

36

Image Formation

CHAPTER OUTLINE

- 36.1 Images Formed by Flat Mirrors
- 36.2 Images Formed by Spherical Mirrors
- 36.3 Images Formed by Refraction
- 36.4 Thin Lenses
- 36.5 Lens Aberrations
- 36.6 The Camera
- 36.7 The Eye
- 36.8 The Simple Magnifier
- 36.9 The Compound Microscope
- 36.10 The Telescope

ANSWERS TO QUESTIONS

Q36.1 The mirror shown in the textbook picture produces an inverted image. It actually reverses top and bottom. It is not true in the same sense that “Most mirrors reverse left and right.” Mirrors don’t actually flip images side to side—we just assign the labels “left” and “right” to images as if they were real people mimicking us. If you stand face to face with a real person and raise your left hand, then he or she would have to raise his or her *right* hand to “mirror” your movement. Try this while facing a mirror. For sake of argument, let’s assume you are facing north and wear a watch on your left hand, which is on the western side. If you raise your left hand, you might say that your image raises its right hand, based on the labels we assign to other people. But your image raises its western-side hand, which is the hand with the watch.

- Q36.2** With a concave spherical mirror, for objects beyond the focal length the image will be real and inverted. For objects inside the focal length, the image will be virtual, upright, and magnified. Try a shaving or makeup mirror as an example.
- Q36.3** With a convex spherical mirror, all images of real objects are upright, virtual and smaller than the object. As seen in Question 36.2, you only get a change of orientation when you pass the focal point—but the focal point of a convex mirror is on the non-reflecting side!
- Q36.4** The mirror equation and the magnification equation apply to plane mirrors. A curved mirror is made flat by increasing its radius of curvature without bound, so that its focal length goes to infinity. From $\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = 0$ we have $\frac{1}{p} = -\frac{1}{q}$; therefore, $p = -q$. The virtual image is as far behind the mirror as the object is in front. The magnification is $M = -\frac{q}{p} = \frac{p}{p} = 1$. The image is right side up and actual size.
- Q36.5** Stones at the bottom of a clear stream always appears closer to the surface because light is refracted away from the normal at the surface. Example 36.8 in the textbook shows that its apparent depth is three quarters of its actual depth.

Q36.6 For definiteness, we consider real objects ($p > 0$).

(a) For $M = -\frac{q}{p}$ to be negative, q must be positive. This will happen in $\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$ if $p > f$, if the object is farther than the focal point.

(b) For $M = -\frac{q}{p}$ to be positive, q must be negative.
From $\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$ we need $p < f$.

(c) For a real image, q must be positive.
As in part (a), it is sufficient for p to be larger than f .

(d) For $q < 0$ we need $p < f$.

(e) For $|M| > 1$, we consider separately $M < -1$ and $M > 1$.

If $M = -\frac{q}{p} < -1$, we need $\frac{q}{p} > 1$ or $q > p$
or $\frac{1}{q} < \frac{1}{p}$

From $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$, $\frac{1}{p} + \frac{1}{p} > \frac{1}{f}$ or $\frac{2}{p} > \frac{1}{f}$

or $\frac{p}{2} < f$ or $p < 2f$.

Now if $-\frac{q}{p} > 1$ or $-q > p$ or $q < -p$

we may require $q < 0$, since then $\frac{1}{p} = \frac{1}{f} - \frac{1}{q}$ with $\frac{1}{f} > 0$

gives $\frac{1}{p} > -\frac{1}{q}$ as required or $-p > q$.

For $q < 0$ in $\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$ we need $p < f$.

Thus the overall condition for an enlarged image is simply $p < 2f$.

(f) For $|M| < 1$, we have the reverse of part (e), requiring $p > 2f$.

Q36.7 Using the same analysis as in Question 36.6 except $f < 0$.

(a) Never.

(b) Always.

(c) Never, for light rays passing through the lens will always diverge.

(d) Always.

(e) Never.

(f) Always.

Q36.8 We assume the lens has a refractive index higher than its surroundings. For the biconvex lens in Figure 36.27(a), $R_1 > 0$ and $R_2 < 0$. Then all terms in $(n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ are positive and $f > 0$. For the other two lenses in part (a) of the figure, R_1 and R_2 are both positive but R_1 is less than R_2 . Then $\frac{1}{R_1} > \frac{1}{R_2}$ and the focal length is again positive.

For the biconcave lens and the plano-concave lens in Figure 36.27(b), $R_1 < 0$ and $R_2 > 0$. Then both terms are negative in $\frac{1}{R_1} - \frac{1}{R_2}$ and the focal length is negative. For the middle lens in part (b) of the figure, R_1 and R_2 are both positive but R_1 is greater than R_2 . Then $\frac{1}{R_1} < \frac{1}{R_2}$ and the focal length is again negative.

Q36.9 Both words are inverted. However OXIDE has up-down symmetry whereas LEAD does not.

Q36.10 An infinite number. In general, an infinite number of rays leave each point of any object and travel in all directions. Note that the three principal rays that we use for imaging are just a subset of the infinite number of rays. All three principal rays can be drawn in a ray diagram, provided that we extend the plane of the lens as shown in Figure Q36.10.

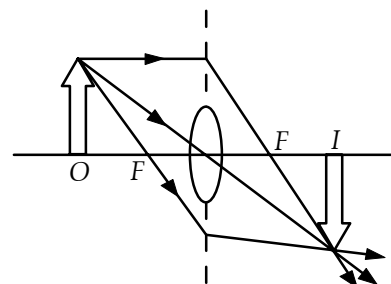


FIG. Q36.10

Q36.11 In this case, the index of refraction of the lens material is less than that of the surrounding medium. Under these conditions, a biconvex lens will be diverging.

Q36.12 Chromatic aberration arises because a material medium's refractive index can be frequency dependent. A mirror changes the direction of light by reflection, not refraction. Light of all wavelengths follows the same path according to the law of reflection, so no chromatic aberration happens.

Q36.13 This is a convex mirror. The mirror gives the driver a wide field of view and an upright image with the possible disadvantage of having objects appear diminished. Your brain can then interpret them as farther away than the objects really are.

Q36.14 As pointed out in Question 36.11, if the converging lens is immersed in a liquid with an index of refraction significantly greater than that of the lens itself, it will make light from a distant source diverge. This is not the case with a converging (concave) mirror, as the law of reflection has nothing to do with the indices of refraction.

- Q36.15** As in the diagram, let the center of curvature C of the fishbowl and the bottom of the fish define the optical axis, intersecting the fishbowl at vertex V . A ray from the top of the fish that reaches the bowl surface along a radial line through C has angle of incidence zero and angle of refraction zero. This ray exits from the bowl unchanged in direction. A ray from the top of the fish to V is refracted to bend away from the normal. Its extension back inside the the fishbowl determines the location of the image and the characteristics of the image. The image is upright, virtual, and enlarged.

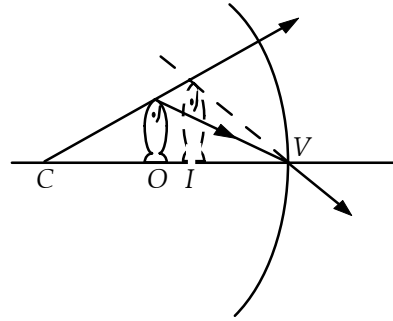


FIG. Q36.15

- Q36.16** Because when you look at the **AMBULANCE** in your rear view mirror, the apparent left-right inversion clearly displays the name of the **AMBULANCE** behind you. Do not jam on your brakes when a **MIAMI** city bus is right behind you.
- Q36.17** The entire image is visible, but only at half the intensity. Each point on the object is a source of rays that travel in all directions. Thus, light from all parts of the object goes through all unblocked parts of the lens and forms an image. If you block part of the lens, you are blocking some of the rays, but the remaining ones still come from all parts of the object.
- Q36.18** With the meniscus design, when you direct your gaze near the outer circumference of the lens you receive a ray that has passed through glass with more nearly parallel surfaces of entry and exit. Thus, the lens minimally distorts the direction to the object you are looking at. If you wear glasses, turn them around and look through them the wrong way to maximize this distortion.
- Q36.19** The eyeglasses on the left are diverging lenses that correct for nearsightedness. If you look carefully at the edge of the person's face through the lens, you will see that everything viewed through these glasses is reduced in size. The eyeglasses on the right are converging lenses, which correct for farsightedness. These lenses make everything that is viewed through them look larger.
- Q36.20** The eyeglass wearer's eye is at an object distance from the lens that is quite small—the eye is on the order of 10^{-2} meter from the lens. The focal length of an eyeglass lens is several decimeters, positive or negative. Therefore the image distance will be similar in magnitude to the object distance. The onlooker sees a sharp image of the eye behind the lens. Look closely at the left side of Figure Q36.19 and notice that the wearer's eyes seem not only to be smaller, but also positioned a bit behind the plane of his face—namely where they would be if he was not wearing glasses. Similarly, in the right half of Figure Q36.19, his eyes seem to be in front of the plane of his face and magnified. We as observers take this light information coming from the object through the lens and perceive or photograph the image as if it were an object.
- Q36.21** In the diagram, only two of the three principal rays have been used to locate images to reduce the amount of visual clutter. The upright shaded arrows are the objects, and the correspondingly numbered inverted arrows are the images. As you can see, object 2 is closer to the focal point than object 1, and image 2 is farther to the left than image 1.

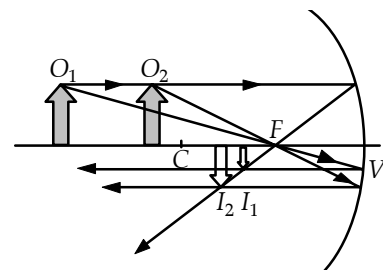


FIG. Q36.21

- Q36.22** Absolutely. Only absorbed light, not transmitted light, contributes internal energy to a transparent object. A clear lens can stay ice-cold and solid as megajoules of light energy pass through it.
- Q36.23** One can change the f number either by changing the focal length (if using a “zoom” lens) or by changing the aperture of the camera lens. As the f number increases, the exposure time required increases also, as both increasing the focal length or decreasing the aperture decreases the light intensity reaching the film.
- Q36.24** Make the mirror an efficient reflector (shiny). Make it reflect to the image even rays far from the axis, by giving it a parabolic shape. Most important, make it large in diameter to intercept a lot of solar power. And you get higher temperature if the image is smaller, as you get with shorter focal length; and if the furnace enclosure is an efficient absorber (black).
- Q36.25** For the explanation, we ignore the lens and consider two objects. Hold your two thumbs parallel and extended upward in front of you, at different distances from your nose. Alternately close your left eye and your right eye. You see both thumbs jump back and forth against the background of more distant objects. Parallax by definition is this apparent motion of a stationary object (one thumb) caused by motion of the observer (jumping from right eye to left eye). Your nearer thumb jumps by a larger angle against the background than your farther thumb does. They will jump by the same amount only if they are equally distant from your face. The method of parallax for adjusting one object so that it is the same distance away from you as another object will work even if one ‘object’ is an image.
- Q36.26** The artist’s statements are accurate, perceptive, and eloquent. The image you see is “almost one’s whole surroundings,” including things behind you and things farther in front of you than the globe is, but nothing eclipsed by the opaque globe or by your head. For example, we cannot see Escher’s index and middle fingers or their reflections in the globe.
- The point halfway between your eyes is indeed the focus in a figurative sense, but it is not an optical focus. The principal axis will always lie in a line that runs through the center of the sphere and the bridge of your nose. Outside the globe, you are at the center of your observable universe. If you wink at the ball, the center of the looking-glass world hops over to the location of the image of your open eye.
- Q36.27** The three mirrors, two of which are shown as M and N in the figure to the right, reflect any incident ray back parallel to its original direction. When you look into the corner you see image I_3 of yourself.

