Theory of group delay ripple generated by chirped fiber gratings

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Abstract: The theory of the group delay ripple generated by apodized chirped fiber gratings is developed using the analogy between noisy gratings and superstructure Bragg gratings. It predicts the fundamental cutoff of the high frequency spatial noise of grating parameters in excellent agreement with the experimental data. We find simple general relationship between the high-frequency ripple in the grating period and the group delay ripple. In particular, we show that the amplitude of a single-frequency group delay ripple component changes with grating period chirp, *C*, as $C^{-3/2}$ and is proportional to the grating index modulation, while its phase shift and period changes as C^{-1}.

OCIS codes: (060.2340) Fiber optics components; (230.1480) Bragg reflectors.

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Group delay ripple (GDR) is the principal impediment restraining the practical implementation of chirped fiber Bragg gratings (CFBG), which are emerging as potential candidates for per-channel tunable dispersion compensation devices [1]. Analysis and calculations of GDR in CFBGs has been addressed in several publications [1-6]. In particular, the authors of [4-6] conducted the general study of random spatial variations of strength and period of gratings. They showed that these variations could have significant effect on group delay characteristics. In [3] the effects of fiber diameter variation for the ultra-long fiber gratings, which have been proposed for broadband dispersion compensation,

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are studied. The previous analysis was essentially based on results of numerical simulations or statistical averaging, and no clear relation between the shape of the GDR and the spatial ripple of the grating parameters was obtained. In this paper we develop a theory, which yields further understanding, predicts new effects, and establishes a simple relationship between the high-frequency spatial noise in grating parameters and the corresponding GDR. The gratings, which are commonly used for single channel dispersion compensation are apodized and linearly chirped having around 0.03-0.1 nm/cm chirp and 0.5 μm period. The theory developed here explains most of the characteristic features of GDR generated by such apodized CFBG, which are observed experimentally and in numerical modeling. We demonstrate that the noise of grating parameters with spatial frequencies exceeding a certain threshold is cut off and does not appear in the GDR spectrum. Consequently, the frequency of the observed GDR cannot exceed a certain threshold magnitude. This result manifests as a fundamental cutoff in the GDR and along with intrinsic interest has relevant practical importance. In particular, it shows that minimizing GDR does not require reduction of the high frequency components of the spatial noise of grating parameters. Rather it emphasizes the importance of suppressing the noise components with the periods having the order few tens of millimeter and up. Also, our theory explains the gradual suppression of the high GDR frequency along the reflection bandwidth. Confirming numerical observations and statistical analysis reported earlier [5,6] we prove that the high frequency GDR is proportional to the amplitude of grating modulation. We show, that a single-frequency component of GDR, as a function of grating period chirp C, is proportional to $C^{-3/2}$ rather than C^{-1} as predicted by the classical ray approximation.

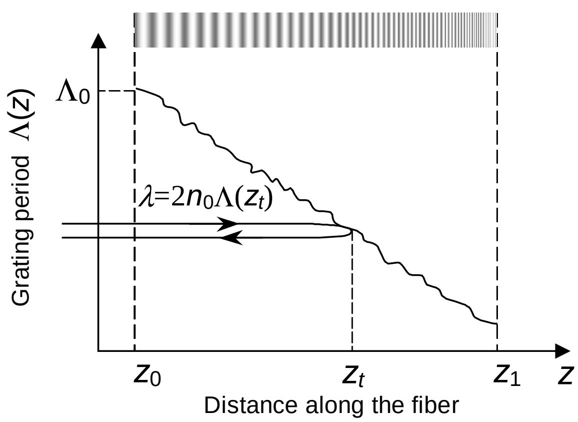

Fig 1. Reflection of light from chirped Bragg fiber grating

Without the loss of generality of our approach, it is assumed that there is no spatial ripple in the grating parameters other than in the period $\Lambda(z)$,

$$\begin{aligned}\Lambda(z) &= \Lambda_0 + C(z - z_0) + \Delta\Lambda(z),\\ \Delta\Lambda(z) &= \sum_{q>0} \Delta\Lambda_q \exp[iq(z - z_0)] + c.c.,\end{aligned} \tag{1}$$

where, for convenience, we use the spectral representation of the grating period noise, $\Delta\Lambda(z)$. The spatial ripple of the other grating parameters, like the amplitude of index modulation and of the DC component of refractive index, can be described similarly.

