Concave Gratings for Astronomical Spectrographs and Spectrometers

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Concave gratings possess a number of characteristics which should make them of interest to earth-bound stellar spectroscopists. Evaluation of a number of specific spectrograph and spectrometer designs indicates that concave grating instruments can be extremely efficient, and therefore are particularly suitable for nebular spectrographs and broad-band spectrophotometry.

Introduction

In recent years much effort has gone into the development of astronomical detectors with high quantum efficiencies. The so-called trialkali photocathodes (S-20 response) now respond to 20% of the incident photons at the wavelength of peak sensitivity ($\sim\lambda 4200$ Å).[1] Hiltner[2] and others have described image intensifiers with these cathodes and with multiplier stages yielding total gains of the order of 10^4. In most instances these modern detectors are used in conjunction with fast optical systems such as solid Schmidt cameras, Baker super-Schmidt cameras, and exotic lens systems. It is therefore rather surprising to find that many visible light astronomical spectrographs and spectrometers, particularly those of high resolution, and even many low resolution instruments, transmit less than a third of the light passing through the entrance slit. A typical stellar spectrograph[3] used on a Cassegrain telescope (two or three reflections) is a rather bulky affair which has an image rotator (one transmission), an inverted Cassegrain collimator (two reflections), a blazed grating (70% efficient), a Schmidt correcting lens (two transmissions), a camera mirror (one reflection), and a field flattening lens (one transmission). Under good conditions, each reflecting surface and each transmitting component will reflect or transmit about 90% of the light. Thus the total efficiency of the spectrograph just described will be $(0.90)^7 \times 0.70 \times 100 = 33\%$. If the surfaces of the optical components drop to 80% efficiency, then the entire spectrograph will pass less than 15% of the incoming radiation. The substitution of a train of prisms for the grating would lower the total transmission even further. These facts are most disheartening to an astronomer facing a one hundred hour exposure stretched over several midwinter nights.

Satellite astronomy has introduced the techniques of ultraviolet spectroscopy to astronomers. One of the more intriguing devices used in this field is the concave grating which immediately reduces (or eliminates entirely) additional reflections or transmissions in a spectrograph. Not only do the smaller number of reflections permit more compact instruments, but also they make possible highly efficient systems with good resolution. The immensely successful use of concave gratings in rockets by Tousey,[4] by Rense,[5] and by others has produced superb solar spectrograms at wavelengths below $\lambda 3000$.

The visible light stellar astronomer and the ultraviolet physicist have at least one basic and rather difficult problem in common: to provide enough photons at the detector. The astronomer has few photons to start with; the ultraviolet physicist must work with inefficient reflecting surfaces and low transmission optics. Both must design efficient analyzing systems.

The concave grating was first introduced by Rowland,[6] and although there have been many subsequent articles on its use, Rowland's fundamental ideas remain basic to nearly all modern applications. Rowland noted that the grooves of a concave grating should be ruled in such a way that they appear parallel and equally spaced when viewed from an infinite distance. The *Rowland circle*, as it is now known, is a circle with a radius exactly half the radius of curvature of the grating, with its center on a line perpendicular to the grating center, and with its plane perpendicular to the rulings of the grating. Rowland showed that if the

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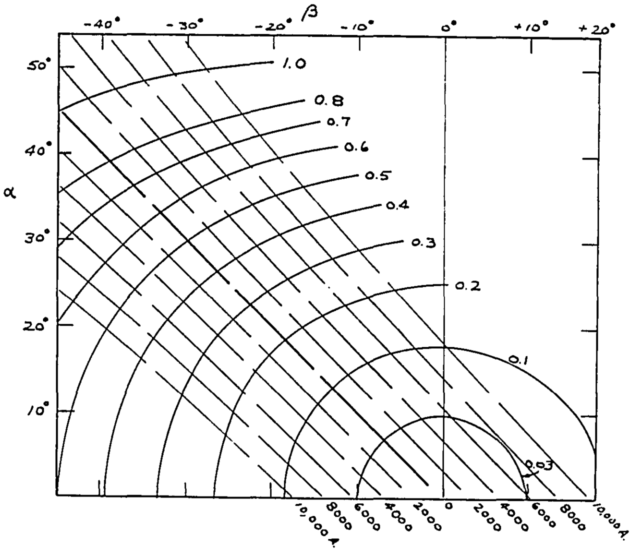