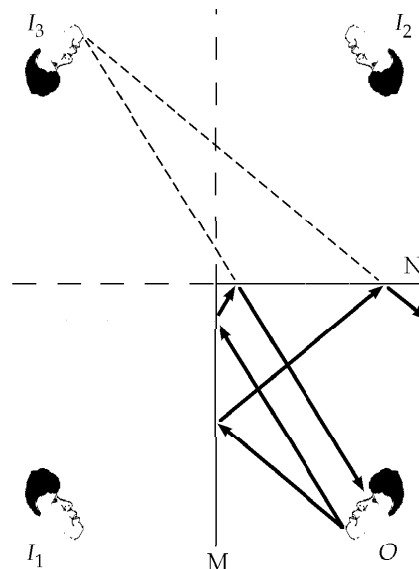


FIG. Q36.21

- Q36.28** You have likely seen a Fresnel mirror for sound. The diagram represents first a side view of a band shell. It is a concave mirror for sound, designed to channel sound into a beam toward the audience in front of the band shell. Sections of its surface can be kept at the right orientations as they are pushed around inside a rectangular box to form an auditorium with good diffusion of sound from stage to audience, with a floor plan suggested by the second part of the diagram.

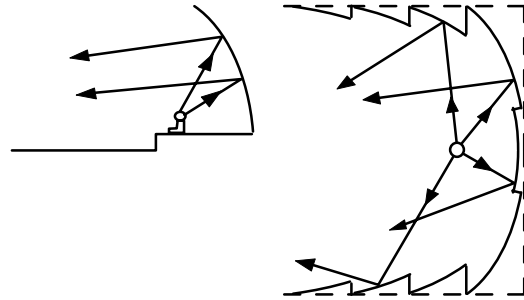


FIG. Q36.28

SOLUTIONS TO PROBLEMS

Section 36.1 Images Formed by Flat Mirrors

- P36.1** I stand 40 cm from my bathroom mirror. I scatter light, which travels to the mirror and back to me in time

$$\frac{0.8 \text{ m}}{3 \times 10^8 \text{ m/s}} = \boxed{\sim 10^{-9} \text{ s}}$$

showing me a view of myself as I was at that look-back time. I'm no Dorian Gray!

- P36.2** The virtual image is as far behind the mirror as the choir is in front of the mirror. Thus, the image is 5.30 m behind the mirror. The image of the choir is $0.800 \text{ m} + 5.30 \text{ m} = 6.10 \text{ m}$ from the organist. Using similar triangles:

$$\frac{h'}{0.600 \text{ m}} = \frac{6.10 \text{ m}}{0.800 \text{ m}}$$

or
$$h' = (0.600 \text{ m}) \left(\frac{6.10 \text{ m}}{0.800 \text{ m}} \right) = \boxed{4.58 \text{ m}}.$$

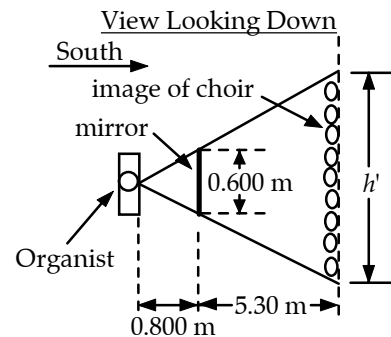


FIG. P36.2

P36.3 The flatness of the mirror is described

by $R = \infty, f = \infty$

and $\frac{1}{f} = 0$.

By our general mirror equation,

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = 0$$

or $q = -p$.

Thus, the image is as far behind the mirror as the person is in front. The magnification is then

$$M = \frac{-q}{p} = 1 = \frac{h'}{h}$$

so $h' = h = 70.0$ inches.

The required height of the mirror is defined by the triangle from the person's eyes to the top and bottom of his image, as shown. From the geometry of the triangle, we see that the mirror height must be:

$$h' \left(\frac{p}{p-q} \right) = h' \left(\frac{p}{2p} \right) = \frac{h'}{2}.$$

Thus, the mirror must be at least 35.0 inches high.

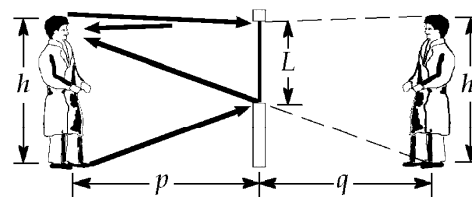


FIG. P36.3

P36.4 A graphical construction produces 5 images, with images I_1 and I_2 directly into the mirrors from the object O ,

and (O, I_3, I_4)

and (I_2, I_1, I_5)

forming the vertices of equilateral triangles.

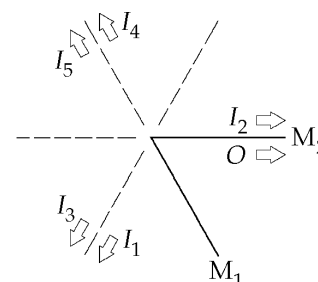


FIG. P36.4

- P36.5**
- (1) The first image in the left mirror is 5.00 ft behind the mirror, or 10.0 ft from the position of the person.
 - (2) The first image in the right mirror is located 10.0 ft behind the right mirror, but this location is 25.0 ft from the left mirror. Thus, the second image in the left mirror is 25.0 ft behind the mirror, or 30.0 ft from the person.
 - (3) The first image in the left mirror forms an image in the right mirror. This first image is 20.0 ft from the right mirror, and, thus, an image 20.0 ft behind the right mirror is formed. This image in the right mirror also forms an image in the left mirror. The distance from this image in the right mirror to the left mirror is 35.0 ft. The third image in the left mirror is, thus, 35.0 ft behind the mirror, or 40.0 ft from the person.

*P36.6 (a) The flat mirrors have

$$R \rightarrow \infty$$

$$\text{and } f \rightarrow \infty.$$

The upper mirror M_1 produces a virtual, actual sized image I_1 according to

$$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f} = \frac{1}{\infty} = 0$$

$$q_1 = -p_1$$

$$\text{with } M_1 = -\frac{q_1}{p_1} = +1.$$

As shown, this image is above the upper mirror. It is the object for mirror M_2 , at object distance

$$p_2 = p_1 + h.$$

The lower mirror produces a virtual, actual-size, right-side-up image according to

$$\frac{1}{p_2} + \frac{1}{q_2} = 0$$

$$q_2 = -p_2 = -(p_1 + h)$$

$$\text{with } M_2 = -\frac{q_2}{p_2} = +1 \text{ and } M_{\text{overall}} = M_1 M_2 = 1.$$

Thus the final image is at distance $p_1 + h$ behind the lower mirror.

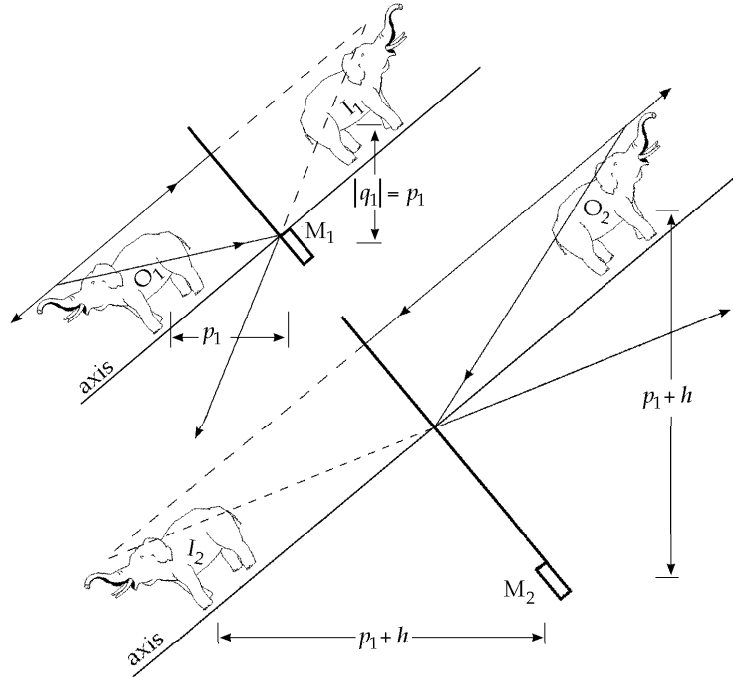


FIG. P36.6

- (b) It is virtual.
- (c) Upright
- (d) With magnification +1.
- (e) It does not appear to be reversed left and right. In a top view of the periscope, parallel rays from the right and left sides of the object stay parallel and on the right and left.

Section 36.2 Images Formed by Spherical Mirrors

P36.7 For a concave mirror, both R and f are positive.

We also know that $f = \frac{R}{2} = 10.0 \text{ cm}$.

$$(a) \quad \frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{10.0 \text{ cm}} - \frac{1}{40.0 \text{ cm}} = \frac{3}{40.0 \text{ cm}}$$

and

$$q = 13.3 \text{ cm}$$

$$M = \frac{q}{p} = -\frac{13.3 \text{ cm}}{40.0 \text{ cm}} = -0.333$$

The image is 13.3 cm in front of the mirror, real, and inverted.

$$(b) \quad \frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{10.0 \text{ cm}} - \frac{1}{20.0 \text{ cm}} = \frac{1}{20.0 \text{ cm}}$$

and

$$q = 20.0 \text{ cm}$$

$$M = \frac{q}{p} = -\frac{20.0 \text{ cm}}{20.0 \text{ cm}} = -1.00$$

The image is 20.0 cm in front of the mirror, real, and inverted.

$$(c) \quad \frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{10.0 \text{ cm}} - \frac{1}{10.0 \text{ cm}} = 0$$

Thus, $q = \text{infinity}$.

No image is formed. The rays are reflected parallel to each other.

P36.8 $\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = -\frac{1}{0.275 \text{ m}} - \frac{1}{10.0 \text{ m}}$ gives $q = -0.267 \text{ m}$.

Thus, the image is virtual.

$$M = \frac{-q}{p} = -\frac{-0.267}{10.0 \text{ m}} = 0.0267$$

Thus, the image is upright (+M) and diminished ($|M| < 1$).

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- P36.9** (a) $\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$ gives $\frac{1}{30.0 \text{ cm}} + \frac{1}{q} = \frac{2}{(-40.0 \text{ cm})}$
 $\frac{1}{q} = -\frac{2}{40.0 \text{ cm}} - \frac{1}{30.0 \text{ cm}} = -0.0833 \text{ cm}^{-1}$ so $q = \boxed{-12.0 \text{ cm}}$
 $M = \frac{-q}{p} = -\frac{(-12.0 \text{ cm})}{30.0 \text{ cm}} = \boxed{0.400}$.
- (b) $\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$ gives $\frac{1}{60.0 \text{ cm}} + \frac{1}{q} = \frac{2}{(-40.0 \text{ cm})}$
 $\frac{1}{q} = -\frac{2}{40.0 \text{ cm}} - \frac{1}{60.0 \text{ cm}} = -0.0666 \text{ cm}^{-1}$ so $q = \boxed{-15.0 \text{ cm}}$
 $M = \frac{-q}{p} = -\frac{(-15.0 \text{ cm})}{60.0 \text{ cm}} = \boxed{0.250}$.
- (c) Since $M > 0$, the images are **upright**.

P36.10 With radius 2.50 m, the cylindrical wall is a highly efficient mirror for sound, with focal length

$$f = \frac{R}{2} = 1.25 \text{ m}.$$

In a vertical plane the sound disperses as usual, but that radiated in a horizontal plane is concentrated in a sound image at distance q from the back of the niche, where

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \quad \text{so} \quad \frac{1}{2.00 \text{ m}} + \frac{1}{q} = \frac{1}{1.25 \text{ m}}$$

$$q = \boxed{3.33 \text{ m}}.$$

- P36.11** (a) $\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$ becomes $\frac{1}{q} = \frac{2}{60.0 \text{ cm}} - \frac{1}{90.0 \text{ cm}}$
 $q = \boxed{45.0 \text{ cm}}$ and $M = \frac{-q}{p} = -\frac{45.0 \text{ cm}}{90.0 \text{ cm}} = \boxed{-0.500}$.
- (b) $\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$ becomes $\frac{1}{q} = \frac{2}{60.0 \text{ cm}} - \frac{1}{20.0 \text{ cm}}$
 $q = \boxed{-60.0 \text{ cm}}$ and $M = \frac{-q}{p} = -\frac{(-60.0 \text{ cm})}{(20.0 \text{ cm})} = \boxed{3.00}$.

- (c) The image (a) is real, inverted and diminished. That of (b) is virtual, upright, and enlarged. The ray diagrams are similar to Figure 36.15(a) and 36.15(b) in the text, respectively.