It is instructive to consider first the description of the GDR based on the classical ray approximation, which, as it is shown below, is inadequate for the gratings of our concern. In this approximation, GDR appears only due to the ripple in the position of turning point z_t, and the grating period noise elsewhere does not matter (see Fig. 1). Simple calculation using Eq. (1) defines GDR in the form demonstrating its inverse proportionality to the chirp C:

$$\Delta\tau(\lambda) = \frac{2n_0}{c_0 C}\Delta\Lambda(z_t),\ \ z_t = z_0 + (\lambda - 2n_0\Lambda_0)/2n_0 C\ , \qquad (2)$$

where n_0 is the effective refractive index and c_0 is the speed of light.

By contrast, for the GDR of gratings considered in this paper the contribution of the turning point ripple is irrelevant. Rather, the GDR is predominantly generated by the interference of the waves reflected from the weak non-homogeneities of grating parameters along the whole grating length traveled by the light. In terms of noise component with frequency q (see Eq. (1)) this reflection is nothing but the secondary Bragg reflection governed by the coupled wave equations [2] with $(2\pi/q)$-periodic coefficients.

A simple explanation of the properties of GDR is given in Fig. 2, which is a band diagram [7,8] of a CFBG with a weak long-period modulation representing a frequency component of the spatial noise $\Delta\Lambda(z)$ defined by Eq. (1). This modulation produces narrow and weak reflection sidebands, or photonic bandgaps, shaded in Fig. 2, spaced by interval $\Delta\lambda_q \propto q$ (superstructure effect [8]). The GDR arises due to the interference of the light reflected from the main band and its sideband. There are two different wavelength regimes:

a.(*above cutoff*): If the light is reflected from the main reflection band only, without intersecting the sidebands then no GDR is observed.

b.(*below cutoff*): If the light also crosses the sideband then the interference of the wave reflected from the main band and the sideband causes GDR.

