

Thermo-optic effects affecting the high pump power end pumped solid state lasers: Modeling and analysis

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Abstract

A complete solution of the thermal effect for continuous wave laser diode array fiber coupled high pump power end pumped actively solid state laser is presented. In this paper, the thermal loading resulted from energy transfer up-conversion effect for laser operated under different pulse repetition frequency has been taken into account in the overall thermal effects. The thermal loading transfer the absorbed pump power into non-uniform temperature distribution in the laser rod, and thus results in thermal strain, thermal stress, thermal dispersion of index, and thermally induced diffraction losses. For a practical laser cavity configuration containing the internal optical elements, we use spherical aberration theory to give a better explanation of experimental results via calculating diffraction losses quantitatively. According to analysis of thermal focal length experimental data, the results show that our calculated value of thermal loading under fixed Nd³⁺ doped concentration laser crystal is not a constant but a function of pump power instead. The theoretical predicted curve of thermal focal length based on our model is in good agreement with experimental measurements. At low pump power, the thermal loading is a linear function varying with pump power, but it begins to saturate at pump powers exceeding 6 W. This phenomenon is consistent with our physical intuitions.

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1. Introduction

Nd:GdVO₄ is an isomorph of Nd:YVO₄, since they belongs to the same space group of the laser crystal [1]. Its advantages compared with Nd:YVO₄ are a broader absorption near 808 nm and larger emission cross section, and a twice higher thermal conductivity of 11.1 W/mK along $\langle 110 \rangle$ direction [1] and 12.3 W/mK along $\langle 001 \rangle$ direction [2]. This is very important for quasi-three-level lasers because first of all it leads to a smaller temperature gradient and therefore to a smaller thermal lens [1]. By

using Nd:GdVO₄ as the gain medium and KTP as the frequency doubling crystal, the laser operating for diode pumped actively both at 532 nm of Q-switching regimes [2,3] and continuous wave (CW) regimes [4,5] have made a great progress. When optically pumped, many dynamic physic processes take place in the Nd:GdVO₄ crystal like the one photon absorption and the natural fluorescence. There are other higher order processes that can take place inside Nd:GdVO₄ such as energy transfer up-conversion (ETU). The energy transfer up-conversion is a nonradiative process and is also termed the Auger process [6]. The Auger effect is a mechanism of nonradiative recombination of electron–hole or excited-state pairs. Through Auger recombination of exciton pairs, two nearby excited ions Nd³⁺ in the ⁴F_{3/2} interact with each other, making one ion jump to

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a lower state, either $^4I_{13/2}$ or $^4I_{11/2}$, and the other to be excited to the higher states $^4G_{7/2}$ or $^2G_{9/2}$ respectively using the energy released by the first ion. The process has already been reported for Nd:GdVO₄ [7,8]. A yellow fluorescence was observed by Jesnse et al. [9] and attributed to energy transfer up-conversion from $^4F_{3/2}$ to $^2G_{9/2}$ or $^4G_{7/2}$. Fig. 1 gives a partial energy level diagram for Nd³⁺ in GdVO₄ crystal. This higher order process can significantly affect the laser dynamics.

Liu et al. had demonstrated an end-pumped frequency-doubled acousto-optically Q-switched Nd:GdVO₄ laser from which the green output at power level of several Watts was obtained [10]. The dependence of the initial and residual inversion population density on ETU effect, thermal effect, and Q-switching repetition frequency had not yet been considered in their model. The differences between the theoretical predictions and the experimental results become significantly remarkable at high pump power and output level. The behavior of second harmonic generation output pulse of Q-switched intracavity frequency doubled solid state lasers between lasing and non-lasing can be connected to obtain the exact solution of population inversion density in the energized amplifying medium. Then, an overall laser output performance can be predicted accurately and ETU effect under different pulse repetition frequency is obtained [24]. However, The thermal effect degrades the laser performance especially at high output power. Therefore, thermally induced diffraction losses cannot be neglected in the high pumping power

even for the lower dopant concentrate crystal (for example, Nd is 0.5 at.% in the experiments). There is the little discrepancy between the previous theoretical model [24] and the experimental measurement [10].

In this paper, the authors present some extended modeling to better explain experimental results as previously published by other authors. In particular, thermal effects and energy transfer up-conversion effect are considered. The self-quenching effect, cross relation quenching, and the presence of ion clusters (That exist in Er³⁺-ions doped fiber laser, but it have not seen reported in any experimental measurements for Nd³⁺-ions doped laser crystal.) are ignored in our analysis. We calculate the influence of thermal diffraction losses on the lasers, and account for the losses on laser output power via thermally induced spherical aberration of diffraction theory. Also the model is presented for calculating the thermal focal length of end-pumped actively solid state laser especially in explanation how the thermal loading affecting the measurement of thermal focal length.

2. Thermal model

End pumping of solid-state laser is a standard method to produce single transverse mode laser radiation. For high power laser operation the thermal lens of the laser crystal is an important parameter for optimization of the system. The change in temperature within a laser rod causes a thermal distortion of the laser beam due to a temperature stress dependent variation of the refractive index. In an addition, the generated stresses induce birefringence. An optical beam propagating along the rod axis suffers a quadratic spatial phase variation. This perturbation is equivalent to the effect of a spherical lens. When considering the high power case of end-pumped lasers, fiber-coupled laser-diodes with circular beam profiles are often used as pump sources. To derive the closed-form expressions of the steady-state temperature profiles, we write the heat equation of the temperature distribution by neglecting the anisotropies in the cylindrical coordinate as follows:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} = -\frac{Q(r, z)}{K_c} \quad (1)$$

where $Q(r, z)$ is the heat density distribution in the laser crystal which is proportional to the spatial variation of the pump power. For a fiber-coupled pump beam, $Q(r, z)$ may be approximately described as a top-hat distribution [11,12].

$$Q(r, z) = \frac{\xi P_{in}}{\pi w_p^2} \frac{\alpha e^{-\alpha z}}{(1 - e^{-\alpha \ell})} \Theta(w_p^2 - r^2) \quad (2)$$

where P_{in} is the pump power, ℓ is the length of the active medium, α is the absorption coefficient at the pump wavelength, and $\Theta()$ denotes the Heaviside step function. w_p is the pump-beam waist in the active medium. On the basis of the paraxial approximation, the pump beam waist w_p should be replaced by $w_p(z)$

$$w_p(z) = w_{p0} + \theta_p |z - z_0| \quad (3)$$

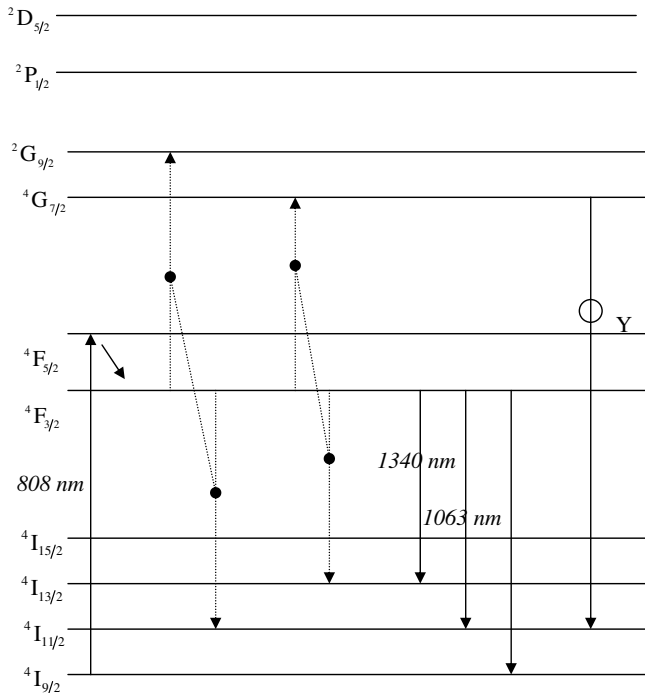


Fig. 1. A partial energy level diagram for Nd³⁺ in GdVO₄ crystal. The solid circles beside with dashed lines represent the ETU pairs. The stimulated emission is represented by the downward arrow line [7,8]. The downward line with open circle indicates yellow fluorescent transition [9].