Fig. 1. The amount of astigmatism (solid lines) and the wavelengths (dashed lines) for a 300 grooves/mm grating as a function of the angles of incidence (α) and of diffraction (β). The unit of astigmatism is the length of the grating ruling.

entrance slit were located anywhere on the circle and perpendicular to it, then the final spectrum would also lie on the circle. Furthermore, the equation describing the performance of the plane grating applies, namely

$$\pm n\lambda = d(\sin\alpha + \sin\beta), \tag{1}$$

where n is the order, d the ruling spacing, and α and β are the angles of incidence and diffraction, respectively. If the incident and diffracted rays are on opposite sides of the grating normal, then $\sin\alpha$ and $\sin\beta$ will be of different sign. If the incident and diffracted rays are on opposite sides of the zero order reflection, n is negative. From Eq. (1) there follow the expressions for angular dispersion,

$$d\lambda/d\beta = d\cdot\cos\beta/n, \tag{2}$$

and for linear dispersion,

$$d\lambda/dx = d\cdot\cos\beta/nR. \tag{3}$$

Here R is the radius of the Rowland circle. It is not difficult to show that the distances from the grating center to the entrance slit and to the final focus are very nearly $R\cdot\cos\alpha$ and $R\cdot\cos\beta$, respectively. The derivations of these and other relationships are given in such standard texts on spectroscopy as those by Sawyer,[7] by Harrison, Lord, and Loofbourow,[8] and in an exhaustive paper by Beutler.[9]

The most serious disadvantage of a simple concave grating spectrograph is the astigmatism introduced. Specifically this causes a point of light at the entrance slit to be drawn out at the focus into a line perpendicular to the Rowland circle. As was first shown by Runge and Mannkopf,[10] the length of this line

$$z = (\sin^2\beta + \sin\alpha \tan\alpha \cos\beta)\, l, \tag{4}$$

where l is the length of the grating rulings. Figure 1 shows curves of the quantity in parentheses (solid lines) as functions of α and β. Crossing through this graph are dashed curves indicating values of wavelengths from 0 to 10,000 Å for a 300 lines per millimeter grating. It is at once clear that in visible light spectrographs the length of the lines cannot be less than seven-thousandths the length of the grating rulings. For best performance one would generally choose to work with approximately equal (algebraically) values of α and β.

Several schemes have been devised to reduce or eliminate astigmatism. Green and Loring[11] use a sphero-cylindrical lens in front of the plate, while Smyth[12] introduces a spherical mirror before the entrance slit. A variation of Smyth's method has been successfully used by Rense and Violett[13] in a rocket spectrograph.

For a concave grating *spectrometer*, at the focus of which is an exit slit and a photomultiplier, moderate astigmatism is of little consequence, as long as all the light strikes the photocathode and avoids areas of low sensitivity. Usually it is desirable to image the objective of the telescope on the photosensitive surface of the detector. As will be seen in a later section, this condition can be fulfilled to a good approximation. A short astigmatic image would actually be a blessing to a stellar astronomer who, in using a *spectrograph*, usually desires a somewhat widened spectrum of his stellar point of light. For example, a 50 mm square grating with 300 lines/mm produces a spectrum which is about 350 μ wide* at λ4000. Later we will consider the several means available for reducing this astigmatism.

A second disadvantage of concave grating spectrographs is the curved focal surface. It should be noted, however, that photographic emulsions are available on glass plates having a thickness of 0.010 in. (0.25 mm), and to bend such plates to a radius of curvature of half meter is not difficult. For less exacting use commercially available film can, of course, be used.