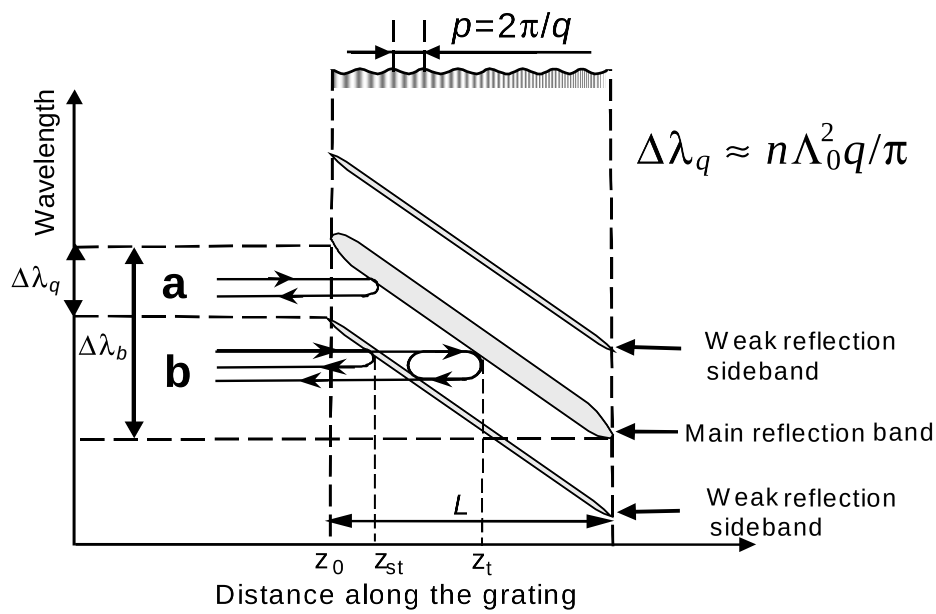

Fig. 2. Physical picture of the GDR cutoff effect

Thus, the formation of sidebands is accompanied by GDR if the ripple period is below the cutoff magnitude. It is also clear from Fig. 2 that if the main band is partly transparent then, even in region *a*, the grating will exhibit GDR because of interference with the wave reflected from the upper sideband. Below we assume that the grating is strongly reflective and the GDR produced by this interference is negligible. Also, it is assumed that the grating is apodized so that the reflections near edge z_0 are small. As it was mentioned, the corresponding noise in amplitude of grating modulation can be considered similarly.

We calculate the GDR using the WKB approximation for the unperturbed solution of the coupled wave equations [7], and the first order perturbation theory for the noise. The developed theory is described in the Appendix where one can find derivation of the following Eqs. (3)-(5). The secondary Bragg reflection point from the sideband, z_{st}, corresponding to the noise component with frequency q (see Fig. 2) is defined by equation:

$$2k_{eff}(z_{st}) = q, \quad k_{eff}(z) = \frac{\pi}{2n_0\Lambda_0^2}\sqrt{\left[\Delta\lambda - 2n_0C(z-z_0)\right]^2 - \Lambda_0^2\Delta n^2(z)}\,. \tag{3}$$

Here $k_{eff}(z)$ is the effective (envelope) wavenumber, $\Delta\lambda$ is the detuning of wavelength of incident light, which is calculated from the high-wavelength edge of the reflection band, and $\Delta n(z)$ is the index modulation. It is seen from Fig. 2 that whenever z_{st} is outside the interval (z_0, z_t) the component with frequency q does not generate GDR. Eq. (3) defines the cutoff spatial frequency, q_c, and the cutoff GDR frequency, ν_c, in the form:

$$q_c = \frac{\pi\Delta\lambda}{n_0\Lambda_0^2}, \; \nu_c = \frac{\pi\Delta\lambda}{2Cn_0^2\Lambda_0^2}\,. \tag{4}$$

Eqs. (3) and (4) determine the frequency range, $q \sim \pi\Delta\lambda_b/(2n_0\Lambda_0^2)$, where the developed theory is valid. Here $\Delta\lambda_b$ is the reflection bandwidth, which can be expressed through the grating length L by the equation $\Delta\lambda_b = 2n_0CL$. For example, for the bandwidth $\Delta\lambda_b = 1\,\text{nm}$ our theory is valid for the spatial noise frequencies of the order 4 mm^{-1} and, respectively, for the periods of the order 1 mm. This high-frequency interval is, in fact, the interval of our interest where the following expression for the GDR is found:

$$\begin{aligned}\Delta\tau(\Delta\lambda) &= \sum_{0<q<q_c}\Delta\tau_q \exp\left[\frac{iq\Delta\lambda}{2n_0C}\right] + c.c.,\\ \Delta\tau_q &= \frac{2i\pi\Delta n}{c_0(2C)^{3/2}}\exp\left(-\frac{iq^2\Lambda_0^2}{4\pi C}\right)\Delta\Lambda_q.\end{aligned} \tag{5}$$