Here w_{p0} is the radius at the beam waist, θ_p and z_0 are the far field half-angle and focal plane of the pump beam in the laser medium [12]. Here ξ defined in Eq. (2) is the ratio of thermal load power to pump power, namely $P_{\text{load}}/P_{\text{in}}$, which represents how much pump power is converted into heat spreading in the laser rod. The heat generation can be approximately divided into two parts from the point view of quantum process. (1) Heat generation from ETU effect: the ion in excited state transfer its energy to another excited ion, and then drop to ground state (such as ${}^4F_{3/2} \rightarrow {}^4I_{11/2}$ or ${}^4I_{13/2}$) without heat generation. The nearby ion receives energy and transfer to higher excited state (called ETU effect, such as ${}^4F_{3/2} \rightarrow {}^2G_{9/2}$ or ${}^4G_{7/2}$), then drop to excited state (such as ${}^2G_{9/2} \rightarrow {}^4F_{3/2}$) via multi-phonon emission which results in the vibration of crystal lattices and heat generation. The energy of multi-phonon emission is the same as the energy transition from ${}^4F_{3/2} \rightarrow {}^4I_{11/2}$ or ${}^4I_{13/2}$, as indicated in Fig. 1. Therefore, the heat generation due to the ETU effect is the same as the absorbed energy from ground state to excited state and which is proportional to the absorbed pump power, namely, $P_{\text{abs}} \times F_{\text{etu}}$. According to the definition of thermal load, it can be expressed by $\xi_{\text{etu}} = F_{\text{etu}}$. (2) Another portion of heat generation is due to quantum defect. The explanation is that the laser in lasing period results in part of the energy unutilized, and generation heat by these unutilized energy, since the pump wavelength is not the same as laser output wavelength. The mathematical expression is simplified as $\xi_{\text{QD}} = (1 - F_{\text{etu}}) \times (1 - \lambda_p/\lambda_l)$, where $\lambda_p = 808$ nm is the pump wavelength and $\lambda_l = 1064$ nm is the laser output wavelength. F_{etu} is a function of repetition frequency and pump power, so as to the thermal load ξ . The total thermal load is the summation of the above two parts [13]

$$\xi(f, P_{\text{abs}}) = F_{\text{etu}}(f, P_{\text{abs}}) + (1 - F_{\text{etu}}(f, P_{\text{abs}})) \times \left(1 - \frac{\lambda_p}{\lambda_l}\right) \quad (4)$$

The solution of Eqs. (1) and (2) gives the steady state temperature distribution at any point along the radius r of laser rod. For conventional edge cooling, the thermal boundary conditions of the rod-end can be approximated as adiabatic due to the fact that the cooling-surface convection coefficient is orders of magnitude larger than the natural convection coefficient on the end. Detailed derivation of solutions see Appendix A, which is given by [11,12,20,25]

$$\begin{aligned} \Delta T(r, z) \equiv T(r, z) - T(r_b, z) &= \frac{\xi P_{\text{in}}}{4\pi K_c} \frac{\alpha e^{-\alpha z}}{(1 - e^{-\alpha l})} \\ &\times \left\{ \left[\left(1 - \frac{r^2}{w_p^2}\right) + \ln\left(\frac{r_b^2}{w_p^2}\right) \right] \Theta(w_p^2 - r^2) \right. \\ &\left. + \ln\left(\frac{r_b^2}{r^2}\right) \Theta(r^2 - w_p^2) \right\} \end{aligned} \quad (5)$$

where K_c is heat conductivity, and r_b is the radius of the laser rod. Note that the radial temperature profile exhibits parabolic behavior outside the pump region. This indicates that there are quadratic and higher order phase aberrations

in the region for $r > w_p$ region. The steady state equilibrium equation of thermal-induced strains for a non-uniformly heated solid is described as [14]

$$\frac{1-v}{1+v} \nabla(\nabla \cdot u) - \frac{1-2v}{1+v} \nabla \times \nabla \times u = \alpha_T \nabla(T) \quad (6)$$

where v is the Poisson's ratio, α_T is the thermal expansion coefficient. The displacement vector u is giving rise to the elastic tensor $u_{r,\theta}$. If u_i is small and for small deformations, the strains and displacements is given by [14]

$$u_{ik} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} \right) \quad (7)$$

where u_{ik} is the strain tensor. It is not straightforward to solve Eq. (6), since it is difficult to obtain a complete solution. For simplification, we consider an ideal case to obtain the approximate solution by assuming that the strain tensor is equal to the plane tensor. (Also we will see this approximation solution give a fairly good explanation for experiments in most case.) So u_r is a function of r and not related to z . In the same way, u_z is determined by z and not related to r . Therefore, strain tensor matrix element is reduced to be diagonal. Therefore, we use the plane stress approximation to solve the equilibrium equations, and obtain useful scaling relationships (derivations see Appendix B). The error of this approximation has been determined to be typically less than 10% [15]. Based on this approximation, from Eq. (6), the radial displacement u_r introduced by the radial temperature distribution $T(r)$ satisfy the following differential equation:

$$\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} = \frac{1+v}{1-v} \alpha_T \frac{\partial T(r)}{\partial r} \quad (8)$$

The surface-cooling does not totally result in the temperature flowing along radial direction, but also partly contribute to absorb the pump power heat dissipation along the axis direction in the laser rod, namely $T(z)$. We then give the governing differential equation of the axis displacement u_z as follows (derivations see Appendix B):

$$\frac{d^2 u_z}{dz^2} = \frac{1+v}{1-v} \alpha_T \frac{\partial T(z)}{\partial z} \quad (9)$$

By integrating the Eq. (8), we obtain the solution of radial displacement u_r :

$$u_r = \frac{1+v}{1-v} \alpha_T \frac{1}{r} \int_0^r r T dr + c_1 r + \frac{c_2}{r} \quad (10)$$

Using Hooke's law between the relations of the plane stress σ_{rr} , $\sigma_{\theta\theta}$ and the plane strain u_{rr} , $u_{\theta\theta}$, and satisfying the boundary conditions $r = 0$, $u_r = 0$, and $r = r_b$, $\sigma_{rr} = 0$, the radial and azimuthally stress tensor plane strain u_{rr} , $u_{\theta\theta}$ components are given by [16]

$$\sigma_{rr} = \frac{E}{1-v^2} [u_{rr} + v u_{\theta\theta} - (1+v) \alpha_T T] \quad (11)$$

$$\sigma_{\theta\theta} = \frac{E}{1-v^2} [u_{\theta\theta} + v u_{rr} - (1+v) \alpha_T T] \quad (12)$$

Note that the strain in Eqs. (11) and (12) is partly due to thermal expansion of temperature and partly due to the

stress. Here E is Young's modulus, σ_{rr} , u_{rr} are the stress and strain along the radial component, and $\sigma_{\theta\theta}$, $u_{\theta\theta}$ are the stress and strain along azimuthally axis component. The displacement u_r and u_z are giving rise to the elastic strain tensor u_{rr} , $u_{\theta\theta}$ and u_{zz} which can be obtained from Eq. (7) in the cylindrical coordinates (r, θ, z)

$$u_{rr} = \frac{\partial u_r}{\partial r}, \quad u_{\theta\theta} = \frac{u_r}{r}, \quad u_{zz} = \frac{\partial u_z}{\partial z} \quad (13)$$

From the above theory, the expressions for radial, tangential, and axial stress components and strain components in the crystal under the condition of plane strain in the case of free ends are obtained by Timoshenko and Goodier [16] and Koechner [17].