Before we move to the next section, it should be noted that today concave replica gratings are readily available in a wide variety at reasonable costs. For an example, Bausch & Lomb lists a half meter radius of curvature grating with a ruled area of 28 × 40 mm and with 600 grooves/mm blazed at λ4267 Å. This grating is tripartite; i.e., the blaze angle of the diffracting grooves was changed twice during the ruling so that the curvature of the grating does not alter the

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* It should be noted that both the words *width* and *height* are used to denote the dimension of a spectrum measured perpendicular to the dispersion. Here we will use the word width, or variations thereof.

wavelength of the blaze markedly over the grating surface. There usually is some loss of resolution, however, in tripartite gratings. As yet, only spherical concave gratings are available. Except for the final section of this article, in which the potentialities of ellipsoidal and toroidal gratings are examined, all discussions will be limited to the use of spherical gratings.

In the following section we discuss different possible concave grating spectrograph designs suitable for astronomical work in the visible spectrum. The third section contains a description of mountings for spectrometers intended for similar use. In the final section we consider the advantages of nonspherical concave gratings.

Spectrographs

Many excellent articles on concave grating spectrographs have appeared elsewhere[14] or will appear soon.[15] The astronomer wishing to investigate more thoroughly the properties of the various mounting arrangements described in the following paragraphs is advised to refer to these.

Except for the Wadsworth mounting, all concave grating spectrographs in general use today are based on the principles first described by Rowland.[6] Individual mountings, which will be discussed briefly below, are distinguished by their different means of adjustment to allow different parts of the spectrum to be photographed or to permit one to work near the grating normal where the dispersion is linear. Since earthbound astronomers frequently work over hardly more than a single octave of spectral frequencies, and usually need not make such adjustments, only brief mention will be made of each mounting. More complete descriptions of their performance may be found in the original articles referred to or in the standard textbooks on spectroscopy cited above.

Interestingly, the earliest mounting used, that of *Rowland*,[16] is relatively complicated compared to the currently more popular mountings. Both grating and plate holder move together along perpendicular axes which lie in the plane of the Rowland circle. These axes have their origin at the entrance slit; therefore, the incoming light always is directed along the grating axis. The main virtue of this mounting is that it keeps the plateholder near the grating normal; its major disadvantage is that it requires precise motion of components. A few years later, Abney[17] described a simpler mounting in which grating and plateholder are fixed at the ends of a diameter of the Rowland circle and whereby the entrance slit swings at the end of a radius of the Rowland circle. However, the changing direction of the incoming beam presents design complications for astronomical use. Probably the most frequently used mounting today and in many respects the simplest is the *Paschen* or the *Paschen-Runge* mounting.[18] Here the entrance slit and grating remain fixed; different spectral regions are recorded simply by moving the position of the photographic plate. In the *radius* or *Beutler*[7] mounting the entrance slit and the plateholder remain fixed while the grating slides along the Rowland circle at the end of a radius. More complicated but more compact is the *Eagle*[19] mounting in which the diffracted ray nearly doubles back along the paths of the incident ray of the grating. A small right angled mirror or prism is occasionally used in this mounting, and the entrance slit is the component placed at the "Newtonian focus." To shift wavelength range requires altering the distance of the grating from the plateholder and rotating the grating simultaneously. Because the angle of diffraction is positive in the Eagle mounting, astigmatism can be reduced to a minimum.