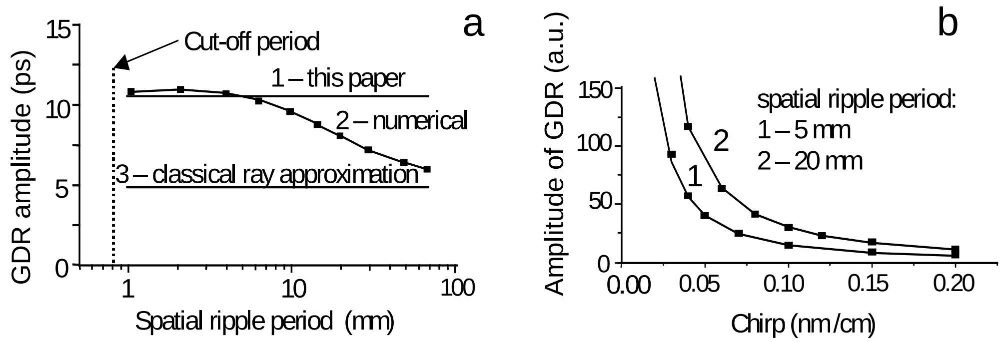

Fig 3. a – comparison of the single-harmonic GDR amplitude calculated from Eq. (4) with numerical calculations and with classical ray approximation Eq. (2) for the amplitude of spatial ripple 0.0025 nm; b – fitting the numerically calculated GDR amplitude vs. chirp dependence (squares) by $C^{-3/2}$ power law (solid lines) for the spatial period ripple 5 mm and 20 mm .

Fig. 3a shows the comparison of the absolute value of GDR harmonic amplitude, $2|\Delta\tau_q|$, given by this equation with numerical calculation for *L*=100 mm, Λ_0= 540 nm, *C*=0.05 nm/cm, and $\Delta n_0 = 2\cdot 10^{-4}$. The agreement is very good in the interval of the spatial ripple period between the cut-off period equal to 0.8 mm and up to 1cm. This figure also shows that the GDR value calculated from Eq. (2) is wrong. This approximation is, obviously, inadequate when the spatial frequency *q* and effective wavenumber k_{eff} are of the same order of magnitude. Interestingly, according to Eq. (5) a single harmonic of GDR depends on chirp as $|\Delta\tau_q| \sim C^{-3/2}$ but not as $|\Delta\tau_q| \sim C^{-1}$, which follows from Eq. (2). Fig. 3b demonstrates that the $C^{-3/2}$ dependence is very accurate for the periods of spatial ripple up to 2 cm. The linear dependence on the index modulation, Δn, follows from Eq. (5) and can be understood from the linear dependence of the GDR on the amplitude of reflection coefficient from the sideband, which is proportional to Δn.

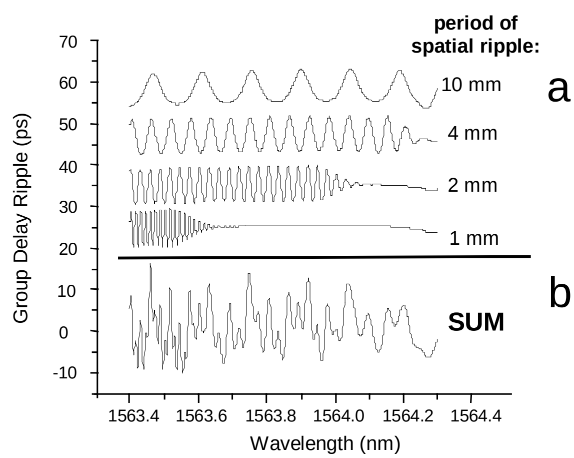

Fig. 4. Ripple in grating parameters and corresponding GDR

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The predicted cutoff effect is confirmed by the numerical solution of the coupled wave equation for CFBG with the parameters indicated above. Fig. 4.a demonstrates the cutoff effect for GDR generated by spatial harmonic ripples having different periods. The sum of these harmonics creates GDR, which has gradually disappearing high frequency components (Fig. 4.b). In this example the noise components with the periods less than 0.8 mm do not affect the GDR at all. Eq. (4) defines the GDR-free bandwidth in the form $\Delta\lambda_q = n\Lambda_0^2 q/\pi$ in agreement with numerical simulations. Fig. 4 explains the gradual suppression of the high frequency GDR, which is usually observed experimentally [1,2] and, in particular, is seen from Fig.5. Fig. 5a demonstrates typical group delay and GDR measurements for one of the gratings written from a phase mask fabricated at OFS Labs. Fig. 5b shows the Fourier spectrums of this GDR calculated for different bandwidths $\Delta\lambda$ measured from the high-wavelength edge of the reflection band. It is seen that the experimental cutoff frequencies are in excellent agreement with the ones predicted by Eq. (4). Note, that the adiabatically small GDR generated by imperfect apodization [2] has the same cut-off frequencies. However, its numerically estimated value has the 0.1 picosecond order and is negligible compared to the effect of spatial noise considered.