$$\sigma_{rr} = \frac{\alpha_T E}{1-\nu} \left(\frac{1}{r_b^2} \int_0^{r_b} r T dr - \frac{1}{r^2} \int_0^r r T dr \right) \quad (14)$$

$$\sigma_{\theta\theta} = \frac{\alpha_T E}{1-\nu} \left(-T + \frac{1}{r_b^2} \int_0^{r_b} r T dr + \frac{1}{r^2} \int_0^r r T dr \right) \quad (15)$$

$$\sigma_{zz} = \frac{\alpha_T E}{1-\nu} \left(-T + \frac{2}{r_b^2} \int_0^{r_b} r T dr \right) \quad (16)$$

$$u_r = \frac{1+\nu}{1-\nu} \alpha_T \frac{1}{r} \int_0^r r T dr + (1-\nu) \alpha_T \frac{r}{r_b^2} \int_0^{r_b} r T dr \quad (17)$$

$$u_{zz} = \frac{1+\nu}{1-\nu} \alpha_T T \quad (18)$$

Introducing the temperature distribution Eq. (5) into Eqs. (14) and (15), the stress distribution are solved as

$$\begin{aligned} \sigma_{rr} = & \frac{\alpha_T E}{1-\nu} \frac{\xi P_{in}}{4\pi K_c} \frac{\alpha e^{-\alpha z}}{1-e^{-\alpha \ell}} \\ & \times \left\{ \left[-\frac{w_p^2}{4r_b^2} + \frac{r^2}{4w_p^2} - \ln\left(\frac{r_b}{w_p}\right) + K_1 \right] \Theta(w_p^2 - r^2) \right. \\ & \left. + \left[\frac{w_p^2}{4r^2} - \frac{w_p^2}{2r_b^2} - \ln\left(\frac{r_b}{r}\right) - \frac{w_p^2}{2r_b^2} \ln\left(\frac{r_b^2}{w_p^2}\right) + K_2 \right] \Theta(r^2 - w_p^2) \right\} \end{aligned} \quad (19)$$

$$\begin{aligned} \sigma_{\theta\theta} = & \frac{\alpha_T E}{1-\nu} \frac{\xi P_{in}}{4\pi K_c} \frac{\alpha e^{-\alpha z}}{1-e^{-\alpha \ell}} \\ & \times \left\{ \left[-\frac{w_p^2}{4r_b^2} + \frac{3r^2}{4w_p^2} - \ln\left(\frac{r_b}{w_p}\right) + K_1 \right] \Theta(w_p^2 - r^2) \right. \\ & \left. + \left[1 - \frac{w_p^2}{4r^2} - \ln\left(\frac{r_b}{r}\right) - \frac{w_p^2}{2r_b^2} - \frac{w_p^2}{r_b^2} \ln\left(\frac{r_b}{w_p}\right) + K_2 \right] \Theta(r^2 - w_p^2) \right\} \end{aligned} \quad (20)$$

$$\begin{aligned} \sigma_{zz} = & \frac{\alpha_T E}{1-\nu} \frac{\xi P_{in}}{4\pi K_c} \frac{\alpha e^{-\alpha z}}{1-e^{-\alpha \ell}} \\ & \times \left\{ \left[-1 + \frac{w_p^2}{2r_b^2} + \frac{r^2}{w_p^2} + \left(\frac{w_p^2}{r_b^2} - 1 \right) \ln\left(\frac{r_b}{w_p}\right) + K_1 \right] \Theta(w_p^2 - r^2) \right. \\ & \left. + \left[1 - \ln\left(\frac{r_b}{r}\right)^2 + \frac{w_p^2}{r_b^2} \left(-1 - \ln\left(\frac{r_b^2}{w_p^2}\right) \right) + K_2 \right] \Theta(r^2 - w_p^2) \right\} \end{aligned} \quad (21)$$

where K_1 and K_2 are integration constants, which are adjusted to satisfy the boundary condition that can be deter-

mined by numerical calculation. Eq. (19) show that the crystal is under compression (negative stress magnitudes) except the position at $r = r_b$, where $\sigma_{rr} = 0$. Note that for the above expressions, if we use the approximation $\ln\left(\frac{r_b^2}{w_p^2}\right) \approx \frac{r_b^2}{w_p^2} - 1$ and manually add an integration constant $\frac{r_b^2}{4w_p^2}$, then Eqs. (19) and (20) are reduced as

$$\begin{aligned} \sigma_{rr} = & \alpha_T E \frac{\xi P_{in}}{4\pi K_c} \frac{\alpha e^{-\alpha z}}{1-e^{-\alpha \ell}} \\ & \times \left\{ \left[-\frac{w_p^2}{4r_b^2} + \frac{r^2}{4w_p^2} - \ln\left(\frac{r_b}{w_p}\right) \right] \Theta(w_p^2 - r^2) \right. \\ & \left. + \left[\frac{w_p^2}{4r^2} - \frac{w_p^2}{4r_b^2} - \ln\left(\frac{r_b}{r}\right) \right] \Theta(r^2 - w_p^2) \right\} \end{aligned} \quad (22)$$

$$\begin{aligned} \sigma_{\theta\theta} = & \alpha_T E \frac{\xi P_{in}}{4\pi K_c} \frac{\alpha e^{-\alpha z}}{1-e^{-\alpha \ell}} \\ & \times \left\{ \left[-\frac{w_p^2}{4r_b^2} + \frac{3r^2}{4w_p^2} - \ln\left(\frac{r_b}{w_p}\right) \right] \Theta(w_p^2 - r^2) \right. \\ & \left. + \left[1 - \frac{w_p^2}{4r^2} - \ln\left(\frac{r_b}{r}\right) - \frac{w_p^2}{4r_b^2} \right] \Theta(r^2 - w_p^2) \right\} \end{aligned} \quad (23)$$

Eqs. (22) and (23) are consistent with the results derived by Chen [11]. It is of interest to determine the fracture limited pump power for a given absorption coefficient α , and pump geometry. From Eq. (23), we find that the maximum tensile stress is given by

$$\sigma_{\max} = \alpha_T E \frac{\xi P_{in}}{4\pi K_c} \frac{\alpha}{1-e^{-\alpha \ell}} \left[1 - \frac{3}{4} \frac{w_p^2}{r_b^2} - \frac{w_p^2}{r_b^2} \ln\left(\frac{r_b}{w_p}\right) + k_2 \right] \quad (24)$$

Under some approximations, i.e., $\ln\left(\frac{r_b}{w_p}\right) \approx \frac{r_b}{w_p} - 1$, $\alpha \ell \gg 1$ and $r_b \gg w_p$, the maximum tensile stress becomes

$$\sigma_{\max} \cong \frac{\alpha_T E \xi P_{in} \alpha}{4\pi K_c} \quad (25)$$

which is consistent with the results [11]. Suppose $\sigma_{\max}$ is the maximum surface stress at which fracture occurs, the absorbed pump power at the thermal fracture limit is given by

$$MAX.P_{in} = \frac{1}{\alpha} \frac{4\pi R}{\xi} \quad (26)$$

where

$$R = \frac{K_c \sigma_{\max}}{\alpha_T E} \quad (27)$$

R is the “thermal shock parameter” which depends on the mechanical and thermal properties of the host material [18]. A larger R indicates a higher permissible absorbed pump power before thermal fracture occurs. For anisotropic crystal such as Nd:YVO₄ and Nd:GdVO₄ crystals, the analytical expression for the thermal shock parameter is difficult to obtain, however, we can use Eqs. (26) and (27) and laser experiments to determine it.