In all of these mountings the concave grating replaces both the collimator and the focusing mirror of the plane grating spectrograph. In doing so, however, astigmatism is introduced which, although it does not spoil resolution, reduces the brightness of the final spectrum. At the cost of a single additional reflection, the *Wadsworth*[20] mounting eliminates or greatly reduces the astigmatism. Here a spherical mirror (or lens) serves as a collimator; astigmatism and coma both become zero at the grating normal. To change spectral region, therefore, both plateholder and grating should be rotated around the center of the grating so that the desired spectrum always appears near the grating normal. A consequence of the use of the collimating mirror is, of course, that the distance from the grating to the final focus is reduced by about a factor of two. Thus, the dispersion of the final spectrum in a Wadsworth mounting is similarly decreased. Focal ratios below about six are not too satisfactory because of increasing amounts of spherical aberration, and according to Beutler,[9] little is gained by going to nonspherical mirrors. However, J. G. Baker has used a mounting with the grating serving as the collimator and with a fast Schmidt camera following it. The optical axes of the grating and the camera coincide in this arrangement.

For visible light astronomical spectrography, it seems that the Paschen or the Wadsworth mounting is most suitable. If the "prewidened" spectra produced by the Paschen design are not objectionable, then high resolution, high dispersion spectra can be simply and inexpensively provided. Operating in the first order, a half meter radius of curvature grating having 600 lines/mm produces a dispersion at the grating normal of 33.3 Å/mm in the Paschen mounting. If the entrance slit is located so that the Hδ line of the Balmer series (λ = 4101 Å) falls at the grating normal, then the width of the spectrum is 0.063 times the length of the grating rulings. An $f/15$ telescope requires a grating approximately 35 mm square which, therefore, would yield at Hδ a spectrum 2.2 mm wide and would have a theoretical resolution of about 20,000 or 0.2 Å. At 33.3 Å/mm, the size of the emulsion grain would limit the resolution at four- or five-tenths of an angstrom. A grating of larger dimensions would be needed if a telescope of smaller focal ratio were used; the resulting spectrum would be wider and there would be no increase in resolution, all else remaining equal. A larger number of lines per millimeter would increase the dispersion and hence the resolution, but since the angle of incidence would also increase for a given wavelength, the width of the spectrum would increase correspondingly (see Fig. 1). A convenient rule of thumb is that the width of the spectrum increases as the square of the number of grating rulings per millimeter. On the other hand it should be noted that the width of the spectrum is independent of the radius of curvature of the grating.

If high resolution with not unreasonably wide spectra is desired, then clearly the best Paschen mounting spectrograph would contain a medium sized grating with a large radius of curvature but with as few lines per millimeter as possible. If practicable, both incident and diffracted rays should be on the same side of the grating normal. For example, an $f/67$ Coudé focus might employ a Paschen spectrograph having a 100 line/mm grating with a radius of curvature of 10 meters. At normal incidence the first order dispersion of such an instrument would be 10 Å/mm, the resolving power 15,000 ($\Delta\lambda = 0.3$ Å at Hδ), and the width of the spectrum 0.25 mm. The diameter of the grating would be 15 cm, a size not difficult to obtain today. Working in the nth order changes all these quantities by a factor of n except the width of the spectrum which increases approximately as n^2. Therefore, the usual choice would be to work in the first order.

Because of the saving of reflections, the best use for a Paschen mounting in earth-bound astronomical spectrography is for faint source, low dispersion work. An interesting instrument for nebular and stellar spectrography might be designed around an 11.5 cm radius of curvature grating having 150 rulings/mm. A normal incidence Paschen mounting at the focus of an $f/5$ reflector would produce a first order spectrum with a dispersion of 580 Å/mm and a width of 0.09 mm at Hδ. The grating would be only 24 mm square. Clearly, the resolution would be limited by photographic grain and typically would be about 10 Å. At the focus of an $f/5$, 60-in. (153-cm) reflector, the diameter of a star image might be 0.05 mm on a night of good "seeing"; a slit just wide enough to admit an entire image of this size would result in a spectrum having a resolution of some 30 Å. Such an instrument would likely be unsurpassed in efficiency; possibly a spectrograph employing a low density prism with convex surfaces and with antireflection coatings might be faster.