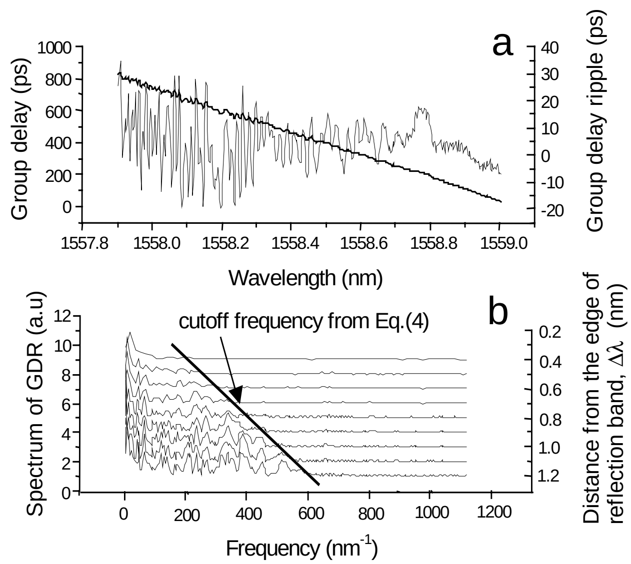

Fig.5. a – Experimentally measured group delay and GDR of a typical CFBG; b – Fourier spectrum of this GDR calculated for different bandwidths $\Delta\lambda$ measured from the high-wavelength edge of the reflection band and demonstrating the cutoff frequencies coincident with the ones predicted by Eq. (4).

Eq. (5) is convenient for investigating the statistical properties of grating noise. For example, assuming that $\Delta\Lambda_q$ are random statistically independent functions and ignoring the relatively small oscillatory components, we get for the standard deviation of the GDR, $\sqrt{\overline{\Delta\tau^2}} \propto \sqrt{\Delta\lambda}\Delta n / C^{3/2}$. This result, which is similar to the one obtained in [6], demonstrates the $C^{-3/2}$ dependence of the local standard deviation of GDR. In general, as follows from Eq.(5), the $C^{-3/2}$ dependence is valid for the single harmonic amplitude only and the arbitrary grating may not possess this property because the phase of each frequency component in Eq. (5) also depends on *C*. In particular, this may explain the controversial, at first glance, results for the chirp dependence obtained in the previous publications (C^{-1} in [5]

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and $C^{-3/2}$ in [6]). However, Eq. (5) shows that the linear dependence of high-frequency GDR on the index modulation is valid for any particular grating. Notice that this result is not trivial and is valid for the high-frequency ripple only. Actually, Eq.(2), describing the low-frequency GDR, does not depend on the index modulation amplitude at all. Interestingly, the $\Delta\lambda^{1/2}$ dependence of the GDR standard deviation is directly related to the cutoff effect. This dependence follows from Eq. (5) after statistical averaging and may not be valid for a specific grating.

In summary, we have investigated the effect of spatial noise of the apodized CFBGs on the GDR and found that the high frequency noise is cutting off. The developed theory is based on the analogy between the noisy gratings and the superstructure Bragg gratings and on the isolation of the high-frequency components in the spectral representation of the GDR. The cutoff effect is demonstrated here for the spatial noise in the grating period. It can be similarly shown that Eq. (4) holds for the spatial noise in any grating parameter. The results obtained are important both for basic understanding and for the further improvement of the grating performance.

