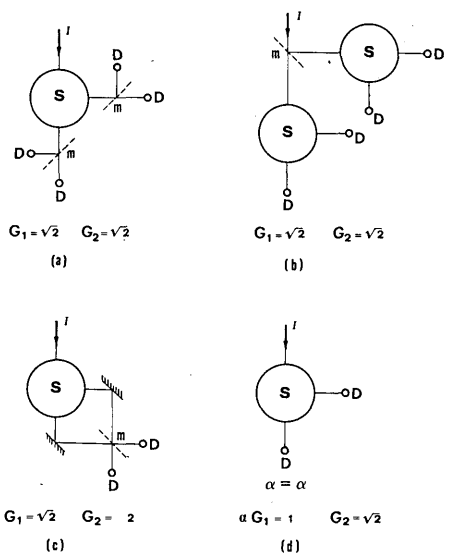

Fig. 1. Instrumental layouts which increase the performances of a spectrometer: I, radiation input; S, spectrometer; m, modulator for signal coding; D, detector; α, signal coding efficiency; G_1, G_2, gain in SNR (see Secs. II and III).

Table III. Efficiency Factor due to Signal Filtering

Sampling method	Type of integration	Notes	DS	FS	HS
Continuous movement and discrete sampling	Real integration	$t_m = \tau$	[a]	$E = \operatorname{sinc}(\pi\nu t_m)\cos(\pi\nu(t_m + 2t_0))$	[a]
	RC integration	—	[a]	$E = \left[\dfrac{\cosh\left(\frac{t_m}{\tau}\right) - \cos(2\pi\nu t_m)}{\sinh(t_m/\tau)}\right]^{1/2} \cdot \dfrac{\cos(2\pi\nu t_0) - 2\pi\nu\tau\sin(2\pi\nu t_0)}{1 + (2\pi\nu\tau)^2}$ [b]	[a]
Step and sample	Real integration	$t_m = \tau + t_s$	$E = 1$	$E = \cos(2\pi\nu t_0)$	$E = 1$
	RC integration	$t_s \ll t_m$	[a]	$E = \dfrac{(1 - l^{-t_m/\tau})(e^{t_m/\tau}\cos(2\pi\nu t_0) - \cos(2\pi\nu(t_0 + t_m)))}{2\left[\sinh\left(\frac{t_m}{\tau}\right)\left(\cosh\left(\frac{t_m}{\tau}\right) - \cos(2\pi\nu t_m)\right)\right]^{1/2}}$ [b]	[a]

Where τ is the integration time, t_m is the sampling period, equal to the total measurement time divided by N, t_s is the stepping time, t_0 is the sampling error.

[a] In these cases the efficiency factor due to signal filtering is equal to 1, and the only effect is the broadening of the instrumental function due to the convolution of the static instrumental function with the integration function (expressed in the optical frequency domain). A correction may be introduced to reduce the instrumental function broadening. In this case E is smaller than 1 (see Ref. 12).

[b] These efficiency factors are obtained extending the calculations reported in Ref. 11.

(2) optical losses: losses due to the optical components (e.g., not ideal reflectivity of mirrors or transparency of windows and filters);

(3) geometrical losses: walk-off losses and shadowing.

Furthermore, also those instrumental effects which in principle cannot be extracted from the FFs, because they vary with the FFs, can be accounted for by a relative efficiency factor calculated by comparison with the ideal case, using the *ad hoc* computing method. Examples of this type of efficiency factors are: grating efficiency; alignment errors; phase errors in FS; signal filtering.

On the other hand, the relative efficiency factors often require a long enumeration of cases for a full description of each possibility. An example of this is given by the signal filtering (frequency dependent attenuation of the signal due to the integration function) as shown in Table III. Therefore, in those cases in which a general approach is desirable, the use of the factor $\beta(\nu)$ instead of the efficiency factors appears more effective.

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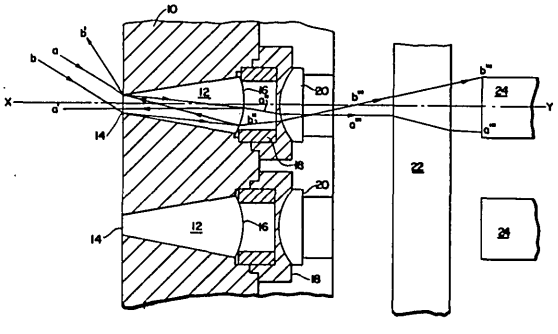
Patents *continued from page 3947*

4,153,368 8 May 1979 (Cl. 356–225)

Electrooptical detector protection device.