The Wadsworth mounting is to be preferred if a stigmatic image is desired; not only will an unwidened spectrum be produced, but also, of course, the variation in spectrum along a slit may be studied. Since astigmatism is eliminated, considerable advantage will result, therefore, by going to large gratings with many lines per millimeter.

In summary, Paschen mountings can be used for astronomical spectrography with outstanding success under proper conditions. There is no more efficient spectrograph if the spectra produced are widened no more than necessary. At the cost of a single additional reflection, stigmatic spectra can be produced, thereby making it possible to use gratings that will give a higher dispersion and resolving power than is possible with the Paschen mounting.

Spectrometers

Many of the advantages and disadvantages discussed in the preceding section apply also to spectrometers. In considering spectrometer designs, however, thought must be given to the ease of achieving the wavelength scan. All of the mountings described above are rather complex as far as changing the wavelength at the exit slit is concerned. Scanning is achieved by moving two components (Rowland, Wadsworth), by both translating and rotating one component (Eagle), or by moving the detector (Paschen, Rowland, Wadsworth). Two mountings require the direction of the incoming beam to be altered (Abney, radius). Further descriptions of these and of other concave grating spectrometers have been given in an excellent article by Namioka.[21]

Because of these disadvantages two relatively recent mountings have been proposed, both of which we will describe in moderate detail. Both only approximate Rowland's criteria for in-focus spectra, and therefore neither can be used for very high-resolution spectroscopy over a wide range of wavelength. The first and more popular design is the *Seya-Namioka*[22, 23] mounting wherein the angle between incident and diffracted ray is held constant at 70°15′. Wavelength scan is achieved by rotating the grating about its vertical axis. Under these conditions, as Seya first suggested and Namioka later investigated, aberrations are reduced to a minimum over a wide range of wavelengths. Best focus results when the distance from each slit to the grating is $0.818\,R$.

The second design is the *Johnson-Onaka*[24, 25] mounting in which the grating rotates about some exterior point whose precise position has been calculated by these investigators. This mounting has not achieved the popularity of the Seya-Namioka mounting largely because it is limited to moderately small wavelength ranges. The primary disadvantage of the Seya-Namioka mounting is the large amount of astigmatism which