For a paraxial coherent beam propagating in the z direction, the optical path difference (OPD) is given by [19]

$$\frac{d\varphi(r, z)}{dz} = \frac{\partial n}{\partial T} T(r, z) + (n - 1)u_{zz} + \sum_{i,j=1}^3 \frac{\partial n}{\partial u_{ij}} u_{ij} \quad (28)$$

where the first term results from the thermal dispersion $\frac{\partial n}{\partial T}$, the second term is caused by the axial strain dispersion u_{zz} and the third term represents the strain-induced birefringence with the strain tensor u_{ij} . In fact, the contribution to the OPD from thermal stress-induced birefringence is small [19,20], and considering the fact that the beam passes through the rod twice, the total OPD is accounted from the center of the laser rod along its radial direction and can be expressed as follows:

$$\varphi(r) = 2 \int_0^r \left[\frac{\partial n}{\partial T} + (n - 1) \left(\frac{1 + \nu}{1 - \nu} \right) \alpha_T \right] T(r, z) dz \quad (29)$$

The round-trip loss inside the resonator due to thermal-induced spherical aberration was theoretical calculated by Fox and Li [21]. From the diffraction theory of aberration [22], Schulz and Henion gave a convenient and reasonable estimation for diffraction losses δ_d , and it can be in terms of Strehl intensity ratio R , namely, $\delta_d = 1 - R$ [23]. This wave aberration result from the pump light, and convert into heat dissipation which results in the thermal-induced spherical aberration loss δ_d in the laser rod, and interact with the fundamental E-M (electro-magnetic) coherent wave in the optical resonant cavity. Therefore, it seriously degrades the output performance of laser system. When considering the spherical wave aberration losses of photon in the laser crystal interaction with coherent electromagnetic wave in the resonator, it should be weighted by the Gaussian field amplitude. TEM₀₀ Gaussian field amplitude is proportional to $\exp(-r^2/w_L^2)$, where w_L is the beam waist of the laser cavity mode. Chen et al. gave a simple expression as [12]

$$\delta_d = \left(\frac{2\pi}{\lambda} \right)^2 \Delta\Phi^2 \quad (30)$$

with

$$\Delta\Phi^2 = \left[\frac{2}{w_L^2} \int_0^{r_b} \Delta\varphi^2 \exp\left(-\frac{r^2}{w_L^2}\right) r dr \right] - \left[\frac{2}{w_L^2} \int_0^{r_b} \Delta\varphi \exp\left(-\frac{r^2}{w_L^2}\right) r dr \right]^2 \quad (31)$$

where λ is the laser wavelength and $\Delta\varphi$ is the wave aberration referred to a reference sphere, i.e.

$$\Delta\varphi = \varphi(r) - \varphi(0) - \kappa r^2 \quad (32)$$

Here κ is a constant adjusted to give the best quadratic approximation resulting in the smallest value of the diffraction losses. It can be determined by numerical calculation. Eqs. (30)–(32) states that when the aberrations are small, the diffraction losses can be evaluated from the mean-square deformation of the wave aberration.

3. Analysis and comparison with experimental results

We give quantitative calculations of laser properties considering the experimental setup conducted by Liu et al. [10], and take it as an example to examine our extended thermal model. Liu et al. adjusted the Q-switching repetition frequency to 25 kHz, and the maximum SHG average power was found to be 2.18 W with incident pumping power of 10 W. In such circumstance, the Nd:GdVO₄ crystal experienced a high temperature gradient internally but its peripheral surface maintained at 15 °C since the cooling system take away the dissipation heat spreading in the laser rod (as the paper described, 0.5 at.% Nd:GdVO₄ crystal was wrapped with Indifum foil and copper which was cooled by using TEC to prevent from thermal fracture). The Nd:GdVO₄ crystal is 0.5 at.%, a-cut, with dimension of 3 mm × 3 mm × 4 mm [10]. The material parameters for 0.5 at.% Nd:GdVO₄ crystal required are: $n = 2.19$, $\alpha = 6.9 \text{ cm}^{-1}$ [2], $K_c = 11.7 \times 10^{-2} \text{ W/K cm}$, $\xi = 0.278$ (Ref. Eq. (4) and using $F_{\text{etu}} = 0.05$ from Fig. 9 of Ref. [24]), and pumping parameters $\omega_p = 0.26 \text{ mm}$. As the paper described, in order to prevent from the thermal fracture and to reduce the thermal loading with enables effective thermal management of the crystal, Liu et al. chose the lightly doped crystal i.e., 0.5 at.% Nd:GdVO₄ in the experiments. Actually, Nd-doped concentration can greatly affect the performance of a laser and this relation was confirmed by Liu et al.'s experiments. Substituting these parameters into Eq. (5), we calculate the radial temperature distribution which is shown in Fig. 2. The result shows that the temperature in the center of the crystal reaches 75 °C. Therefore, there is a temperature gradient of 65 °C across the longitudinal cut section area, which generates stresses between the center and its peripheral surface. But according to our analysis of calculated data, the stress distribution between the center and the peripheral surface resulted from such temperature gradient is still under the maximum limit of thermal fracture for the 0.5 at.% Nd:GdVO₄ crystal. Proof as follows: plugging the thermal parameters $\partial n/\partial T = 4.7 \times 10^{-6}/\text{K}$, $\alpha_T = 4.43 \times 10^{-6}/\text{K}$, $\nu = 0.28$ into Eqs. (19)–(21) in order to estimate the maximum value of the stress caused by this temperature distribution, the theoretical thermal stress as a function of radius is presented in Fig. 3. From the figure, taking the square summation of the orthogonal stresses components of $\sigma_{\theta\theta}$ and σ_{zz} , we find that the peripheral surface of the 0.5 at.% Nd:GdVO₄ crystal suffers a stress about 100 kg/cm² (there is 14.696 psi in 1.033 kg/cm²). According to the general analysis of numerical thermal shock parameter for different laser materials by Koechner [18], it follows that the laser crystal is far away from the maximum limit of thermal fracture (The photograph of laser crystal (not Nd:GdVO₄ crystal, but Nd:YVO₄ crystal) with thermal fracture has been verified from laser experimental Lab. [11]). On the other hand, our estimation value of thermal loading $\xi = 0.278$ could increase until the stress fracture occurred. This indicates a higher permissible thermal

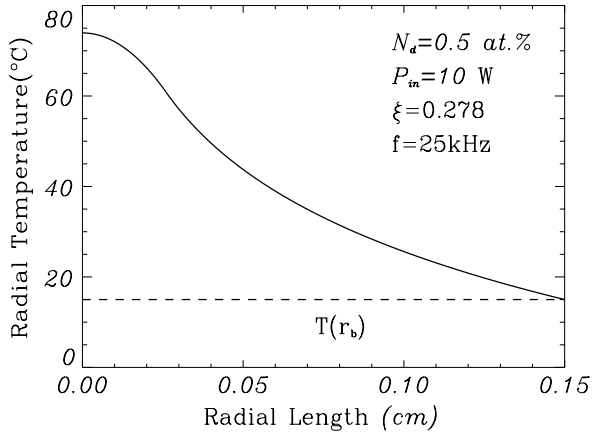


Fig. 2. Calculated temperature profile at $z = 0$ (the laser crystal surface). The profiles are computed for a fiber-coupled laser-diode array pumped Nd:GdVO₄ at 808 nm for 10 W of pump power. The thermal load is assumed about 27.8%, which converted pump power into heat.