G. FALBEL and P. E. SPANGENBERG. Assigned to the U.S.A. as represented by the Secretary of the Navy. Filed 12 Dec. 1977.

An optical device is described that simultaneously accepts a large field of view and provides reasonably uniform attenuation over this large field to protect the detector from intense radiation, e.g., high-energy lasers. The optical element is a truncated solid silicon cone having a semireflecting concave surface opposite the truncation, with a radius equal to its distance. R.L.H.



4,154,503 15 May 1979 (Cl. 350–96.34)

Infrared transmitting glasses.

A. H. LETTINGTON and J. A. SAVAGE. Assigned to The Secretary of State for Defence in Her Britannic Majesty's Government of the United Kingdom of Great Britain and Northern Ireland. Filed 29 June 1977.

This invention describes a new IR transmitting glass with low dispersion for use in the 8–12-μm and 3–5-μm ranges. The material is $G_EA_SS_E$ with T_E added in place of S_E to form low absorption components comprised of bundles of fibers. An example is presented illustrating the use of this material in IR imaging system. A.M.

4,154,590 15 May 1979 (Cl. 65–30 R)

Preparation of photochromic gradient lenses.

A. F. MENYHART. Assigned to American Optical Corp. Filed 7 Nov. 1977. Division of Ser. No. 657,006, 10 Feb. 1976.

This invention describes a method for producing opthalmic lenses exhibiting gradient photochromic behavior in a conventional production furnace, wherein one portion of the lens blank is raised to a temperature exceeding the glass strain point but not the softening point, and other portions are heated to variable temperatures decreasing from the strain point. A locally variable extinction coefficient is provided through well-controlled development of a proper size distribution of silver halide particles in unnucleated glass. A.M.

4,155,627 22 May 1979 (Cl. 350–162 R)

Color diffractive subtractive filter master recording comprising a plurality of superposed two-level relief patterns on the surface of a substrate.

M. T. GALE, H. W. LEHMANN, and R. W. WIDMER. Assigned to RCA Corp. Filed 21 Dec. 1977. Continuation of Ser. No. 705,931, 16 July 1976, abandoned.

A process is described for fabricating three superposed two-level relief patterns on the surface of a substrate for the manufacture of diffractive subtractive color filters in embossed plastic. This is done by linearly adding by evaporation, or by linearly subtracting by sputter or plasma etching, an additional two-level pattern to an existing multilevel pattern. Superposed fine-detail multilevel pattern diffraction gratings having specified line spacing dimensions less than 10 μm, and depth dimensions less than 5 μm can be fabricated in this manner. A.M.

4,155,628 22 May 1979 (Cl. 350–174)

Optical multiplexer/demultiplexer with interferometer elements.

H. R. SCHLOSSBERG. Filed 20 Apr. 1978.

An optical multiplexer/demultiplexer is described that uses an array of Fabry-Perot resonators to select specific frequency intervals. By suitably mixing and separating the outputs of these interferometer elements, it is claimed that a high-efficiency system for multiplexing and demultiplexing may be obtained. R.L.H.

4,155,631 22 May 1979 (Cl. 350–310)

Apparatus for compensating for thermally introduced distortions in reflecting surfaces.

E. C. BORSARE, and E. V. LOCKE. Assigned to W. J. Schafer Associates, Inc. Filed 14 Apr. 1977.

This invention describes a method for compensating for thermally introduced distortions in a laser beam by means of prestressing a reflective surface such that the beam-induced heating of the surface causes a relaxation of the prestress designed to produce a surface distortion substantially equal and opposite to that caused by the heating. The optimized mirror geometry is set forth for a 20-kW CO_2 laser incident upon a mirror structure made of molybdenum. Resulting optical surface distortions for test cases run were $\lambda/100$ rms for a uniform beam and $\lambda/16$ rms for nonuniform beam intensity distribution. A.M.

4,155,734 22 May 1979 (Cl. 65–30 R)

Preparation of photochromic gradient lenses.

D. A. KROHN. Assigned to American Optical Corp. Filed 7 Nov. 1977. Division of Ser. No. 657,006, 10 Feb. 1976.

This invention describes a method for producing opthalmic lenses exhibiting gradient photochromic behavior in a conventional production furnace, wherein one portion of the lens blank is raised to a temperature exceeding the glass strain point but not the softening point, and other portions are heated to variable temperatures decreasing from the strain point. A locally variable extinction coefficient is provided through well-controlled development of a proper size distribution of silver halide particles in unnucleated glass. A.M.

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