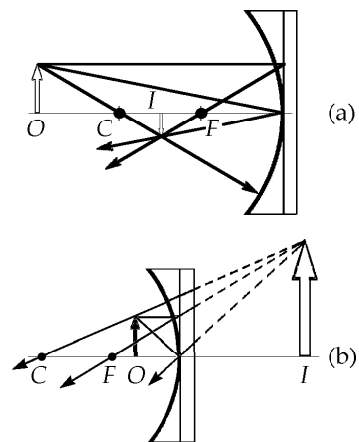


FIG. P36.11

P36.12 For a concave mirror, R and f are positive. Also, for an erect image, M is positive. Therefore,

$$M = -\frac{q}{p} = 4 \text{ and } q = -4p.$$

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} \text{ becomes } \frac{1}{40.0 \text{ cm}} = \frac{1}{p} - \frac{1}{4p} = \frac{3}{4p}; \text{ from which, } p = \boxed{30.0 \text{ cm}}.$$

***P36.13** The ball is a convex mirror with $R = -4.25 \text{ cm}$ and

$$f = \frac{R}{2} = -2.125 \text{ cm. We have}$$

$$M = \frac{3}{4} = -\frac{q}{p}$$

$$q = -\frac{3}{4}p$$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{p} + \frac{1}{-(3/4)p} = \frac{1}{-2.125 \text{ cm}}$$

$$\frac{3}{3p} - \frac{4}{3p} = \frac{1}{-2.125 \text{ cm}}$$

$$3p = 2.125 \text{ cm}$$

$$\boxed{p = 0.708 \text{ cm}} \text{ in front of the sphere.}$$

The image is upright, virtual, and diminished.

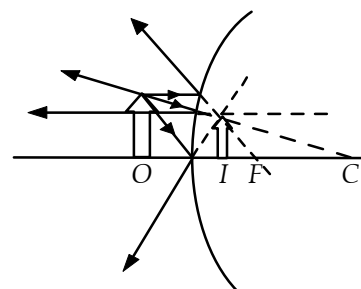


FIG. P36.13

***P36.14** (a) $M = -4 = -\frac{q}{p}$ $q = 4p$

$$q - p = 0.60 \text{ m} = 4p - p$$

$$p = 0.2 \text{ m}$$

$$q = 0.8 \text{ m}$$

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{0.2 \text{ m}} + \frac{1}{0.8 \text{ m}}$$

$$f = \boxed{160 \text{ mm}}$$

(b) $M = +\frac{1}{2} = -\frac{q}{p}$ $p = -2q$

$$|q| + p = 0.20 \text{ m} = -q + p = -q - 2q$$

$$q = -66.7 \text{ mm}$$

$$p = 133 \text{ mm}$$

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{R} = \frac{1}{0.133 \text{ m}} + \frac{1}{-0.0667 \text{ m}}$$

$$R = \boxed{-267 \text{ mm}}$$

***P36.15** $M = -\frac{q}{p}$
 $q = -Mp = -0.013(30 \text{ cm}) = -0.39 \text{ cm}$
 $\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{2}{R}$
 $\frac{1}{30 \text{ cm}} + \frac{1}{-0.39 \text{ cm}} = \frac{2}{R}$
 $R = \frac{2}{-2.53 \text{ m}^{-1}} = -0.790 \text{ cm}$

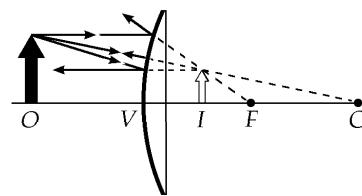


FIG. P36.15

The cornea is convex, with radius of curvature 0.790 cm.

***P36.16** With
 $M = \frac{h'}{h} = \frac{+4.00 \text{ cm}}{10.0 \text{ cm}} = +0.400 = -\frac{q}{p}$
 $q = -0.400p$
 the image must be virtual.

(a) It is a convex mirror that produces a diminished upright virtual image.

(b) We must have
 $p + |q| = 42.0 \text{ cm} = p - q$
 $p = 42.0 \text{ cm} + q$
 $p = 42.0 \text{ cm} - 0.400p$
 $p = \frac{42.0 \text{ cm}}{1.40} = 30.0 \text{ cm}$

The mirror is at the 30.0 cm mark.

(c) $\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{1}{30 \text{ cm}} + \frac{1}{-0.4(30 \text{ cm})} = \frac{1}{f} = -0.0500/\text{cm}$

$f = -20.0 \text{ cm}$

The ray diagram looks like Figure 36.15(c) in the text.

P36.17 (a) $q = (p + 5.00 \text{ m})$ and, since the image must be real,

$$M = -\frac{q}{p} = -5 \quad \text{or} \quad q = 5p.$$

Therefore, $p + 5.00 \text{ m} = 5p$

or $p = 1.25 \text{ m}$ and $q = 6.25 \text{ m}.$

From $\frac{1}{p} + \frac{1}{q} = \frac{2}{R},$ $R = \frac{2pq}{p+q} = \frac{2(1.25)(6.25)}{1.25+6.25}$
 $= \text{2.08 m(concave)}$

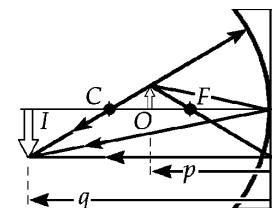


FIG. P36.17

(b) From part (a), $p = 1.25 \text{ m}$; the mirror should be 1.25 m in front of the object.

P36.18 Assume that the object distance is the same in both cases (i.e., her face is the same distance from the hubcap regardless of which way it is turned). Also realize that the near image ($q = -10.0$ cm) occurs when using the convex side of the hubcap. Applying the mirror equation to both cases gives:

$$\begin{aligned} \text{(concave side: } R = |R|, \quad q = -30.0 \text{ cm)} \\ \frac{1}{p} - \frac{1}{30.0} &= \frac{2}{|R|} \\ \text{or} \quad \frac{2}{|R|} &= \frac{30.0 \text{ cm} - p}{(30.0 \text{ cm})p} \end{aligned} \quad (1)$$

$$\begin{aligned} \text{(convex side: } R = -|R|, \quad q = -10.0 \text{ cm)} \\ \frac{1}{p} - \frac{1}{10.0} &= -\frac{2}{|R|} \\ \text{or} \quad \frac{2}{|R|} &= \frac{p - 10.0 \text{ cm}}{(10.0 \text{ cm})p}. \end{aligned} \quad (2)$$

(a) Equating Equations (1) and (2) gives:

$$\begin{aligned} \frac{30.0 \text{ cm} - p}{3.00} &= p - 10.0 \text{ cm} \\ \text{or} \quad p &= 15.0 \text{ cm}. \end{aligned}$$

Thus, her face is 15.0 cm from the hubcap.

(b) Using the above result ($p = 15.0$ cm) in Equation (1) gives:

$$\begin{aligned} \frac{2}{|R|} &= \frac{30.0 \text{ cm} - 15.0 \text{ cm}}{(30.0 \text{ cm})(15.0 \text{ cm})} \\ \text{or} \quad \frac{2}{|R|} &= \frac{1}{30.0 \text{ cm}} \\ \text{and} \quad |R| &= 60.0 \text{ cm}. \end{aligned}$$

The radius of the hubcap is 60.0 cm.

***P36.19** (a) The flat mirror produces an image according to

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{2}{R} \quad \frac{1}{24 \text{ cm}} + \frac{1}{q} = \frac{1}{\infty} = 0 \quad q = -24.0 \text{ m}.$$

The image is 24.0 m behind the mirror, distant from your eyes by

$$1.55 \text{ m} + 24.0 \text{ m} = \boxed{25.6 \text{ m}}.$$

(b) The image is the same size as the object, so $\theta = \frac{h}{d} = \frac{1.50 \text{ m}}{25.6 \text{ m}} = \boxed{0.0587 \text{ rad}}.$

$$(c) \quad \frac{1}{p} + \frac{1}{q} = \frac{2}{R} \quad \frac{1}{24 \text{ m}} + \frac{1}{q} = \frac{2}{(-2 \text{ m})} \quad q = \frac{1}{-(1/1 \text{ m}) - (1/24 \text{ m})} = -0.960 \text{ m}$$

$$\text{This image is distant from your eyes by} \quad 1.55 \text{ m} + 0.960 \text{ m} = \boxed{2.51 \text{ m}}.$$

continued on next page

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(d) The image size is given by $M = \frac{h'}{h} = -\frac{q}{p}$ $h' = -h \frac{q}{p} = -1.50 \text{ m} \left(\frac{-0.960 \text{ m}}{24 \text{ m}} \right) = 0.0600 \text{ m}$.

So its angular size at your eye is $\theta' = \frac{h'}{d} = \frac{0.06 \text{ m}}{2.51 \text{ m}} = \boxed{0.0239 \text{ rad}}$.

(e) Your brain assumes that the car is 1.50 m high and calculate its distance as

$$d' = \frac{h}{\theta'} = \frac{1.50 \text{ m}}{0.0239} = \boxed{62.8 \text{ m}}.$$

P36.20 (a) The image starts from a point whose height above the mirror vertex is given by

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{2}{R} \quad \frac{1}{3.00 \text{ m}} + \frac{1}{q} = \frac{1}{0.500 \text{ m}}. \quad \text{Therefore,} \quad q = 0.600 \text{ m}.$$

As the ball falls, p decreases and q increases. Ball and image pass when $q_1 = p_1$. When this is true,

$$\frac{1}{p_1} + \frac{1}{p_1} = \frac{1}{0.500 \text{ m}} = \frac{2}{p_1} \quad \text{or} \quad p_1 = 1.00 \text{ m}.$$

As the ball passes the focal point, the image switches from infinitely far above the mirror to infinitely far below the mirror. As the ball approaches the mirror from above, the virtual image approaches the mirror from below, reaching it together when $p_2 = q_2 = 0$.

(b) The falling ball passes its real image when it has fallen

$$3.00 \text{ m} - 1.00 \text{ m} = 2.00 \text{ m} = \frac{1}{2}gt^2, \text{ or when } t = \sqrt{\frac{2(2.00 \text{ m})}{9.80 \text{ m/s}^2}} = \boxed{0.639 \text{ s}}.$$

The ball reaches its virtual image when it has traversed

$$3.00 \text{ m} - 0 = 3.00 \text{ m} = \frac{1}{2}gt^2, \text{ or at } t = \sqrt{\frac{2(3.00 \text{ m})}{9.80 \text{ m/s}^2}} = \boxed{0.782 \text{ s}}.$$

Section 36.3 Images Formed by Refraction

P36.21 $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R} = 0 \text{ and } R \rightarrow \infty$

$$q = -\frac{n_2}{n_1}p = -\frac{1}{1.309}(50.0 \text{ cm}) = -38.2 \text{ cm}$$

Thus, the virtual image of the dust speck is 38.2 cm below the top surface of the ice.

P36.22 When $R \rightarrow \infty$, the equation describing image formation at a single refracting surface becomes

$q = -p\left(\frac{n_2}{n_1}\right)$. We use this to locate the final images of the two surfaces of the glass plate. First, find the image the glass forms of the *bottom* of the plate.

$$q_{B1} = -\left(\frac{1.33}{1.66}\right)(8.00 \text{ cm}) = -6.41 \text{ cm}$$

This virtual image is 6.41 cm below the top surface of the glass of 18.41 cm below the water surface. Next, use this image as an object and locate the image the water forms of the bottom of the plate.

$$q_{B2} = -\left(\frac{1.00}{1.33}\right)(18.41 \text{ cm}) = -13.84 \text{ cm} \quad \text{or} \quad 13.84 \text{ cm below the water surface.}$$

Now find image the water forms of the *top* surface of the glass.

$$q_3 = -\left(\frac{1}{1.33}\right)(12.0 \text{ cm}) = -9.02 \text{ cm} \quad \text{or} \quad 9.02 \text{ cm below the water surface.}$$

Therefore, the apparent thickness of the glass is $\Delta t = 13.84 \text{ cm} - 9.02 \text{ cm} = \boxed{4.82 \text{ cm}}$.

P36.23 From Equation 36.8
$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}.$$

Solve for q to find
$$q = \frac{n_2 R p}{p(n_2 - n_1) - n_1 R}.$$

In this case, $n_1 = 1.50$, $n_2 = 1.00$, $R = -15.0 \text{ cm}$

and $p = 10.0 \text{ cm}.$

So
$$q = \frac{(1.00)(-15.0 \text{ cm})(10.0 \text{ cm})}{(10.0 \text{ cm})(1.00 - 1.50) - (1.50)(-15.0 \text{ cm})} = -8.57 \text{ cm}.$$

Therefore, the apparent depth is 8.57 cm.