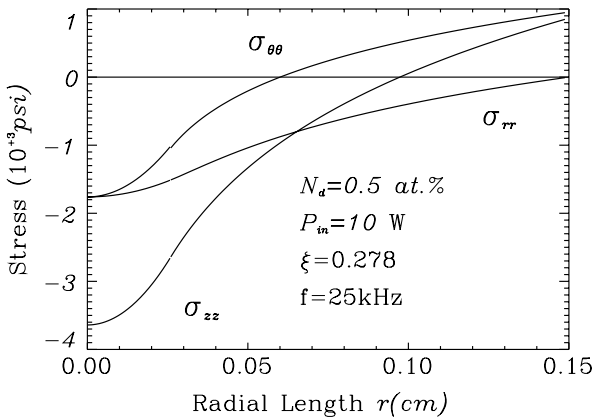


Fig. 3. Radial, tangential and axial stress components distribution varies with the radius of the laser rod.

loading before thermal fracture occurs. The thermal loading of $\xi = 0.278$ actually is within the normal range. In real situation, the laser crystal still worked well in the experiments even though suffering from such a high pump power of $P_{in} = 10$ W. According to Liu et al.'s paper, the thermal management was well controlled and the laser system maintained in thermal equilibrium steady state. Therefore, from our starting point Eq. (1), the initial heat transfer equation is valid in this situation. Turning to Fig. 3, we note that the shape of stress curve is similar to the earlier work by Cousins [15], but it is somewhat different compared with Koechner's [17], since our analysis especially focuses on the LD-array fiber-coupled end-pump configuration system. The only question left is what problem in the experiment by Liu et al. was encountered that make it seem to be very hard to pump more optical radiation power into 0.5 at.% Nd:GdVO₄ laser crystal?

Our simulation may provide part of the answers to this question. The explanation that we deduce may be summarized as follows. Such large stresses and large temperature distribution result in a non-negligible optical path

difference (OPD) in the Nd:GdVO₄ crystal. Applying the brightness theorem [22] and the optimization pump beam quality analysis [12], we obtain the following parameters used in the Liu's experiments: $w_{p0} = 0.222$ mm, $\theta_p = 0.0976$ rad., and $z_0 = 0.214$ mm. Substituting the parameters into Eqs. (3), (4), (25) and (28), a theoretical calculated phase variation is plotted in Fig. 4. The resulted wave aberration in Eq. (32) and thermally-induced spherical aberration losses δ_d in the laser crystal described in Eqs. (30) and (31) are calculated. The thermal loss δ_d in Liu's experiment is estimated for several different pump powers and is shown in Fig. 5. From the figure, we see that the saturation behavior of the thermally induced diffraction losses occur in high pump power region, and the diffraction losses increase quadratic in low pump power region since thermal-induced OPD is proportional to pump power. When pump power P_{in} is 10 W, it results in the thermally induced diffraction losses up to 0.08, which is a really high value of losses. From the laser theory, when the diffraction losses (optical resonator including laser crystal, Q-switching, KTP crystal etc.) is greater than the gain, it is obvious that the overall laser design system is under threshold condition, and the laser is non-lasing. However, in this situation that diffraction loss is as high as 0.08, Liu measured the maximum SHG output power 2.18 W with Q-switching repetition frequency at 25 kHz, while our previous calculation results [24] predicts a value of 2.24 W. The little discrepancy between these two results may be attributed to this high diffraction loss. In fact, the thermal loading ξ is proportional to pump power especially when considering high pump power, then thermal effect results in an extremely high thermally induced diffraction losses value. This means that the quality factor Q -value in laser optical resonator is very low. We also note that the theoretical calculation predict that when $P_{in} = 25$ W and Q-switching repetition frequency f is 52 kHz, the maximum SHG $P_{av} = 7.7$ W is obtained [24], which is an ideal case without considering the thermal effect. Actually, in this situation that $P_{in} = 25$ W, it is difficult for the researchers to maintain the laser in lasing.

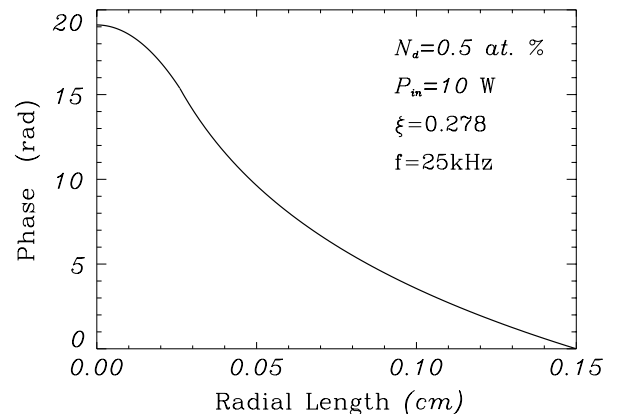


Fig. 4. A plot of the phase variation as a function of the radius in the laser rod.

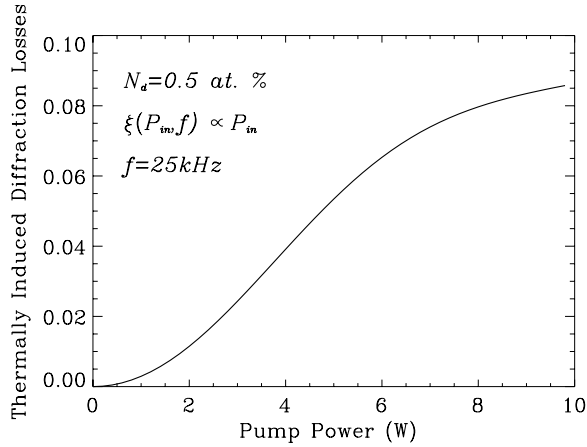


Fig. 5. A plot of the dependence of the thermally induced diffraction losses on the pump power.

Furthermore, it is much more difficult to obtain Q-switched SHG pulse profile from theoretical calculation. Therefore, we can see that Liu's experiment demonstrated a highly well-controlled technique and many difficulties were overcome to take measurement when applying pump power more than 10 W, i.e., $P_{in} = 25$ W.

4. Thermal loading affecting the measurement of thermal focal length

Weber and coworkers conducted the thermal focal length experiment [26] and used Innocenzi's formula [27] to fit the experimental data. However, the result showed that all focal lengths measured are about 30% smaller than the theoretical predicted values. As Koechner mentioned, the thermal focal length formula derived from Innocenzi's is just an approximation for low and medium pump power [28]. A formula similar to Innocenzi's formula was proposed by Tidwell et al. [20]

$$f = \frac{7.6 \cdot r_0^2}{P_{in}} \quad (33)$$

where r_0 is the rod radius. Most of the formulas proposed by previous researchers show that the thermal focal length is inverse proportional to high pump power, but this rule is violated at lower pump power. From previous experiments for thermal focal length measurement by Osterink and Foster [29], Koechner [30], Cousins [15], Pfister et al. [19], Tidwell et al. [20], Innocenzi et al. [27], and Ueda and coworkers [2], we know that the theoretical prediction is not consistent with experimental data. In this paper, we proposed a modified model which can account for the experimental data of the thermal focal length. Our approach does differ from others in that the thermal loading ξ is not a constant but a function of pump power instead, and is proportional to pumping power at lower pump level, namely, $\xi(P_{in})$. Reviewing the published paper [31], Fan defined the fractional thermal loading as the ratio of total heat generated to absorbed energy. He obtained an

estimated value derived from thermal measurements without lasing. The figure in Fan's published paper showed that the fractional thermal loading is a function of concentration for five Nd:YAG samples. The Nd-doped concentration varies from 0.5 at.% to 1.1 at.% and their corresponding thermal loading is suggested to be about 0.248–0.33. Besides, a set of data points showed that 1.04 at.% doping crystal with its fractional thermal loading 0.37–0.43 was measured. Therefore, we deduce that the thermal loading is up to 0.5 from the numerical linear interpolation method, if Nd-doped concentration is 1.3 at.%.