APPENDIX

1. Solution of coupled wave equations by perturbation theory

Propagation of light in fiber grating defined by the refractive index variation

$$n(z) = n_0 + \Delta n \cos\left(\frac{2\pi}{\Lambda_0} z + \frac{2\pi}{\Lambda_0^2} \int^{z} dz (\Lambda(z) - \Lambda_0) \right) \tag{A.1}$$

is governed by the coupled wave equation [2,7]:

$$\begin{aligned} u'(z) &= +i[\delta(z)u(z) + \kappa(z)v(z)] \\ v'(z) &= -i[\delta(z)v(z) + \kappa(z)u(z)] \end{aligned} \tag{A.2}$$

where

$$\delta(z) = \beta - \frac{\pi}{\Lambda_0^2}(\Lambda(z) - \Lambda_0), \quad \kappa(z) = \frac{\pi \Delta n(z)}{2 n_0 \Lambda_0}, \tag{A.3}$$

Δn is the amplitude of index modulation,

$$\beta = \frac{2\pi n_0}{\lambda} - \frac{\pi}{\Lambda_0} \approx -\frac{\pi \Delta\lambda}{2 n_0 \Lambda_0^2} \tag{A.4}$$

is the detuning parameter, and grating period variation $\Lambda(z)$ is defined by Eq. (1).
The effect of the weak spatial grating period noise $\Delta\Lambda(z)$ (see Eq. (1)) can be calculated by solving Eq. (A.2) by perturbation theory. To the first order in power of $\Delta\Lambda(z)$, we find GDR in the form:

$$\Delta\tau = \Delta\tau_1 + \Delta\tau_2,\ \Delta\tau_i = \frac{2\pi n_0}{c_0\Lambda_0^2}\frac{d}{d\beta}\mathrm{Re}\left[\int_{z_0}^{\infty} dx\Delta\Lambda(z)G_i(z)\right] \qquad (A.5)$$
$$G_1 = u_0^+(u_0^+)^* + u_0^-(u_0^-)^*,\ G_2 = r_0^{-1}u_0^+(u_0^-)^* + r_0u_0^-(u_0^+)^*$$

where r_0 is the reflection coefficient in the absence of noise and functions $u_0^{\pm}(z)$, together with $v_0^{\pm}(z) = (u_0^{\mp}(z))^*$, are the solutions of the coupled wave equation in the absence of noise which satisfy the boundary conditions $u^+(z) = \exp(i\beta z)$, and $u^-(z) = 0$ for $z<z_0$.

2. WKB solution in the absence of noise

In the absence of noise, assuming that $\delta(z)=\delta_0(z)$ and $\kappa(z)$ are slowly varying functions, we can solve Eqs. (A.2) using the WKB approximation [7]:

$$\begin{pmatrix} u_0^{\pm}(z) \\ v_0^{\pm}(z) \end{pmatrix} = \frac{e^{\pm i\int_{z_0}^{z} k_{eff}(z)dz,}}{2\sqrt{Q(z)}}\begin{pmatrix} Q(z)\pm 1 \\ Q(z)\mp 1 \end{pmatrix}, \qquad (A.6)$$
$$Q(z) = \sqrt{\frac{\delta_0(z)-\kappa(z)}{\delta_0(z)+\kappa(z)}},\quad k_{eff}(z) = \sqrt{\delta_0{}^2(z)-\kappa^2(z)}$$

Eqs. (A.6) have a singularity and fail in close proximity of the turning point z_t, defined by equation $k_{eff}(z_t) = 0$, where $|z - z_t|$ is less than or of order $\Lambda_0(n_0/\Delta nC)^{1/3}$. In this neighborhood, which is of order 5 mm for the grating parameters considered in the main text, Eqs. (A.6) should be substituted by the accurate solution of the couple wave equations. The noise term in Eq.(1) can contain fast varying components with frequencies q which are comparable or greater than the value of the wave-number k_{eff} in Eqs. (A.5). For these components the WKB approach fails and $\Delta\Lambda(z)$ cannot be directly incorporated into Eqs. (A.6) by substitution $\delta(z)$ for $\delta_0(z)$.