P36.24
$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R} \quad \text{so} \quad \frac{1.00}{\infty} + \frac{1.40}{21.0 \text{ mm}} = \frac{1.40 - 1.00}{6.00 \text{ mm}}$$

and $0.0667 = 0.0667.$

They agree. The image is inverted, real and diminished.

P36.25
$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R} \quad \text{becomes} \quad \frac{1.00}{p} + \frac{1.50}{q} = \frac{1.50 - 1.00}{6.00 \text{ cm}} = \frac{1}{12.0 \text{ cm}}$$

(a)
$$\frac{1.00}{20.0 \text{ cm}} + \frac{1.50}{q} = \frac{1}{12.0 \text{ cm}} \quad \text{or} \quad q = \frac{1.50}{[(1.00/12.0 \text{ cm}) - (1.00/20.0 \text{ cm})]} = \boxed{45.0 \text{ cm}}$$

(b)
$$\frac{1.00}{10.0 \text{ cm}} + \frac{1.50}{q} = \frac{1}{12.0 \text{ cm}} \quad \text{or} \quad q = \frac{1.50}{[(1.00/12.0 \text{ cm}) - (1.00/10.0 \text{ cm})]} = \boxed{-90.0 \text{ cm}}$$

(c)
$$\frac{1.00}{3.0 \text{ cm}} + \frac{1.50}{q} = \frac{1}{12.0 \text{ cm}} \quad \text{or} \quad q = \frac{1.50}{[(1.00/12.0 \text{ cm}) - (1.00/3.0 \text{ cm})]} = \boxed{-6.00 \text{ cm}}$$

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P36.26 $p = \infty$ and $q = +2R$

$$\frac{1.00}{p} + \frac{n_2}{q} = \frac{n_2 - 1.00}{R}$$

$$0 + \frac{n_2}{2R} = \frac{n_2 - 1.00}{R}$$

so

$$\boxed{n_2 = 2.00}$$

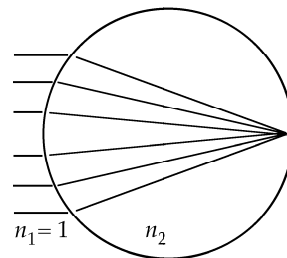


FIG. P36.26

P36.27 For a plane surface, $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$ becomes $q = -\frac{n_2 p}{n_1}$.

Thus, the magnitudes of the rate of change in the image and object positions are related by

$$\left| \frac{dq}{dt} \right| = \frac{n_2}{n_1} \left| \frac{dp}{dt} \right|.$$

If the fish swims toward the wall with a speed of 2.00 cm/s, the speed of the image is given by

$$v_{\text{image}} = \left| \frac{dq}{dt} \right| = \frac{1.00}{1.33} (2.00 \text{ cm/s}) = \boxed{1.50 \text{ cm/s}}.$$

Section 36.4 Thin Lenses

P36.28 Let R_1 = outer radius and R_2 = inner radius

$$\frac{1}{f} = (n - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right] = (1.50 - 1) \left[\frac{1}{2.00 \text{ m}} - \frac{1}{2.50 \text{ cm}} \right] = 0.0500 \text{ cm}^{-1}$$

so $f = \boxed{20.0 \text{ cm}}.$

P36.29 (a) $\frac{1}{f} = (n - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right] = (0.440) \left[\frac{1}{12.0 \text{ cm}} - \frac{1}{(-18.0 \text{ cm})} \right]$

$$f = \boxed{16.4 \text{ cm}}$$

(b) $\frac{1}{f} = (0.440) \left[\frac{1}{18.0 \text{ cm}} - \frac{1}{(-12.0 \text{ cm})} \right]$

$$f = \boxed{16.4 \text{ cm}}$$

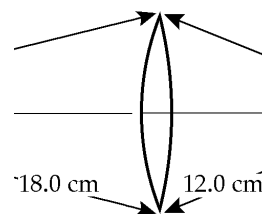


FIG. P36.29

P36.30 For a converging lens, f is positive. We use $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$.

$$(a) \quad \frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{20.0 \text{ cm}} - \frac{1}{40.0 \text{ cm}} = \frac{1}{40.0 \text{ cm}} \quad \boxed{q = 40.0 \text{ cm}}$$

$$M = -\frac{q}{p} = -\frac{40.0}{40.0} = \boxed{-1.00}$$

The image is real, inverted, and located 40.0 cm past the lens.

$$(b) \quad \frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{20.0 \text{ cm}} - \frac{1}{20.0 \text{ cm}} = 0 \quad \boxed{q = \text{infinity}}$$

No image is formed. The rays emerging from the lens are parallel to each other.

$$(c) \quad \frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{20.0 \text{ cm}} - \frac{1}{10.0 \text{ cm}} = -\frac{1}{20.0 \text{ cm}} \quad \boxed{q = -20.0 \text{ cm}}$$

$$M = -\frac{q}{p} = -\frac{(-20.0)}{10.0} = \boxed{2.00}$$

The image is upright, virtual and 20.0 cm in front of the lens.

P36.31 (a) $\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{25.0 \text{ cm}} - \frac{1}{26.0 \text{ cm}} \quad \boxed{q = 650 \text{ cm}}$

The image is real, inverted, and enlarged.

$$(b) \quad \frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{25.0 \text{ cm}} - \frac{1}{24.0 \text{ cm}} \quad \boxed{q = -600 \text{ cm}}$$

The image is virtual, upright, and enlarged.

P36.32 (a) $\frac{1}{p} + \frac{1}{q} = \frac{1}{f} : \quad \frac{1}{32.0 \text{ cm}} + \frac{1}{8.00 \text{ cm}} = \frac{1}{f}$

so $\boxed{f = 6.40 \text{ cm}}$

$$(b) \quad M = -\frac{q}{p} = -\frac{8.00 \text{ cm}}{32.0 \text{ cm}} = \boxed{-0.250}$$

(c) Since $f > 0$, the lens is converging.

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P36.33 We are looking at an enlarged, upright, virtual image:

$$M = \frac{h'}{h} = 2 = -\frac{q}{p} \quad \text{so} \quad p = -\frac{q}{2} = -\frac{(-2.84 \text{ cm})}{2} = +1.42 \text{ cm}$$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \quad \text{gives} \quad \frac{1}{1.42 \text{ cm}} + \frac{1}{(-2.84 \text{ cm})} = \frac{1}{f}$$

$$\boxed{f = 2.84 \text{ cm}}$$

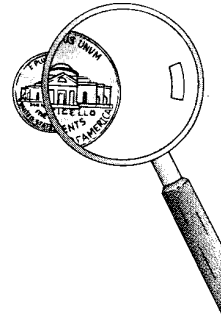


FIG. P36.33

P36.34 (a) $\frac{1}{p} + \frac{1}{q} = \frac{1}{f} : \quad \frac{1}{p} + \frac{1}{-30.0 \text{ cm}} = \frac{1}{12.5 \text{ cm}}$

$$p = 8.82 \text{ cm} \quad M = -\frac{q}{p} = -\frac{(-30.0)}{8.82} = \boxed{3.40, \text{ upright}}$$

(b) See the figure to the right.

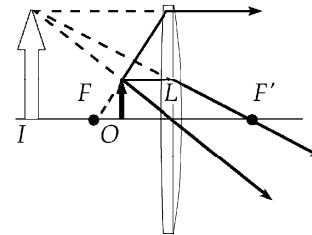


FIG. P36.34(b)

P36.35 $\frac{1}{p} + \frac{1}{q} = \frac{1}{f} : \quad p^{-1} + q^{-1} = \text{constant}$

We may differentiate through with respect to p : $-1p^{-2} - 1q^{-2} \frac{dq}{dp} = 0$

$$\frac{dq}{dp} = -\frac{q^2}{p^2} = -M^2.$$

P36.36 The image is inverted: $M = \frac{h'}{h} = \frac{-1.80 \text{ m}}{0.0240 \text{ m}} = -75.0 = \frac{-q}{p} \quad q = 75.0p.$

(b) $q + p = 3.00 \text{ m} = 75.0p + p \quad p = \boxed{39.5 \text{ mm}}$

(a) $q = 2.96 \text{ m} \quad \frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{0.0395 \text{ m}} + \frac{1}{2.96 \text{ m}} \quad f = \boxed{39.0 \text{ mm}}$

P36.37 (a) $\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \quad \frac{1}{20.0 \text{ cm}} + \frac{1}{q} = \frac{1}{(-32.0 \text{ cm})}$
 so $q = -\left(\frac{1}{20.0} + \frac{1}{32.0}\right)^{-1} = \boxed{-12.3 \text{ cm}}$

The image is 12.3 cm to the left of the lens.

(b) $M = -\frac{q}{p} = -\frac{(-12.3 \text{ cm})}{20.0 \text{ cm}} = \boxed{0.615}$

(c) See the ray diagram to the right.

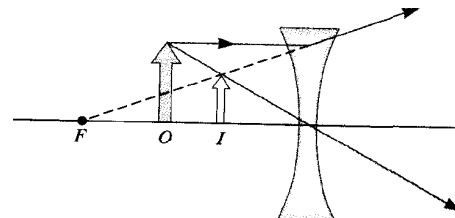


FIG. P36.37

***P36.38**

In

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$p^{-1} + q^{-1} = \text{constant},$$

we differentiate with respect to time

$$-1(p^{-2})\frac{dp}{dt} - 1(q^{-2})\frac{dq}{dt} = 0$$

$$\frac{dq}{dt} = \frac{-q^2}{p^2} \frac{dp}{dt}.$$

We must find the momentary image location q :

$$\frac{1}{20 \text{ m}} + \frac{1}{q} = \frac{1}{0.3 \text{ m}}$$

$$q = 0.305 \text{ m}.$$

$$\text{Now } \frac{dq}{dt} = -\frac{(0.305 \text{ m})^2}{(20 \text{ m})^2} 5 \text{ m/s} = -0.00116 \text{ m/s} = \boxed{1.16 \text{ mm/s toward the lens}}.$$

***P36.39**

$$(a) \quad \frac{1}{p} + \frac{1}{q} = \frac{1}{f} \quad \frac{1}{480 \text{ cm}} + \frac{1}{q} = \frac{1}{7.00 \text{ cm}} \quad q = \boxed{7.10 \text{ cm}}$$

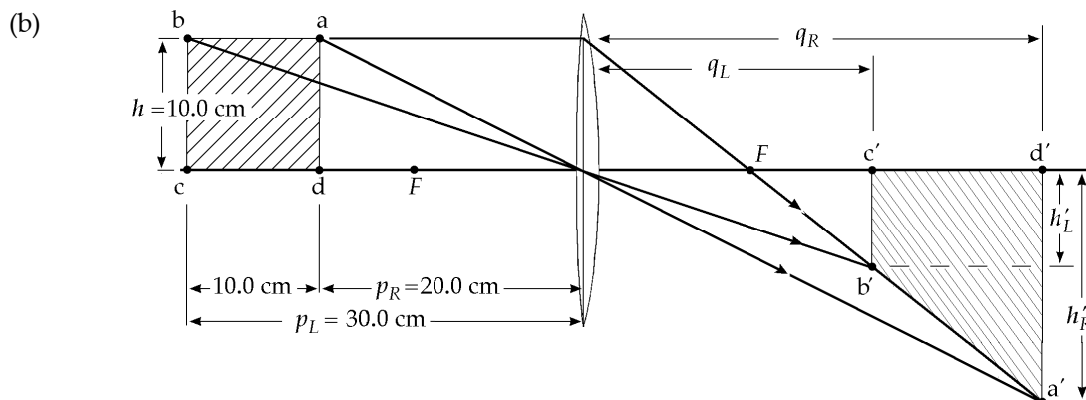
$$(b) \quad M = \frac{h'}{h} = -\frac{q}{p} \quad h' = \frac{-hq}{p} = \frac{-(5.00 \text{ mm})(7.10 \text{ cm})}{480 \text{ cm}} = -0.0740 \text{ mm}$$

$$\text{diameter of illuminated spot} = \boxed{74.0 \text{ } \mu\text{m}}$$

$$(c) \quad I = \frac{\mathcal{P}}{A} = \frac{\mathcal{P}4}{\pi d^2} = \frac{0.100 \text{ W}(4)}{\pi(74.0 \times 10^{-6} \text{ m})^2} = \boxed{2.33 \times 10^7 \text{ W/m}^2}$$

P36.40

$$(a) \quad \frac{1}{f} = (n-1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right] = (1.50-1) \left[\frac{1}{15.0 \text{ cm}} - \frac{1}{(-12.0 \text{ cm})} \right] \quad \text{or} \quad \boxed{f = 13.3 \text{ cm}}$$



The square is imaged as a trapezoid.