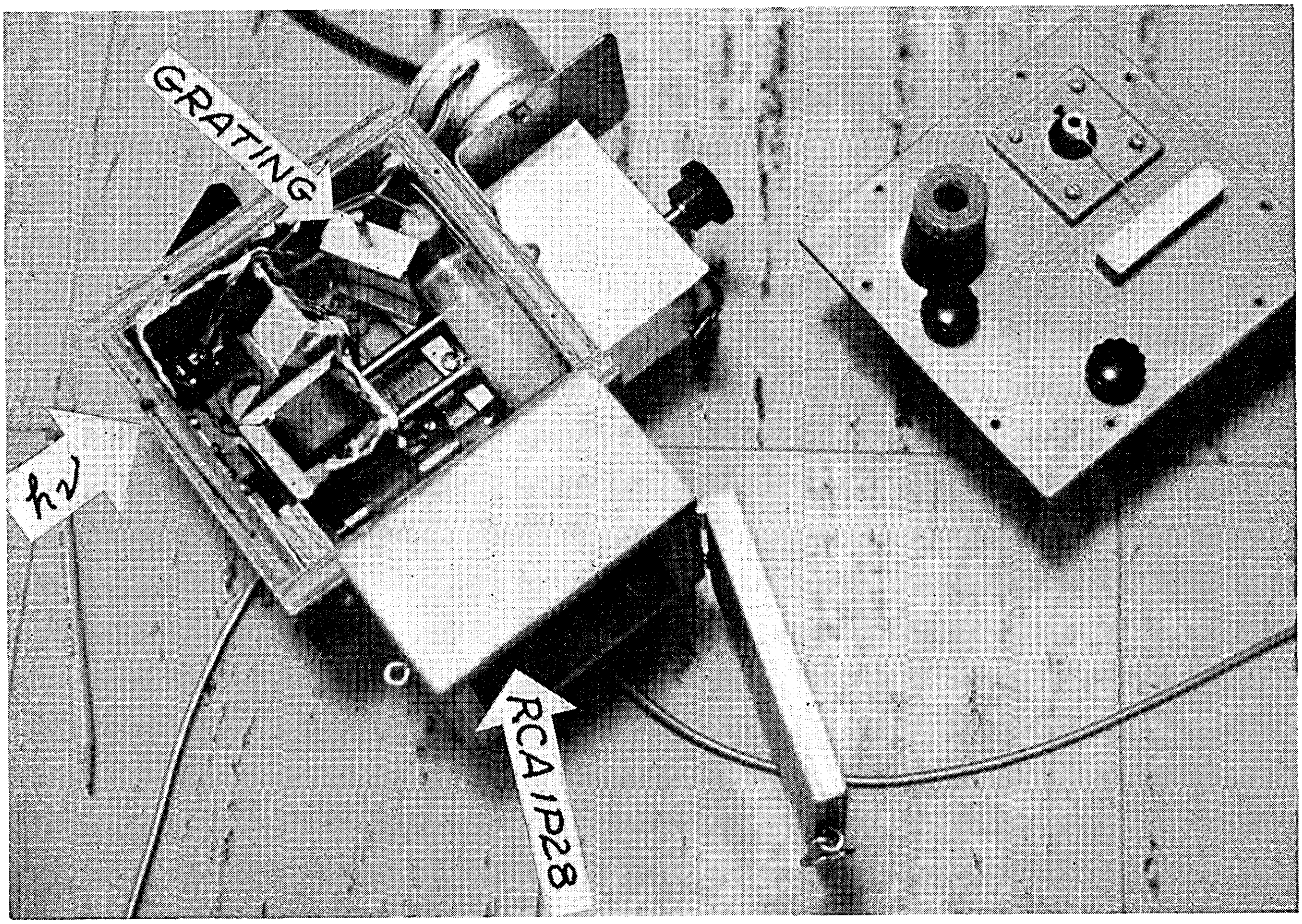

Fig. 2. A Seya-Namioka spectrometer designed and constructed by the author for use at visible and near-ultraviolet wavelengths. The grating with 600 lines/mm has a radius of curvature of 11.5 cm; the detector is an RCA 1P28 photomultiplier. It is normally used at the $f/5$ focus of the 61-in. (155-cm) reflector of Harvard College Observatory.

produces a spectrum whose width for a 600 line/mm grating is approximately two-thirds the length of the grating rulings. Furthermore, according to Namioka,[21] loss of resolution occurs unless the grating width is carefully chosen.

For moderately low-resolution scanners using small gratings, the Seya-Namioka mounting serves admirably. The author has recently put into operation such a spectrometer at the $f/5$ focus of the 61-in. (155-cm) reflector of the Harvard College Observatory.[26] The grating is a Bausch & Lomb replica with a radius of curvature of only 11.5 cm and with 600 rulings/mm. The entire instrument including the RCA 1P28 photomultiplier and its cold box weighs less than ten pounds and fits into a carrying case the size of a bread box. The rather large width of the final spectrum (14 mm) presents no problem since it falls conveniently onto the 25-mm-long cathode of the photomultiplier. No field (or Fabry) lens is used to image the telescope objective on the photocathode in order to prevent guiding errors from producing an erratic output. The confusion disk seen by the photocathode apparently is large enough to smooth out minor driftings of the star in the entrance aperture.