We express the theoretical thermal focal length as a simple function of $\xi(P_{in})$ and P_{in} . Then, Eq. (33) should be modified as follows (detailed derivation see Appendix C):

$$f = \frac{2\pi K_c \bar{w}_p^2}{\xi(P_{in}) \cdot P_{in} \cdot dn/dT} \quad (34)$$

In Eq. (34), the calculated value of thermal loading ξ has included many effects [31,32]. However, the properties of thermal loading is actually attributed to the properties of laser crystal with some complex intrinsic phenomena, see for example Ref. [33,34]. As Koechner mentioned, the effect thermal focal length can be derived from the resonator theory. Thus from the ABCD matrix theory [35–37], one can obtain the critical thermal focal lens from the corresponding maximum threshold pump power [30]. In the experiment conducted by Ueda, at the maximum incident pump power of 8.4 W, the corresponding thermal focal length is about 20.2 cm. To determine the full curve of theoretical thermal focal length varying with pump power, it is needed to execute the measurement of thermal focal length. We use the experiment data from the paper entitled “Efficient and compact intracavity-frequency-doubled Nd:GdVO₄/KTP laser end-pumped by a fiber-coupled laser diode” [2] as an example to verify present model. Ueda used a 1.3 at.% neodymium-doped Nd:GdVO₄ crystal was a-axis cut with a cross sectional area of 3×3 mm² and a thickness of 3 mm. Thus, the material parameters for Nd:GdVO₄ required are $v = 0.28$, $\ell = 3$ mm, $K_c = 0.117$ W/cm K, $dn/dT = 4.7 \times 10^{-6}$ K⁻¹, and $\alpha = 10.2$ cm⁻¹. Ueda et al. use the scanning slit technique [9] to measure the actual beam radii and according to the measured divergence data to estimate the thermal focal length [2]. An appropriate choice for average radius of the pump beam in the laser rod taken from experiments $\bar{w}_p$ is 0.23 mm [38]. Substituting above parameters into Eqs. (C3), (C9) and (C10), one can estimate theoretical value of thermal focal length. The calculated value of thermal loading and Ueda's experimental data of thermal focal length are in the same plot shown in Fig. 6. The result shows that the calculated value of thermal loading is not a constant, but a function of pump power instead. At low pump power, the thermal loading is a linear function varies with pump power, but it saturates with an increasing breakthrough high pump power, especially when pump power is more

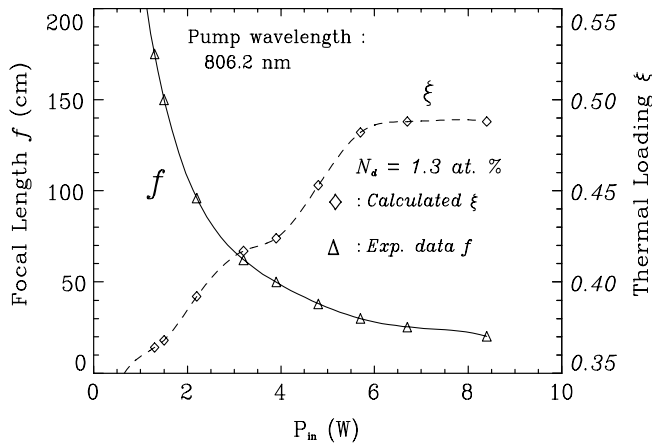


Fig. 6. The theoretical calculated value of thermal loading with its fitting curve according to Eq. (34) are plotted (calculated thermal loading data denote by a diamond symbol with its fitting curve denote by broken line) together with fitting curve f and experimental measured thermal focal length data (denote by triangle symbol).

than 6 W. It is well noted that in the Ueda et al.'s experiments, they used the empirical value of 0.5 for ξ as a parameter to estimate the intracavity frequency doubled CW green output laser pulse. In this circumstance, our calculated value of saturation thermal loading approaches to 0.49.

Reviewing the previous published paper by Chen et al., Figs. 3 and 4 of Ref. [39] shows their theoretical calculations and experimental measurement for the ratio of the thermal loading under lasing and non-lasing conditions, also under different Auger up-conversion rates with different pump powers. Chen et al. believed that their present model can be applied to other Nd-doped laser materials (for our present case is Nd:GdVO₄ used in Ueda's experiments). Based on these two figures, we have the concern that the thermal loading is not a constant, but varies with different pump power. This suggestion is indirectly supported by this experiment. We also note that the approaching curve of thermal loading is somewhat linear. In our approach, the calculated thermal focal length obtained from calculated thermal loading is consistent with Ueda's experimental precision and it is really for practical diode array fiber-coupled end-pumped laser system design needs. On the other hand, the heat dissipating in the laser rod accumulates as pump power increases. The resulted heat should be taken away by TEC cooler (or other water cooling system), and finally the thermal loading goes to maximum value and approaches to saturation. However, the laser cooler system always maintains steady state heat flowing, since the temperature within laser rod and cooling system are maintained in thermal equilibrium. There is no doubt that when one estimates the thermal heat conductivity of the crystal, he would find that the theoretical starting point Eq. (C1) (or Eq. (1)) is valid and can always be applied to similar laser design no matter how the thermal loading is changed.

5. Conclusion

Our theoretical analysis shows that the overall laser performance is affected by pulse repetition rate, energy transfer up-conversion effect and thermal-optic effect. When laser is operating at high pump power, the thermal effect plays the most important role for deteriorating the laser performance and thermally induced diffraction losses have to be considered to predict the laser operation characteristics accurately, especially in determining the SHG pulse profile with its pulse width. A detailed thermal model analysis of fiber-coupled LD array end-pumped configuration is presented. Applying the experimental measurement by Liu et al., we verify the presented model. Furthermore, we proposed a theoretical model for calculating thermally induced focal length of end-pumped actively solid state laser. The curve of theoretical prediction for thermal focal length can be matched with experimental measured data via using appropriate calculated value of thermal loading. The results of Ueda et al.'s experiment are in close agreement with our calculated ones.

Thus we conclude that the standard procedure of calculating the theoretical thermally induced diffraction losses and determining the laser pulse of output profile is

- (A) First, use the experimental measured data to fit the thermal focal length curve, and then determine the thermal loading.
- (B) Plot the thermal loading as a function of pump power. Based on this appropriate thermal loading value taken from experimental measurements, one can calculate the steady state thermal temperature distribution within the laser rod (see Section 2).
- (C) Via these accurate temperature distributions in laser rod, the thermally induced diffraction losses are evaluated (see Section 2 and Refs. [12,21–23]).
- (D) Incorporating calculated diffraction losses factor into the spatial-time dependence rate equations, and solving the equation, one can obtain the theoretical laser output pulse profile and pulse duration [3,40].

Appendix A

For completeness, we list below is the derivations of Eq. (5). Using the method of separating variable solve non-homogeneous partial differential equations, it is a general principle to use the cylindrical coordinates (r, φ, z) with respect to which the boundary considerations has a simple representation. Since the pump system is posed azimuthally symmetric, so it is not depend on φ , the steady state heat transfer equation reduces to the form

$$\frac{1}{R} \frac{1}{r} \frac{\partial R}{\partial r} + \frac{1}{R} \frac{\partial^2 R}{\partial r^2} + \frac{1}{Z} \frac{\partial^2 Z}{\partial Z^2} = -\frac{Q}{K_c R Z} \quad (\text{A1})$$

where Q is the same as expression of Eq. (2), and here we have used the method of separation variables $T(r, z) = R(r)Z(z)$ in Eq. (1). We set

$$\frac{1}{Z} \frac{d^2 Z}{dz^2} = K^2 \quad (\text{A2})$$

and choose

$$Z(z) = \frac{e^{-\alpha z}}{w_p^2(z)} \quad (\text{A3})$$

This yields two ordinary linear differential equations

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial R}{\partial r} \right) + K^2 R = 0 \quad \text{where } r > w_p \quad (\text{A4})$$

and

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial R}{\partial r} \right) + K^2 R = -\frac{P_h}{\pi K_c} \frac{\alpha}{(1 - e^{-\alpha \ell})} \quad \text{where } r < w_p \quad (\text{A5})$$

where $P_h = \xi \cdot P_{in}$. We first consider Eq. (A4), which is just the zero order Bessel equation, and its solution is

$$R(r) = C_1 J_0(Kr) + C_2 Y_0(Kr) \quad (\text{A6})$$

With the asymptotic expansion of Bessel functions as follows:

$$\begin{aligned} r \ll 1 J_n(r) &\rightarrow \frac{1}{\Gamma(n+1)} \left(\frac{r}{2} \right)^n \\ &\rightarrow J_0(0) = 1 \\ r \ll 1 Y_n(r) &\rightarrow \begin{cases} \frac{2}{\pi} \left[\ln \left(\frac{r}{2} \right) + 0.5772 \dots \right], & n = 0 \\ -\frac{\Gamma(n)}{\pi} \left(\frac{r}{2} \right)^n, & n \neq 0 \end{cases} \end{aligned} \quad (\text{A7})$$