FIG. P36.40(b)*continued on next page*

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- (c) To find the area, first find q_R and q_L , along with the heights h'_R and h'_L , using the thin lens equation.

$$\frac{1}{p_R} + \frac{1}{q_R} = \frac{1}{f} \quad \text{becomes} \quad \frac{1}{20.0 \text{ cm}} + \frac{1}{q_R} = \frac{1}{13.3 \text{ cm}} \quad \text{or} \quad q_R = 40.0 \text{ cm}$$

$$h'_R = hM_R = h \left(\frac{-q_R}{p_R} \right) = (10.0 \text{ cm})(-2.00) = -20.0 \text{ cm}$$

$$\frac{1}{30.0 \text{ cm}} + \frac{1}{q_L} = \frac{1}{13.3 \text{ cm}} \quad \text{or} \quad q_L = 24.0 \text{ cm}$$

$$h'_L = hM_L = (10.0 \text{ cm})(-0.800) = -8.00 \text{ cm}$$

Thus, the area of the image is: $\text{Area} = |q_R - q_L| |h'_L| + \frac{1}{2} |q_R - q_L| |h'_R - h'_L| = \boxed{224 \text{ cm}^2}$.

- P36.41** (a) The image distance is:

$$q = d - p.$$

Thus, $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ becomes $\frac{1}{p} + \frac{1}{d - p} = \frac{1}{f}$.

This reduces to a quadratic equation: $p^2 + (-d)p + fd = 0$

which yields:

$$p = \frac{d \pm \sqrt{d^2 - 4fd}}{2} = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} - fd}.$$

Since $f < \frac{d}{4}$, both solutions are meaningful and the two solutions are not equal to each other. Thus, there are two distinct lens positions that form an image on the screen.

- (b) The smaller solution for p gives a larger value for q , with a real, enlarged, inverted image.
The larger solution for p describes a real, diminished, inverted image.

- P36.42** To properly focus the image of a distant object, the lens must be at a distance equal to the focal length from the film ($q_1 = 65.0 \text{ mm}$). For the closer object:

$$\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f}$$

becomes $\frac{1}{2000 \text{ mm}} + \frac{1}{q_2} = \frac{1}{65.0 \text{ mm}}$

and $q_2 = (65.0 \text{ mm}) \left(\frac{2000}{2000 - 65.0} \right)$.

The lens must be moved away from the film by a distance

$$D = q_2 - q_1 = (65.0 \text{ mm}) \left(\frac{2000}{2000 - 65.0} \right) - 65.0 \text{ mm} = \boxed{2.18 \text{ mm}}.$$

- *P36.43** In the first arrangement the lens is used as a magnifying glass, producing an upright, virtual enlarged image:

$$M = \frac{h'}{h} = \frac{120 \text{ cm}}{3.6 \text{ cm}} = 33.3 = -\frac{q}{p}$$

$$q = -33.3p = -33.3(20 \text{ cm}) = -667 \text{ cm}$$

For the lens,

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{1}{20 \text{ cm}} + \frac{1}{-667 \text{ cm}} = \frac{1}{f}$$

$$f = 20.62 \text{ cm}$$

In the second arrangement the lens is used as a projection lens to produce a real inverted enlarged image:

$$-\frac{120 \text{ cm}}{3.6 \text{ cm}} = -33.3 = -\frac{q_2}{p_2}$$

$$q_2 = 33.3p_2$$

$$\frac{1}{p_2} + \frac{1}{33.3p_2} = \frac{1}{20.62 \text{ cm}}$$

$$\frac{34.3}{33.3p_2} = \frac{1}{20.62 \text{ cm}}$$

$$p_2 = 21.24 \text{ cm}$$

$$\text{The lens was moved } 21.24 \text{ cm} - 20.0 \text{ cm} = \boxed{1.24 \text{ cm}}.$$

Section 36.5 Lens Aberrations

- P36.44** (a) The focal length of the lens is given by

$$\frac{1}{f} = (n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) = (1.53 - 1.00) \left(\frac{1}{-32.5 \text{ cm}} - \frac{1}{42.5 \text{ cm}} \right)$$

$$f = -34.7 \text{ cm}$$

Note that R_1 is negative because the center of curvature of the first surface is on the virtual image side.

When $p = \infty$

the thin lens equation gives $q = f$.

Thus, the violet image of a very distant object is formed

at $\boxed{q = -34.7 \text{ cm}}.$

The image is $\boxed{\text{virtual, upright and diminished}}.$

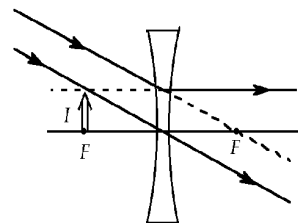
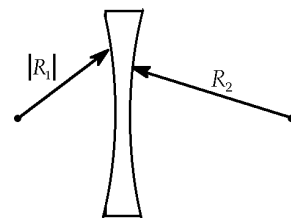


FIG. P36.44

- (b) The same ray diagram and image characteristics apply for red light.

Again, $q = f$

$$\text{and now } \frac{1}{f} = (1.51 - 1.00) \left(\frac{1}{-32.5 \text{ cm}} - \frac{1}{42.5 \text{ cm}} \right)$$

$$\text{giving } f = \boxed{-36.1 \text{ cm}}.$$

- P36.45** Ray h_1 is undeviated at the plane surface and strikes the second surface at angle of incidence given by

$$\theta_1 = \sin^{-1}\left(\frac{h_1}{R}\right) = \sin^{-1}\left(\frac{0.500 \text{ cm}}{20.0 \text{ cm}}\right) = 1.43^\circ.$$

Then, $1.00 \sin \theta_2 = 1.60 \sin \theta_1 = (1.60)\left(\frac{0.500}{20.0 \text{ cm}}\right)$

so

$$\theta_2 = 2.29^\circ.$$

The angle this emerging ray makes with the horizontal is

$$\theta_2 - \theta_1 = 0.860^\circ.$$

It crosses the axis at a point farther out by f_1

where $f_1 = \frac{h_1}{\tan(\theta_2 - \theta_1)} = \frac{0.500 \text{ cm}}{\tan(0.860^\circ)} = 33.3 \text{ cm}.$

The point of exit for this ray is distant axially from the lens vertex by

$$20.0 \text{ cm} - \sqrt{(20.0 \text{ cm})^2 - (0.500 \text{ cm})^2} = 0.00625 \text{ cm}$$

so ray h_1 crosses the axis at this distance from the vertex:

$$x_1 = 33.3 \text{ cm} - 0.00625 \text{ cm} = 33.3 \text{ cm}.$$

Now we repeat this calculation for ray h_2 :

$$\theta = \sin^{-1}\left(\frac{12.0 \text{ cm}}{20.0 \text{ cm}}\right) = 36.9^\circ$$

$$1.00 \sin \theta_2 = 1.60 \sin \theta_1 = (1.60)\left(\frac{12.00}{20.0}\right) \quad \theta_2 = 73.7^\circ$$

$$f_2 = \frac{h_2}{\tan(\theta_1 - \theta_2)} = \frac{12.0 \text{ cm}}{\tan 36.8^\circ} = 16.0 \text{ cm}$$

$$x_2 = (16.0 \text{ cm})\left(20.0 \text{ cm} - \sqrt{(20.0 \text{ cm})^2 - (12.0 \text{ cm})^2}\right) = 12.0 \text{ cm}.$$

Now $\Delta x = 33.3 \text{ cm} - 12.0 \text{ cm} = \boxed{21.3 \text{ cm}}.$

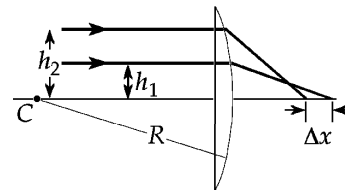


FIG. P36.45

Section 36.6 The Camera

- *P36.46** The same light intensity is received from the subject, and the same light energy on the film is required:

$$IA_1 \Delta t_1 = IA_2 \Delta t_2$$

$$\frac{\pi d_1^2}{4} \Delta t_1 = \frac{\pi d_2^2}{4} \Delta t_2$$

$$\left(\frac{f}{4}\right)^2 \left(\frac{1}{16} \text{ s}\right) = d_2^2 \left(\frac{1}{128} \text{ s}\right)$$

$$d_2 = \sqrt{\frac{128}{16} \frac{f}{4}} = \boxed{\frac{f}{1.41}}$$

Section 36.7 The Eye

P36.47 $P = \frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{\infty} - \frac{1}{0.250 \text{ m}} = -4.00 \text{ diopters} = \boxed{-4.00 \text{ diopters, a diverging lens}}$

P36.48 For starlight going through Nick's glasses, $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$

$$\frac{1}{\infty} + \frac{1}{(-0.800 \text{ m})} = \frac{1}{f} = -1.25 \text{ diopters}.$$

For a nearby object, $\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = -1.25 \text{ m}^{-1}$, so $p = \boxed{23.2 \text{ cm}}$.

P36.49 Consider an object at infinity, imaged at the person's far point:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \quad \frac{1}{\infty} + \frac{1}{q} = -4.00 \text{ m}^{-1} \quad q = -25.0 \text{ cm}.$$

The person's far point is $25.0 \text{ cm} + 2.00 \text{ cm} = 27.0 \text{ cm}$ from his eyes. For the contact lenses we want

$$\frac{1}{\infty} + \frac{1}{(-0.270 \text{ m})} = \frac{1}{f} = \boxed{-3.70 \text{ diopters}}.$$

Section 36.8 The Simple Magnifier

Section 36.9 The Compound Microscope

Section 36.10 The Telescope

P36.50 (a) From the thin lens equation: $\frac{1}{p} + \frac{1}{(-25.0 \text{ cm})} = \frac{1}{5.00 \text{ cm}}$ or $p = \boxed{4.17 \text{ cm}}$.

(b) $M = -\frac{q}{p} = 1 + \frac{25.0 \text{ cm}}{f} = 1 + \frac{25.0 \text{ cm}}{5.00 \text{ cm}} = \boxed{6.00}$

P36.51 Using Equation 36.24, $M \approx -\left(\frac{L}{f_o}\right)\left(\frac{25.0 \text{ cm}}{f_e}\right) = -\left(\frac{23.0 \text{ cm}}{0.400 \text{ cm}}\right)\left(\frac{25.0 \text{ cm}}{2.50 \text{ cm}}\right) = \boxed{-575}$.

P36.52 $M = M_o m_e = M_o \left(\frac{25.0 \text{ cm}}{f_e}\right) \Rightarrow f_e = \left(\frac{M_o}{M}\right)(25.0 \text{ cm}) = \left(\frac{-12.0}{-140}\right)(25.0 \text{ cm}) = \boxed{2.14 \text{ cm}}$

P36.53 $f_o = 20.0 \text{ m} \quad f_e = 0.0250 \text{ m}$

(a) The angular magnification produced by this telescope is: $m = -\frac{f_o}{f_e} = \boxed{-800}$.

(b) Since $m < 0$, the image is inverted.

P36.54 (a) The lensmaker's equation

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

gives

$$q = \frac{1}{1/f - 1/p} = \frac{1}{(p-f)/fp} = \frac{fp}{p-f}.$$

Then,

$$M = \frac{h'}{h} = -\frac{q}{p} = -\frac{f}{p-f}$$

gives

$$\boxed{h' = \frac{hf}{f-p}}.$$

(b) For $p \gg f$, $f-p \approx -p$. Then,

$$h' = \boxed{-\frac{hf}{p}}.$$

(c) Suppose the telescope observes the space station at the zenith:

$$h' = -\frac{hf}{p} = -\frac{(108.6 \text{ m})(4.00 \text{ m})}{407 \times 10^3 \text{ m}} = \boxed{-1.07 \text{ mm}}.$$

P36.55 (b) Call the focal length of the objective f_o and that of the eyepiece $-|f_e|$. The distance between the lenses is $f_o - |f_e|$. The objective forms a real diminished inverted image of a very distant object at $q_1 = f_o$. This image is a virtual object for the eyepiece at $p_2 = -|f_e|$.