The dispersion of the spectrometer averages about 100 Å/mm over the blue portion of the spectrum. On nights of average seeing a star image might be 0.1 mm in diameter making it possible to resolve 10 Å if the entire image is admitted. Use of a narrow entrance slit improves the resolution to slightly more than one angstrom unit. The blaze angle of the grating (7°.35) puts the blaze wavelength at λ3600 Å, permitting the author and his colleagues to concentrate their studies in the region from λ5000 Å down to the atmospheric transmission cutoff near λ3200 Å. The ultraviolet transmitting envelope of the 1P28 photomultiplier has made it possible to operate at these short wavelengths. A photograph of this spectrometer appears in Fig. 2.

In some spectrometer mountings, particularly those where the angles of incidence and diffraction are not large, it might be possible to employ in some way the secondary or horizontal astigmatic focus produced by the grating. In such mountings this focus will fall on the side of the exit slit away from the grating. Its exact position can be calculated from an expression given by Beutler,[9] namely,

$$\frac{1}{r} - \frac{\cos\alpha}{R} + \frac{1}{r'} - \frac{\cos\beta}{R} = 0, \qquad (5)$$

where r and r' are the distances of the object and horizontal image from the grating center, respectively. The grating might image, for example, the telescope mirror or lens, thereby assigning the role of Fabry lens to the grating. Such a design would define and limit, of course, the usable values of α and β.

Seya-Namioka spectrometers are normally limited to a resolution of slightly less than an angstrom; therefore, there is never any need to use gratings with very large radii of curvature in this particular mounting. A high-resolution scanner can be best made using either a Paschen or a radius mounting. Whether the designer prefers to translate a large grating along the Rowland circle (radius mounting) or to move the exit slit and detector along this same circle (Paschen mounting) will decide between the two possibilities. Very high resolutions are possible, as we have already noted. Now the problems of astigmatism are no longer serious and much more latitude in design is possible.

Very recently the McPherson Instrument Corporation, Acton, Massachusetts, has developed a spectrometer which might best be described as a highly successful modification of the Johnson-Onaka mounting. Scanning is achieved both by rotating and translating the grating. Specifically, an arm is rigidly attached at one end to the grating and made to follow an accurately milled curve at the other. The grating is allowed to move (and rotate) along only one axis resulting in high resolution. In many ways it resembles the Eagle mounting, but its drive mechanism is simpler.

Ellipsoidal and Toroidal Gratings

That a concave ellipsoidal grating might be an excellent means of improving the final spectrum has been considered by Namioka.[21] Purcell and Tousey[27] have, in effect, used a toroidal grating; their method was to distort physically a focusing mirror into a toroidal shape and let the light thus focused fall on a spherical grating. However, at the present time gratings manufactured in these shapes are not generally available. In the hope that they will soon come on the market, a few words will be added illustrating their value.

Namioka[21] has reproduced a graph which shows strikingly the large decrease in astigmatism resulting from the use of an ellipsoidal grating in a Seya-Namioka mounting. Specifically, Namioka defines the semiaxes of the grating ellipsoid as a, b, and c. Semiaxis a is measured in the direction of the grating normal; b is the other semiaxis in the Rowland circle plane; c is perpendicular to the Rowland circle plane. With $c/b = 0.814$, and $R = b^2/a$, astigmatism will be zero at λ3800 Å and will increase slowly on either side of this wavelength until at λ6000 Å the width of the spectrum will be 0.03 times the length of the grating rulings. As mentioned earlier, the width of the same spectrum formed with a spherical grating would be about two-thirds the ruling length. Clearly, such an improvement in gratings would be of great interest to stellar spectroscopists.

The particular value of a is not critical in the above discussion. Therefore, toroidal mirrors can also improve the performance of a concave grating. A moderately thin grating blank mounted in a rigid holder with torsion screws at the top and bottom of the grating (above and below the Rowland circle plane) should produce the desired result.

It is hoped that ellipsoidal (or toroidal) concave gratings will be soon available at moderate prices. The spherical astigmatism is the only major disadvantage of concave grating spectrographs used for faint source analysis, and its elimination will make available to the experimental astronomer a dispersive component with even more potentialities than the spherical concave grating.

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