Solutions for the proportion of $r > w_p$ is:

$$R(r) = C'_2 \ln \frac{Kr}{2}, \quad (r > w_p) \quad (\text{A8})$$

Now to solve the Eq. (A5), we must first solve its homogeneous equation. That is just the same as solving for Eq. (A4), only the solutions of variable r in Eq. (A8) is replaced by w_p , we obtain homogeneous solution is

$$R_h = C'_2 \ln \left(\frac{Kr}{2} \right) \Big|_{r=w_p} = C'_2 \ln \left(\frac{Kw_p}{2} \right) \quad (\text{A9})$$

The particular solution is satisfying the Eq. (A10) when the case K is zero in Eq. (A5) (this means that the temperature distribution T is approximately independent of $Z(z)$):

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial R}{\partial r} \right) = \frac{-P_h}{\pi K_c} \frac{\alpha}{(1 - e^{-\alpha \ell})} \quad (\text{A10})$$

This yields the particular solution is

$$R_p = \frac{-P_h \alpha r^2}{4\pi K_c (1 - e^{-\alpha \ell})} \quad (\text{A11})$$

and the general solutions for the proportion of $r \leq w_p$ is the summation of $R_h + R_p$:

$$R_h + R_p = C'_2 \ln \frac{Kw_p}{2} - \frac{P_h \alpha r^2}{4\pi K_c (1 - e^{-\alpha \ell})}; \quad r \leq w_p \quad (\text{A12})$$

we summarize them as follows:

$$T(r, z) = R(r)Z(z)$$

$$\Rightarrow T(r, z) = \begin{cases} C'_2 \frac{e^{-\alpha z}}{w_p^2(z)} \ln \frac{Kw_p}{2} - \frac{P_h \alpha r^2}{4\pi K_c (1 - e^{-\alpha \ell})} \frac{e^{-\alpha z}}{w_p^2(z)}, & r \leq w_p \\ B'_2 \ln \left(\frac{Kr}{2} \right) \times \frac{e^{-\alpha z}}{w_p^2(z)}, & r > w_p \end{cases} \quad (\text{A13})$$

where C'_2 and B'_2 are any constants to be determined. From the definition $\Delta T(r, z) \equiv T(r, z) - T(r_b, z)$, the portion of $r > w_p$ for $\Delta T(r, z)$ is

$$\Delta T(r, z) = B'_2 \frac{e^{-\alpha z}}{w_p^2(z)} \ln \left(\frac{r}{r_b} \right) \quad (\text{A14})$$

Similarly, we can write the portion of $r \leq w_p$ for $\Delta T(r, z)$ is

$$\Delta T(r, z) = C'_2 \frac{e^{-\alpha z}}{w_p^2(z)} \ln \frac{Kw_p}{2} - \frac{P_h \alpha r^2}{4\pi K_c (1 - e^{-\alpha \ell})} \frac{e^{-\alpha z}}{w_p^2(z)} - T(r_b, z) \quad (\text{A15})$$

With substituting the approximate boundary condition at $r = w_p$ and $r = r_b$, the term $T(r_b, z)$ in the Eq. (A15) should be replaced by the following:

$$T(r_b, z) = \left[C'_2 \ln \frac{Kr_b}{2} \frac{e^{-\alpha z}}{w_p^2(z)} - \frac{P_h \alpha w_p^2}{4\pi K_c (1 - e^{-\alpha \ell})} \frac{e^{-\alpha z}}{w_p^2(z)} \right] \quad (\text{A16})$$

Plugging Eq. (A16) into Eq. (A15), yields

$$\begin{aligned} \Delta T(r, z) &= C'_2 \frac{e^{-\alpha z}}{w_p^2(z)} \ln \left(\frac{w_p}{r_b} \right) \\ &\quad + \frac{P_h \alpha e^{-\alpha z}}{4\pi K_c (1 - e^{-\alpha \ell}) w_p^2(z)} (w_p^2 - r^2) \end{aligned} \quad (\text{A17})$$

From this, we obtain

$$\Delta T(r, z) = \begin{cases} C'_2 \frac{e^{-\alpha z}}{w_p^2(z)} \ln \left(\frac{w_p}{r_b} \right) + \frac{P_h \alpha e^{-\alpha z}}{4\pi K_c (1 - e^{-\alpha \ell})} (w_p^2 - r^2), & r \leq w_p \\ B'_2 \frac{e^{-\alpha z}}{w_p^2(z)} \ln \left(\frac{r}{r_b} \right), & r > w_p \end{cases} \quad (\text{A18})$$

Using the boundary condition at $r = w_p$, $\Delta T(r, z)$ demands a smoothly variation function, that is

$$C'_2 \frac{e^{-\alpha z}}{w_p^2(z)} \ln \left(\frac{w_p}{r_b} \right) = B'_2 \frac{e^{-\alpha z}}{w_p^2(z)} \ln \left(\frac{w_p}{r_b} \right) \quad (\text{A19})$$

yields

$$C'_2 = B'_2 \quad (\text{A20})$$

Rewrite $\Delta T(r, z)$ as follows:

$$\Delta T(r, z) = \begin{cases} B'_2 \frac{e^{-\alpha z}}{w_p^2(z)} \ln \left(\frac{w_p}{r_b} \right) + \frac{P_h \alpha e^{-\alpha z}}{4\pi K_c (1 - e^{-\alpha \ell})} (w_p^2 - r^2), & r \leq w_p \\ B'_2 \frac{e^{-\alpha z}}{w_p^2(z)} \ln \left(\frac{r}{r_b} \right), & r > w_p \end{cases} \quad (\text{A21})$$

Finally, using the boundary condition: $\Delta T(r, z) = 0$, when $r = r_b$: since the portion of $r > w_p$ is automatically satisfied, we only need to deal with the portion of $r \leq w_p$ as follows:

$$\frac{B'_2}{2} \frac{e^{-\alpha z}}{w_p^2(z)} \ln \left(\frac{w_p}{r_b} \right)^2 + \frac{P_h \alpha e^{-\alpha z}}{4\pi K_c (1 - e^{-\alpha \ell})} (w_p^2 - r^2) \quad (\text{A22})$$

Using the Taylor expansion of logarithm nature function in Eq. (A22), namely, $\ln(1+x) \cong x$, when $|x| \leq 1$, then Eq. (A22) becomes

$$-\frac{B'_2}{2} \frac{e^{-\alpha z}}{w_p^2(z)} \frac{(r_b^2 - w_p^2)}{w_p^2(z)} + \frac{P_h \alpha e^{-\alpha z}}{4\pi K_c (1 - e^{-\alpha \ell})} (w_p^2 - r^2) \quad (\text{A23})$$

Satisfying the boundary condition demanding $\Delta T(r, z) = 0$, when $r = r_b$, we get

$$B'_2 = -\frac{P_h \alpha}{2\pi K_c} \frac{w_p^4}{(1 - e^{-\alpha \ell})} \quad (\text{A24})$$

Finally, the solutions in text Eq. (5) are expressed as

$$\Delta T(r, z) = \begin{cases} \frac{P_h \alpha e^{-\alpha z} w_p^2}{4\pi K_c (1 - e^{-\alpha \ell})} \left[\left(1 - \frac{r^2}{w_p^2}\right) + \ln \frac{r_b^2}{w_p^2} \right], & r \leq w_p \\ \frac{P_h \alpha w_p^2 e^{-\alpha z}}{4\pi K_c (1 - e^{-\alpha \ell})} \ln \left(\frac{r_b^2}{r^2}\right), & r > w_p \end{cases} \quad (\text{A25})$$