For it $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ becomes $\frac{1}{-|f_e|} + \frac{1}{q} = \frac{1}{-|f_e|}$, $\frac{1}{q_2} = 0$

and $\boxed{q_2 = \infty}$.

(a) The user views the image as **virtual**. Letting h' represent the height of the first image, $\theta_o = \frac{h'}{f_o}$ and $\theta = \frac{h'}{|f_e|}$. The angular magnification is

$$m = \frac{\theta}{\theta_o} = \frac{h'/|f_e|}{h'/f_o} = \frac{f_o}{|f_e|}.$$

(c) Here, $f_o - |f_e| = 10.0 \text{ cm}$ and $\frac{f_o}{|f_e|} = 3.00$.

Thus, $|f_e| = \frac{f_o}{3.00}$ and $\frac{2}{3}f_o = 10.0 \text{ cm}$.

$$f_o = \boxed{15.0 \text{ cm}}$$

$$|f_e| = 5.00 \text{ cm} \quad \text{and} \quad f_e = \boxed{-5.00 \text{ cm}}$$

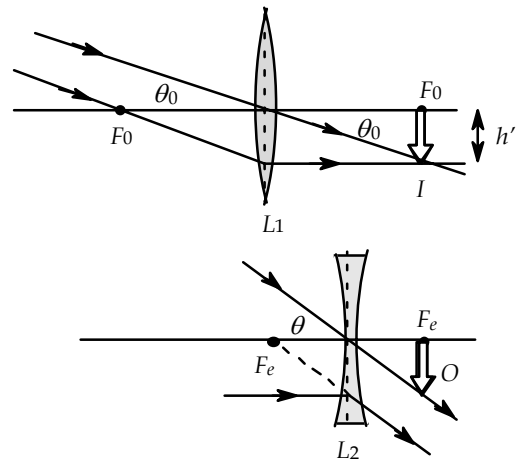


FIG. P36.55

P36.56 Let I_0 represent the intensity of the light from the nebula and θ_0 its angular diameter. With the first telescope, the image diameter h' on the film is given by $\theta_o = -\frac{h'}{f_o}$ as $h' = -\theta_o(2000 \text{ mm})$.

The light power captured by the telescope aperture is $\mathcal{P}_1 = I_0 A_1 = I_0 \left[\frac{\pi(200 \text{ mm})^2}{4} \right]$, and the light energy focused on the film during the exposure is $E_1 = \mathcal{P}_1 \Delta t_1 = I_0 \left[\frac{\pi(200 \text{ mm})^2}{4} \right] (1.50 \text{ min})$.

Likewise, the light power captured by the aperture of the second telescope is

$\mathcal{P}_2 = I_0 A_2 = I_0 \left[\frac{\pi(60.0 \text{ mm})^2}{4} \right]$ and the light energy is $E_2 = I_0 \left[\frac{\pi(60.0 \text{ mm})^2}{4} \right] \Delta t_2$. Therefore, to have the same light energy per unit area, it is necessary that

$$\frac{I_0 \left[\pi(60.0 \text{ mm})^2 / 4 \right] \Delta t_2}{\pi \left[\theta_o (900 \text{ mm})^2 / 4 \right]} = \frac{I_0 \left[\pi(200 \text{ mm})^2 / 4 \right] (1.50 \text{ min})}{\pi \left[\theta_o (2000 \text{ mm})^2 / 4 \right]}.$$

The required exposure time with the second telescope is

$$\Delta t_2 = \frac{(200 \text{ mm})^2 (900 \text{ mm})^2}{(60.0 \text{ mm})^2 (2000 \text{ mm})^2} (1.50 \text{ min}) = \boxed{3.38 \text{ min}}.$$

Additional Problems

P36.57 Only a diverging lens gives an upright diminished image. The image is virtual and

$$d = p - |q| = p + q: \quad M = -\frac{q}{p} \text{ so } q = -Mp \text{ and } d = p - Mp$$

$$p = \frac{d}{1 - M}: \quad \frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{1}{p} + \frac{1}{-Mp} = \frac{-M + 1}{-Mp} = \frac{(1 - M)^2}{-Md}$$

$$f = \frac{-Md}{(1 - M)^2} = \frac{-(0.500)(20.0 \text{ cm})}{(1 - 0.500)^2} = \boxed{-40.0 \text{ cm}}.$$

P36.58 If $M < 1$, the lens is diverging and the image is virtual.

$$M = -\frac{q}{p} \quad \text{so} \quad q = -Mp \quad \text{and}$$

$$p = \frac{d}{1 - M}: \quad \frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{1}{p} + \frac{1}{-Mp} = \frac{(-M + 1)}{-Mp} = \frac{(1 - M)^2}{-Md}$$

If $M > 1$, the lens is converging and the image is still virtual.

$$\text{Now} \quad d = -q - p.$$

We obtain in this case

$$d = p - |q| = p + q$$

$$d = p - Mp$$

$$\boxed{f = \frac{-Md}{(1 - M)^2}}.$$

$$\boxed{f = \frac{Md}{(M - 1)^2}}.$$

P36.59 (a) $\frac{1}{f} = (n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$

$$\frac{1}{-65.0 \text{ cm}} = (1.66 - 1) \left(\frac{1}{50.0 \text{ cm}} - \frac{1}{R_2} \right)$$

$$\frac{1}{R_2} = \frac{1}{50.0 \text{ cm}} + \frac{1}{42.9 \text{ cm}} \quad \text{so} \quad R_2 = \boxed{23.1 \text{ cm}}$$

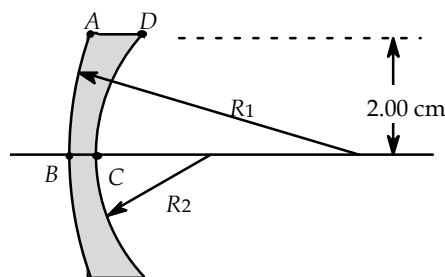


FIG. P36.59

(b) The distance along the axis from B to A is

$$R_1 - \sqrt{R_1^2 - (2.00 \text{ cm})^2} = 50.0 \text{ cm} - \sqrt{(50.0 \text{ cm})^2 - (2.00 \text{ cm})^2} = 0.0400 \text{ cm}.$$

Similarly, the axial distance from C to D is

$$23.1 \text{ cm} - \sqrt{(23.1 \text{ cm})^2 - (2.00 \text{ cm})^2} = 0.0868 \text{ cm}.$$

$$\text{Then, } AD = 0.100 \text{ cm} - 0.0400 \text{ cm} + 0.0868 \text{ cm} = \boxed{0.147 \text{ cm}}.$$

***P36.60** We consider light entering the rod. The surface of entry is convex to the object rays, so $R_1 = +4.50 \text{ cm}$

$$\frac{n_1}{p_1} + \frac{n_2}{q_1} = \frac{n_2 - n_1}{R_1} \quad \frac{1.33}{100 \text{ cm}} + \frac{1.50}{q_1} = \frac{1.50 - 1.33}{4.50 \text{ cm}}$$

$$\frac{1.50}{q_1} = 0.0378/\text{cm} - 0.0133/\text{cm} = 0.0245/\text{cm} \quad q_1 = 61.3 \text{ cm}$$

The first image is real, inverted and diminished. To find its magnification we can use two similar triangles in the ray diagram with their vertices meeting at the center of curvature:

$$\frac{h_1}{100 \text{ cm} + 4.5 \text{ cm}} = \frac{|h'_1|}{61.3 \text{ cm} - 4.5 \text{ cm}} \quad \frac{h'_1}{h_1} = -0.543.$$

Now the first image is a real object for the second surface at object distance from its vertex

$$75.0 \text{ cm} + 4.50 \text{ cm} + 4.50 \text{ cm} - 61.3 \text{ cm} = 22.7 \text{ cm}$$

$$\frac{1.50}{22.7 \text{ cm}} + \frac{1.33}{q_2} = \frac{1.33 - 1.50}{-4.50 \text{ cm}}$$

$$\frac{1.33}{q_2} = 0.0378/\text{cm} - 0.0660/\text{cm} = -0.0282/\text{cm}$$

$$q_2 = -47.1 \text{ cm}$$

(a) The final image is inside the rod, 47.1 cm from the second surface.

(b) It is virtual, inverted, and enlarged. Again by similar triangles meeting at C we have

$$\frac{h_2}{22.7 \text{ cm} - 4.5 \text{ cm}} = \frac{h'_2}{47.1 \text{ cm} - 4.5 \text{ cm}} \quad \frac{h'_2}{h_2} = 2.34.$$

$$\text{Since } h_2 = h'_1, \text{ the overall magnification is } M_1 M_2 = \frac{h'_1}{h_1} \frac{h'_2}{h_2} = \frac{h'_2}{h_1} = (-0.543)(2.34) = -1.27.$$

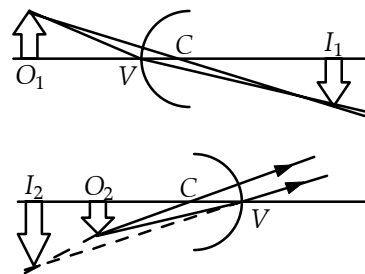


FIG. P36.60

***P36.61** (a) $\frac{1}{q_1} = \frac{1}{f_1} - \frac{1}{p_1} = \frac{1}{5 \text{ cm}} - \frac{1}{7.5 \text{ cm}} \quad \therefore q_1 = 15 \text{ cm}$

$$M_1 = -\frac{q_1}{p_1} = -\frac{15 \text{ cm}}{7.5 \text{ cm}} = -2$$

$$M = M_1 M_2 \quad \therefore 1 = (-2)M_2$$

$$\therefore M_2 = -\frac{1}{2} = -\frac{q_2}{p_2} \quad \therefore p_2 = 2q_2$$

$$\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_2} \quad \therefore \frac{1}{2q_2} + \frac{1}{q_2} = \frac{1}{10 \text{ cm}} \quad \therefore q_2 = 15 \text{ cm}, p_2 = 30 \text{ cm}$$

$$p_1 + q_1 + p_2 + q_2 = 7.5 \text{ cm} + 15 \text{ cm} + 30 \text{ cm} + 15 \text{ cm} = \boxed{67.5 \text{ cm}}$$

(b) $\frac{1}{p'_1} + \frac{1}{q'_1} = \frac{1}{f_1} = \frac{1}{5 \text{ cm}}$

Solve for q'_1 in terms of p'_1 : $q'_1 = \frac{5p'_1}{p'_1 - 5}$ (1)

$$M'_1 = -\frac{q'_1}{p'_1} = -\frac{5}{p'_1 - 5}, \text{ using (1).}$$

$$M' = M'_1 M'_2 \quad \therefore M'_2 = \frac{M'}{M'_1} = -\frac{3}{5}(p'_1 - 5) = -\frac{q'_2}{p'_2}$$

$$\therefore q'_2 = \frac{3}{5}p'_2(p'_1 - 5) \quad (2)$$

Substitute (2) into the lens equation $\frac{1}{p'_2} + \frac{1}{q'_2} = \frac{1}{f_2} = \frac{1}{10 \text{ cm}}$ and obtain p'_2 in terms of p'_1 :

$$p'_2 = \frac{10(3p'_1 - 10)}{3(p'_1 - 5)}. \quad (3)$$

Substituting (3) in (2), obtain q'_2 in terms of p'_1 :

$$q'_2 = 2(3p'_1 - 10). \quad (4)$$

Now, $p'_1 + q'_1 + p'_2 + q'_2 = \text{a constant}$.

Using (1), (3) and (4), and the value obtained in (a):

$$p'_1 + \frac{5p'_1}{p'_1 - 5} + \frac{10(3p'_1 - 10)}{3(p'_1 - 5)} + 2(3p'_1 - 10) = 67.5.$$

This reduces to the quadratic equation

$$21p_1'^2 - 322.5p_1' + 1212.5 = 0,$$

which has solutions $p'_1 = 8.784 \text{ cm}$ and 6.573 cm .