3. Group delay ripple

However, independently of the value of q, wherever WKB approximation for $u_0^{\pm}(z)$ is valid, one can substitute Eqs. (A.6) and Eq. (1) into Eqs. (A.5), which yields for the GDR:

$$\Delta\tau = \Delta\tau_1 + \Delta\tau_2$$
$$\Delta\tau_1 = \frac{\pi n_0}{c_0\Lambda_0^2}\sum_{q>0}\Delta\Lambda_q\frac{d}{d\beta}\int_{z_0}^{z_t} dz\frac{\delta_0(z)}{k_{eff}(z)}e^{iq(z-z_0)} + c.c. \qquad (A.7)$$
$$\Delta\tau_2 = \frac{\pi n_0}{2c_0\Lambda_0^2}\sum_{q>0}\Delta\Lambda_q\frac{d}{d\beta}\int_{z_0}^{z_t} dz\frac{\kappa(z)}{k_{eff}(z)}\left(r_0^{-1}e^{iq(z-z_0)+2i\int_{z_0}^{z} k_{eff}(z)dz} + r_0e^{iq(z-z_0)-2i\int_{z_0}^{z} k_{eff}(z)dz}\right) + c.c.$$

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The first item of GDR, $\Delta\tau_1$, does not describe reflection from grating period ripple and can be obtained in WKB approximation, when the noise $\Delta\Lambda(z)$ is directly inserted into Eqs. (A.6). This term dominates for the components of grating period noise with low frequencies $q << k_{eff}$. When the amplitude of grating modulation, Δn, is small we have $k_{eff}(z) \approx \delta_0(z)$ and $\Delta\tau_1$ coincides with the classical ray approximation of Eq. (2). For low frequency components, the second item, $\Delta\tau_2$, is small, because it contains the fast oscillating exponents with frequency $k_{eff} >> q$ under the integral. For higher frequencies, WKB approximation for $\Delta\tau_1$ fails and the total contribution into integral of Eq. (A.4) of the neighborhood of the turning point z_t becomes exponentially small for big q. Exponential dependence on q follows from the general property of Fourier integral of an analytical function $f(z)$ vanishing for $z \to \pm\infty$. For example, if d_p is the distance between the real axis z and the nearest complex pole of $f(z)$ then $\int_{-\infty}^{+\infty} dz e^{iqz} f(z) \sim e^{-qd_p}$ [9]. On the contrary, for high frequencies, when $q \sim k_{eff}$, the term $\Delta\tau_2$, which describes the Bragg reflection from "long-period" noise frequency components, becomes dominant. The main integral in the expression for $\Delta\tau_2$ in Eqs. (A.7) can be calculated by the stationary phase method. While the first term in this integral is strongly oscillating and has negligible contribution, the second one may have a stationary point z_{st} defined by equation

$$2k_{eff}(z_{st}) = q \, . \tag{A.8}$$

This is Eq. (3) of the main text. Neglecting $\kappa(z)$ (or Δn) in the Eq. (A.6) for $k_{eff}(z)$ we find for the turning point and the stationary point, respectively:

$$z_t = \frac{\Lambda_0^2 \beta}{\pi C}, \quad z_{st} = \frac{\Lambda_0^2}{\pi C}\left(\beta - \frac{q}{2}\right). \tag{A.9}$$

It is now easy to show that because the difference $z_t - z_{st}$ cannot exceed the grating length L defined through the bandwidth $\Delta\lambda_b$ as $L = \Delta\lambda_b / C$ the stationary point can only exist if the frequency q is less than the cutoff frequency defined by Eqs. (4). Whenever z_{st} becomes less than z_0 the second integral in Eq. (A.7) vanishes and the resulting GDR experiences the cutoff effect. Otherwise, calculation of $\Delta\tau_2$ by the stationary point method yields the result of Eq. (5).