Appendix B

From Eq. (6)

$$\frac{1-v}{1+v} \nabla(\nabla \cdot \vec{u}) - \frac{1-2v}{1+v} \nabla \times \nabla \times \vec{u} = \alpha_T \nabla(T) \quad (\text{B1})$$

Use the vector identity

$$\nabla \times \nabla \times \vec{u} = \nabla(\nabla \cdot \vec{u}) - \nabla^2 \vec{u} \quad (\text{B2})$$

Eq. (B1) becomes

$$\frac{v}{1+v} \nabla(\nabla \cdot \vec{u}) - \frac{1-2v}{1+v} \nabla^2 \vec{u} = \alpha_T \nabla(T) \quad (\text{B3})$$

Since $\Delta T = \Delta T(r, z)$, we can write

$$\vec{u}(r, z) = u_r(r, z)\hat{r} + u_z(r, z)\hat{z} \quad (\text{B4})$$

and furthermore express Eq. (B4) denoted by symbols like

$$\vec{u} = u_r \hat{r} + u_z \hat{z} \quad (\text{B5})$$

Substituting Eq. (B5) into Eq. (B3), and taking the $\hat{r}$ and $\hat{z}$ components, we obtain a set of coupled simultaneous differential equations:

$$\begin{aligned} \frac{v}{1+v} \left\{ \frac{\partial}{\partial r} \left[\frac{1}{r} \frac{\partial(r u_r)}{\partial r} + \frac{\partial u_z}{\partial z} \right] \right\} + \frac{1-2v}{1+v} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_r}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} + \frac{\partial^2 u_r}{\partial z^2} \right] &= \alpha_T \frac{\partial}{\partial r} T \frac{v}{1+v} \left\{ \frac{\partial}{\partial z} \left[\frac{1}{r} \frac{\partial(r u_r)}{\partial r} + \frac{\partial u_z}{\partial z} \right] \right\} \\ + \frac{1-2v}{1+v} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2} \right] &= \alpha_T \frac{\partial}{\partial z} T \end{aligned} \quad (\text{B6})$$

Using the scaling relationships, then Eq. (B6) is reduced to

$$\begin{aligned} \frac{v}{1+v} \left\{ \frac{\partial}{\partial r} \left[\frac{1}{r} \frac{\partial(r u_r)}{\partial r} \right] \right\} + \frac{1-2v}{1+v} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_r}{\partial r} \right) \right] \\ = \alpha_T \frac{\partial}{\partial r} T \frac{v}{1+v} \left(\frac{\partial^2 u_z}{\partial z^2} \right) + \frac{1-2v}{1+v} \frac{\partial^2 u_z}{\partial z^2} = \alpha_T \frac{\partial}{\partial z} T \end{aligned} \quad (\text{B7})$$

It is obviously seen that Eq. (B7) is just the simplified results when assuming $\nabla \times \nabla \times \vec{u} = 0$ in Eq. (B1). Thus, we obtain the Eqs. (8) and (9).

Appendix C

We keep the parallel analysis with Innocenzi et al. to calculate the thermal lensing effect [27]. Here we ignore the thermal focusing effect, since the focusing effect caused by a distortion of the flatness of the crystal ends are negligible [30]. The pump power is absorbed by laser rod and converted to a heat source $q(r)$ that un-uniformly spread out entire laser rod. A steady state temperature distribution $T(r)$ is established within the laser rod and governed by heat conduction equation [41]:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} = \frac{-q(r)}{K_c} \quad (\text{C1})$$

For a fiber-coupled pump beam, the heat source may be approximately described as a top-hat distribution, and the thermal loading ξ in Eq. (2) is replaced by $\xi(P_{in})$ in Eq. (3)

$$q(r) = \begin{cases} Q, & r \leq w_p \\ 0, & r > w_p \end{cases} \quad (\text{C2})$$

$$\text{Here } Q = \frac{\xi P_{in} \alpha}{\pi \ell (1 - e^{-\alpha \ell})} \int_0^\ell \frac{e^{-\alpha z}}{w_p^2(z)} dz \quad (\text{C3})$$

$$w_p^2(z) = [w_{p0} + \theta_p |z - z_0|]^2 \quad (\text{C4})$$

With satisfied the thermal-isolated condition on the circumferential surface of the laser rod, the solutions of Eq. (C1) is

$$\Delta T \equiv T(r) - T(r_b) = \begin{cases} \frac{Q}{4K_c} \left(1 - \frac{r^2}{w_p^2(z)} \right) + \frac{Q}{2K_c} \ln \left(\frac{r_b}{w_p(z)} \right), & r \leq w_p \\ \frac{Q}{2K_c} \ln \left(\frac{r_b}{r} \right), & r > w_p \end{cases} \quad (\text{C5})$$

This thermal radial gradient introduces a radial variation of the refractive index, the change of the refractive index can be separated into a temperature and a stress dependent variation. $n(r) = n_0 + \Delta n_T(r) + \Delta n_s(r)$. Here $n(r)$ is total change of the refractive index.

$$\begin{aligned} \Delta n_T(r) &= [T(r) - T(r_b)] \cdot \frac{dn}{dT} = \Delta T \cdot \frac{dn}{dT} \\ &= \frac{dn}{dT} \left[\frac{Q}{4K_c} \left(1 - \frac{r^2}{w_p^2(z)} \right) + \frac{Q}{2K_c} \ln \left(\frac{r_b}{w_p(z)} \right) \right] \\ &\quad \times \Theta(w_p^2(z) - r^2) + \frac{dn}{dT} \left[\frac{Q}{2K_c} \ln \left(\frac{r_b}{r} \right) \right] \Theta(r^2 - w_p^2(z)) \end{aligned} \quad (\text{C6})$$

This shows that not only a quadratic variation with radius r , but also varies with logarithm r with out the pump region. Therefore, an optical beam propagating along the rod axis suffers not only a quadratic spatial aberration. To arrive at the phase change associated with $\Delta n(r)_T$,

Innocenzi et al. [27] provides a method, extracted the appropriate term from the expression for $\Delta T(r)$ and neglected the natural logarithm term. This means that ΔT can be simplified as

$$\Delta T \propto 1 - \frac{r^2}{w_p^2(z)} + \ln \frac{r_b^2}{w_p^2(z)} \approx 1 - \frac{r^2}{w_p^2(z)} + \frac{r_b^2}{w_p^2(z)} - 1$$

$$\approx \frac{r_b^2}{w_p^2(z)} - \frac{r^2}{w_p^2(z)} \quad (C7)$$

Substitution Eq. (C7) into Eq. (C6), then

$$\Delta n_T(r) = \frac{Q}{4K_c} \cdot \frac{dn}{dT} \cdot \frac{r_b^2 - r^2}{w_p^2(z)} \approx -\frac{Q}{4K_c} \frac{dn}{dT} \frac{r^2}{w_p^2(z)} \quad (C8)$$

The total phase change $\Delta\Phi$ that is accumulated by the laser light for a single pass through the laser rod is give by

$$\Delta\Phi(r) = k \int_0^\ell \Delta n(r) dz (\Delta n \equiv n - n_0)$$

$$= k \int_0^\ell \Delta n_T(r) dz = -\frac{Q}{4K_c} \frac{dn}{dT} r^2 k \int_0^\ell \frac{1}{w_p^2(z)} dz \quad (C9)$$

According to “light transmission optics” and the theory of “wave optics of thin lens” by Marcuse [42], a ray through a spherical lens f , the lens is action as a phase transformer, and then the phase change is

$$\Delta\Phi(r) = k \frac{r^2}{2f} \quad (C10)$$

With the aid of Eq. (C10), and assume the crystal has perfect flatness in its end face, the total focal length is expressed and reduced as Eq. (34).

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