Case 1: $p'_1 = 8.784 \text{ cm}$

$$\therefore p'_1 - p_1 = 8.784 \text{ cm} - 7.5 \text{ cm} = 1.28 \text{ cm}.$$

From (4): $q'_2 = 32.7 \text{ cm}$

$$\therefore q'_2 - q_2 = 32.7 \text{ cm} - 15 \text{ cm} = 17.7 \text{ cm}.$$

Case 2: $p'_1 = 6.573 \text{ cm}$

$$\therefore p'_1 - p_1 = 6.573 \text{ cm} - 7.5 \text{ cm} = -0.927 \text{ cm}.$$

From (4): $q'_2 = 19.44 \text{ cm}$

$$\therefore q'_2 - q_2 = 19.44 \text{ cm} - 15 \text{ cm} = 4.44 \text{ cm}.$$

From these results it is concluded that:

The lenses can be displaced in two ways. The first lens can be moved 1.28 cm farther from the object and the second lens 17.7 cm toward the object. Alternatively, the first lens can be moved 0.927 cm toward the object and the second lens 4.44 cm toward the object.

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P36.62 $\frac{1}{q_1} = \frac{1}{f_1} - \frac{1}{p_1} = \frac{1}{10.0 \text{ cm}} - \frac{1}{12.5 \text{ cm}}$

so $q_1 = 50.0 \text{ cm}$ (to left of mirror).

This serves as an object for the lens (a virtual object), so

$$\frac{1}{q_2} = \frac{1}{f_2} - \frac{1}{p_2} = \frac{1}{(-16.7 \text{ cm})} - \frac{1}{(-25.0 \text{ cm})} \text{ and } q_2 = -50.3 \text{ cm},$$

meaning 50.3 cm to the right of the lens. Thus, the final image is located

25.3 cm to right of mirror.

$$M_1 = -\frac{q_1}{p_1} = -\frac{50.0 \text{ cm}}{12.5 \text{ cm}} = -4.00$$

$$M_2 = -\frac{q_2}{p_2} = -\frac{(-50.3 \text{ cm})}{(-25.0 \text{ cm})} = -2.01$$

$$M = M_1 M_2 = \boxed{8.05}$$

Thus, the final image is virtual, upright, 8.05 times the size of object, and 25.3 cm to right of the mirror.

P36.63 We first find the focal length of the mirror.

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{10.0 \text{ cm}} + \frac{1}{8.00 \text{ cm}} = \frac{9}{40.0 \text{ cm}} \quad \text{and} \quad f = 4.44 \text{ cm}.$$

Hence, if $p = 20.0 \text{ cm}$, $\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{4.44 \text{ cm}} - \frac{1}{20.0 \text{ cm}} = \frac{15.56}{88.8 \text{ cm}}.$

Thus, $q = \boxed{5.71 \text{ cm}}$, real.

***P36.64** A telescope with an eyepiece decreases the diameter of a beam of parallel rays. When light is sent through the same device in the opposite direction, the beam expands. Send the light first through the diverging lens. It will then be diverging from a virtual image found like this:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \quad \frac{1}{\infty} + \frac{1}{q} = \frac{1}{-12 \text{ cm}}$$

$$q = -12 \text{ cm}.$$

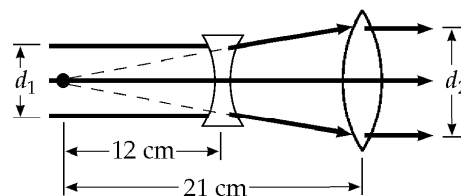


FIG. P36.64

Use this image as a real object for the converging lens, placing it at the focal point on the object side of the lens, at $p = 21 \text{ cm}$. Then

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \quad \frac{1}{21 \text{ cm}} + \frac{1}{q} = \frac{1}{21 \text{ cm}}$$

$$q = \infty.$$

The exiting rays will be parallel. The lenses must be $21.0 \text{ cm} - 12.0 \text{ cm} = 9.00 \text{ cm}$ apart.

By similar triangles, $\frac{d_2}{d_1} = \frac{21 \text{ cm}}{12 \text{ cm}} = \boxed{1.75 \text{ times}}.$

P36.65 A hemisphere is too thick to be described as a thin lens. The light is undeviated on entry into the flat face. We next consider the light's exit from the second surface, for which $R = -6.00$ cm.

The incident rays are parallel, so $p = \infty$.

Then,

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

becomes

$$0 + \frac{1}{q} = \frac{1.00 - 1.56}{-6.00 \text{ cm}}$$

and

$$\boxed{q = 10.7 \text{ cm}}.$$

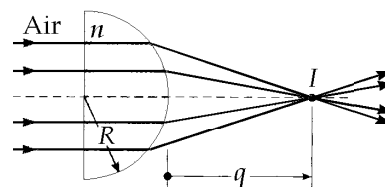


FIG. P36.65

P36.66 (a) $I = \frac{\mathcal{P}}{4\pi r^2} = \frac{4.50 \text{ W}}{4\pi(1.60 \times 10^{-2} \text{ m})^2} = \boxed{1.40 \text{ kW/m}^2}$

(b) $I = \frac{\mathcal{P}}{4\pi r^2} = \frac{4.50 \text{ W}}{4\pi(7.20 \text{ m})^2} = \boxed{6.91 \text{ mW/m}^2}$

(c) $\frac{1}{p} + \frac{1}{q} = \frac{1}{f} :$ $\frac{1}{7.20 \text{ m}} + \frac{1}{q} = \frac{1}{0.350 \text{ m}}$

so $q = 0.368 \text{ m}$

and $M = \frac{h'}{3.20 \text{ cm}} = -\frac{q}{p} = -\frac{0.368 \text{ m}}{7.20 \text{ m}}$

$$h' = \boxed{0.164 \text{ cm}}$$

(d) The lens intercepts power given by $\mathcal{P} = IA = (6.91 \times 10^{-3} \text{ W/m}^2) \left[\frac{\pi}{4} (0.150 \text{ m})^2 \right]$

and puts it all onto the image where $I = \frac{\mathcal{P}}{A} = \frac{(6.91 \times 10^{-3} \text{ W/m}^2) \left[\pi (15.0 \text{ cm})^2 / 4 \right]}{\pi (0.164 \text{ cm})^2 / 4}$

$$I = \boxed{58.1 \text{ W/m}^2}.$$

P36.67 From the thin lens equation, $q_1 = \frac{f_1 p_1}{p_1 - f_1} = \frac{(-6.00 \text{ cm})(12.0 \text{ cm})}{12.0 \text{ cm} - (-6.00 \text{ cm})} = -4.00 \text{ cm}.$

When we require that $q_2 \rightarrow \infty$, the thin lens equation becomes $p_2 = f_2.$

In this case, $p_2 = d - (-4.00 \text{ cm}).$

Therefore, $d + 4.00 \text{ cm} = f_2 = 12.0 \text{ cm}$ and $d = \boxed{8.00 \text{ cm}}.$

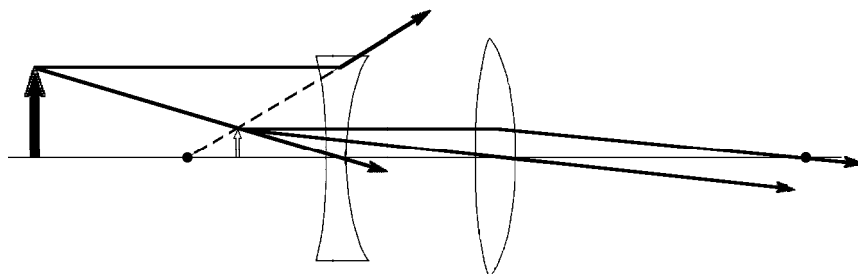


FIG. P36.67

***P36.68** The inverted real image is formed by the lens operating on light directly from the object, on light that has not reflected from the mirror.

For this we have $M = -1.50 = -\frac{q}{p}$ $q = 1.50p$

$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ $\frac{1}{p} + \frac{1}{1.50p} = \frac{1}{10 \text{ cm}} = \frac{2.50}{1.50p}$ $p = 10 \text{ cm} \left(\frac{2.5}{1.5} \right) = 16.7 \text{ cm}$

Then the object is distant from the mirror by $40.0 \text{ cm} - 16.7 \text{ cm} = 23.3 \text{ cm}.$

The second image seen by the person is formed by light that first reflects from the mirror and then goes through the lens. For it to be in the same position as the inverted image, the lens must be receiving light from an image formed by the mirror at the same location as the physical object. The formation of this image is described by

$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ $\frac{1}{23.3 \text{ cm}} + \frac{1}{23.3 \text{ cm}} = \frac{1}{f}$ $f = \boxed{11.7 \text{ cm}}.$

P36.69 For the mirror, $f = \frac{R}{2} = +1.50 \text{ m}.$ In addition, because the distance to the Sun is so much larger than any other distances, we can take $p = \infty.$

The mirror equation, $\frac{1}{p} + \frac{1}{q} = \frac{1}{f},$ then gives $q = f = \boxed{1.50 \text{ m}}.$

Now, in $M = -\frac{q}{p} = \frac{h'}{h}$

the magnification is nearly zero, but we can be more precise: $\frac{h}{p}$ is the angular diameter of the object.

Thus, the image diameter is

$h' = -\frac{hq}{p} = (-0.533^\circ) \left(\frac{\pi \text{ rad}}{180^\circ} \right) (1.50 \text{ m}) = -0.0140 \text{ m} = \boxed{-1.40 \text{ cm}}.$

P36.70 (a) For the light the mirror intercepts,

$$\mathcal{P} = I_0 A = I_0 \pi R_a^2$$

$$350 \text{ W} = (1000 \text{ W/m}^2) \pi R_a^2$$

and $R_a = \boxed{0.334 \text{ m or larger}}.$

(b) In $\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{2}{R}$

we have $p \rightarrow \infty$

so $q = \frac{R}{2}$

$$M = \frac{h'}{h} = -\frac{q}{p}$$

so $h' = -q \left(\frac{h}{p} \right) = -\left(\frac{R}{2} \right) \left[0.533^\circ \left(\frac{\pi \text{ rad}}{180^\circ} \right) \right] = -\left(\frac{R}{2} \right) (9.30 \text{ m rad})$

where $\frac{h}{p}$ is the angle the Sun subtends. The intensity at the image is

then $I = \frac{\mathcal{P}}{\pi h'^2/4} = \frac{4I_0 \pi R_a^2}{\pi h'^2} = \frac{4I_0 R_a^2}{(R/2)^2 (9.30 \times 10^{-3} \text{ rad})^2}$

$$120 \times 10^3 \text{ W/m}^2 = \frac{16(1000 \text{ W/m}^2) R_a^2}{R^2 (9.30 \times 10^{-3} \text{ rad})^2}$$

so $\boxed{\frac{R_a}{R} = 0.0255 \text{ or larger}}.$

P36.71 In the original situation,

$$p_1 + q_1 = 1.50 \text{ m}.$$

In the final situation,

$$p_2 = p_1 + 0.900 \text{ m}$$

and

$$q_2 = q_1 - 0.900 \text{ m} = 0.600 \text{ m} - p_1.$$

Our lens equation is

$$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f} = \frac{1}{p_2} + \frac{1}{q_2}.$$

Substituting, we have

$$\frac{1}{p_1} + \frac{1}{1.50 \text{ m} - p_1} = \frac{1}{p_1 + 0.900} + \frac{1}{0.600 - p_1}.$$

Adding the fractions,

$$\frac{1.50 \text{ m} - p_1 + p_1}{p_1(1.50 \text{ m} - p_1)} = \frac{0.600 - p_1 + p_1 + 0.900}{(p_1 + 0.900)(0.600 - p_1)}.$$

Simplified, this becomes

$$p_1(1.50 \text{ m} - p_1) = (p_1 + 0.900)(0.600 - p_1).$$

(a) Thus,

$$p_1 = \frac{0.540}{1.80} \text{ m} = \boxed{0.300 \text{ m}}$$

$$p_2 = p_1 + 0.900 = \boxed{1.20 \text{ m}}$$

(b) $\frac{1}{f} = \frac{1}{0.300 \text{ m}} + \frac{1}{1.50 \text{ m} - 0.300 \text{ m}}$

and $f = \boxed{0.240 \text{ m}}$

(c) The second image is $\boxed{\text{real, inverted, and diminished}}$

with $M = -\frac{q_2}{p_2} = \boxed{-0.250}.$

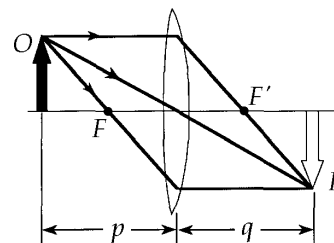


FIG. P36.71

P36.72 (a) The lens makers' equation, $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$
 becomes: $\frac{1}{5.00 \text{ cm}} = (n-1)\left[\frac{1}{9.00 \text{ cm}} - \frac{1}{(-11.0 \text{ cm})}\right]$ giving $n = \boxed{1.99}$.

(b) As the light passes through the lens for the first time, the thin lens equation

$$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f}$$

becomes: $\frac{1}{8.00 \text{ cm}} + \frac{1}{q_1} = \frac{1}{5.00 \text{ cm}}$

or $q_1 = 13.3 \text{ cm}$, and $M_1 = -\frac{q_1}{p_1} = -\frac{13.3 \text{ cm}}{8.00 \text{ cm}} = -1.67$.

This image becomes the object for the concave mirror with:

$$p_m = 20.0 \text{ cm} - q_1 = 20.0 \text{ cm} - 13.3 \text{ cm} = 6.67 \text{ cm}$$

and $f = \frac{R}{2} = +4.00 \text{ cm}$.

The mirror equation becomes: $\frac{1}{6.67 \text{ cm}} + \frac{1}{q_m} = \frac{1}{4.00 \text{ cm}}$

giving $q_m = 10.0 \text{ cm}$

and $M_2 = -\frac{q_m}{p_m} = -\frac{10.0 \text{ cm}}{6.67 \text{ cm}} = -1.50$.

The image formed by the mirror serves as a real object for the lens on the second pass of the light through the lens with:

$$p_3 = 20.0 \text{ cm} - q_m = +10.0 \text{ cm}.$$

The thin lens equation yields: $\frac{1}{10.0 \text{ cm}} + \frac{1}{q_3} = \frac{1}{5.00 \text{ cm}}$

or $q_3 = 10.0 \text{ cm}$

and $M_3 = -\frac{q_3}{p_3} = -\frac{10.0 \text{ cm}}{10.0 \text{ cm}} = -1.00$.

The final image is a real image located $\boxed{10.0 \text{ cm to the left of the lens}}$.

The overall magnification is $M_{\text{total}} = M_1 M_2 M_3 = \boxed{-2.50}$.

(c) Since the total magnification is negative, this final image is $\boxed{\text{inverted}}$.

P36.73 For the objective: $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ becomes $\frac{1}{3.40 \text{ mm}} + \frac{1}{q} = \frac{1}{3.00 \text{ mm}}$ so $q = 25.5 \text{ mm}$.

The objective produces magnification $M_1 = -\frac{q}{p} = -\frac{25.5 \text{ mm}}{3.40 \text{ mm}} = -7.50$.

For the eyepiece as a simple magnifier, $m_e = \frac{25.0 \text{ cm}}{f} = \frac{25.0 \text{ cm}}{2.50 \text{ cm}} = 10.0$

and overall $M = M_1 m_e = \boxed{-75.0}$.

- P36.74** (a) Start with the second lens: This lens must form a virtual image located 19.0 cm to the left of it (i.e., $q_2 = -19.0$ cm). The required object distance for this lens is then

$$p_2 = \frac{q_2 f_2}{q_2 - f_2} = \frac{(-19.0 \text{ cm})(20.0 \text{ cm})}{-19.0 \text{ cm} - 20.0 \text{ cm}} = \frac{380 \text{ cm}}{39.0}.$$

The image formed by the first lens serves as the object for the second lens. Therefore, the image distance for the first lens is

$$q_1 = 50.0 \text{ cm} - p_2 = 50.0 \text{ cm} - \frac{380 \text{ cm}}{39.0} = \frac{1570 \text{ cm}}{39.0}.$$

The distance the original object must be located to the left of the first lens is then given by

$$\frac{1}{p_1} = \frac{1}{f_1} - \frac{1}{q_1} = \frac{1}{10.0 \text{ cm}} - \frac{39.0}{1570 \text{ cm}} = \frac{157 - 39.0}{1570 \text{ cm}} = \frac{118}{1570 \text{ cm}} \quad \text{or} \quad p_1 = \frac{1570 \text{ cm}}{118} = \boxed{13.3 \text{ cm}}.$$

$$(b) \quad M = M_1 M_2 = \left(-\frac{q_1}{p_1} \right) \left(-\frac{q_2}{p_2} \right) = \left[\left(\frac{1570 \text{ cm}}{39.0} \right) \left(\frac{118}{1570 \text{ cm}} \right) \right] \left[\frac{(-19.0 \text{ cm})(39.0)}{380 \text{ cm}} \right] = \boxed{-5.90}$$

- (c) Since $M < 0$, the final image is inverted.

P36.75 (a) $P = \frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{(0.0224 \text{ m})} + \frac{1}{\infty} = \boxed{44.6 \text{ diopters}}$

(b) $P = \frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{(0.330 \text{ m})} + \frac{1}{\infty} = \boxed{3.03 \text{ diopters}}$

- P36.76** The object is located at the focal point of the upper mirror. Thus, the upper mirror creates an image at infinity (i.e., parallel rays leave this mirror).

The lower mirror focuses these parallel rays at its focal point, located at the hole in the upper mirror.

Thus, the image is real, inverted, and actual size.

For the upper mirror:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}: \quad \frac{1}{7.50 \text{ cm}} + \frac{1}{q_1} = \frac{1}{7.50 \text{ cm}} \quad q_1 = \infty.$$

For the lower mirror:

$$\frac{1}{\infty} + \frac{1}{q_2} = \frac{1}{7.50 \text{ cm}} \quad q_2 = 7.50 \text{ cm}.$$

Light directed into the hole in the upper mirror reflects as shown, to behave as if it were reflecting from the hole.

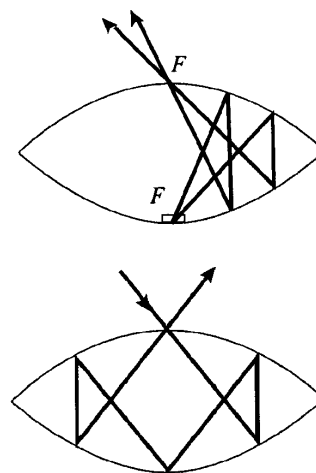


FIG. P36.76

- P36.77 (a) For lens one, as shown in the first figure,

$$\frac{1}{40.0 \text{ cm}} + \frac{1}{q_1} = \frac{1}{30.0 \text{ cm}}$$

$$q_1 = 120 \text{ cm}$$

$$M_1 = -\frac{q_1}{p_1} = -\frac{120 \text{ cm}}{40.0 \text{ cm}} = -3.00$$

This real image $I_1 = O_2$ is a virtual object for the second lens. That is, it is *behind* the lens, as shown in the second figure. The object distance is

$$p_2 = 110 \text{ cm} - 120 \text{ cm} = -10.0 \text{ cm}$$

$$\frac{1}{-10.0 \text{ cm}} + \frac{1}{q_2} = \frac{1}{-20.0 \text{ cm}}:$$

$$q_2 = \boxed{20.0 \text{ cm}}$$

$$M_2 = -\frac{q_2}{p_2} = -\frac{20.0 \text{ cm}}{(-10.0 \text{ cm})} = +2.00$$

$$M_{\text{overall}} = M_1 M_2 = \boxed{-6.00}$$

- (b) $M_{\text{overall}} < 0$, so final image is

inverted.

- (c) If lens two is a converging lens (third figure):

$$\frac{1}{-10.0 \text{ cm}} + \frac{1}{q_2} = \frac{1}{20.0 \text{ cm}}$$

$$q_2 = \boxed{6.67 \text{ cm}}$$

$$M_2 = -\frac{6.67 \text{ cm}}{(-10.0 \text{ cm})} = +0.667$$

$$M_{\text{overall}} = M_1 M_2 = \boxed{-2.00}$$

Again, $M_{\text{overall}} < 0$ and the final image is inverted.

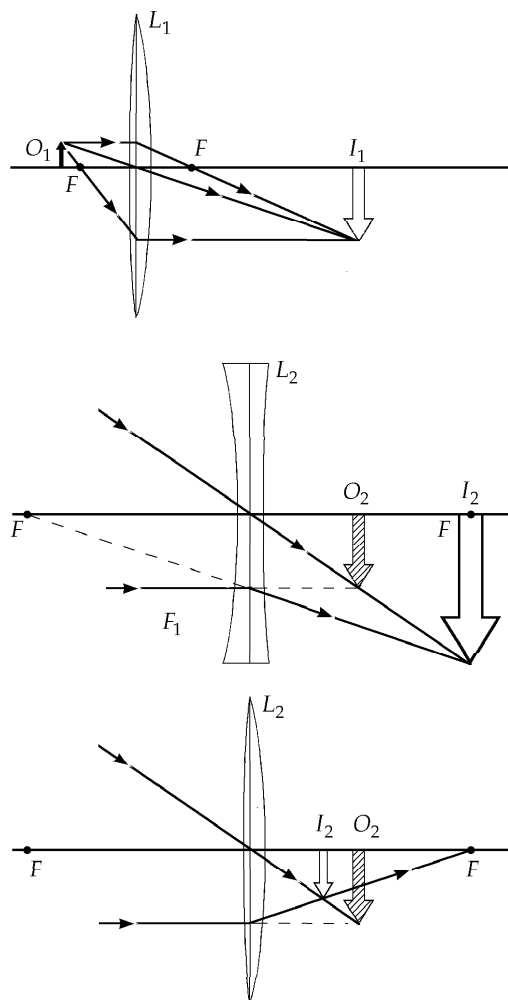


FIG. P36.77

***P36.78** The first lens has focal length described by

$$\frac{1}{f_1} = (n_1 - 1) \left(\frac{1}{R_{11}} - \frac{1}{R_{12}} \right) = (n_1 - 1) \left(\frac{1}{\infty} - \frac{1}{R} \right) = -\frac{n_1 - 1}{R}.$$

For the second lens

$$\frac{1}{f_2} = (n_2 - 1) \left(\frac{1}{R_{21}} - \frac{1}{R_{22}} \right) = (n_2 - 1) \left(\frac{1}{+R} - \frac{1}{-R} \right) = +\frac{2(n_2 - 1)}{R}.$$

Let an object be placed at any distance p_1 large compared to the thickness of the doublet. The first lens forms an image according to

$$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f_1}$$

$$\frac{1}{q_1} = \frac{-n_1 + 1}{R} - \frac{1}{p_1}.$$

This virtual ($q_1 < 0$) image is a real object for the second lens at distance $p_2 = -q_1$. For the second lens

$$\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_2}$$

$$\frac{1}{q_2} = \frac{2n_2 - 2}{R} - \frac{1}{p_2} = \frac{2n_2 - 2}{R} + \frac{1}{q_1} = \frac{2n_2 - 2}{R} + \frac{-n_1 + 1}{R} - \frac{1}{p_1} = \frac{2n_2 - n_1 - 1}{R} - \frac{1}{p_1}.$$

$$\text{Then } \frac{1}{p_1} + \frac{1}{q_2} = \frac{2n_2 - n_1 - 1}{R} \text{ so the doublet behaves like a single lens with } \frac{1}{f} = \frac{2n_2 - n_1 - 1}{R}.$$

ANSWERS TO EVEN PROBLEMS

P36.2	4.58 m	P36.22	4.82 cm
P36.4	see the solution	P36.24	see the solution; real, inverted, diminished
P36.6	(a) $p_1 + h$; (b) virtual; (c) upright; (d) +1; (e) No	P36.26	2.00
P36.8	at $q = -0.267$ m virtual upright and diminished with $M = 0.0267$	P36.28	20.0 cm
P36.10	at 3.33 m from the deepest point of the niche	P36.30	(a) $q = 40.0$ cm real, inverted, actual size $M = -1.00$; (b) $q = \infty$, $M = \infty$, no image is formed; (c) $q = -20.0$ cm upright, virtual, enlarged $M = +2.00$
P36.12	30.0 cm	P36.32	(a) 6.40 cm; (b) -0.250 ; (c) converging
P36.14	(a) 160 mm; (b) $R = -267$ mm	P36.34	(a) 3.40, upright; (b) see the solution
P36.16	(a) convex; (b) At the 30.0 cm mark; (c) -20.0 cm	P36.36	(a) 39.0 mm; (b) 39.5 mm
P36.18	(a) 15.0 cm; (b) 60.0 cm	P36.38	1.16 mm/s toward the lens
P36.20	(a) see the solution; (b) at 0.639 s and at 0.782 s		

380 *Image Formation*

P36.40 (a) 13.3 cm;
(b) see the solution; a trapezoid;
(c) 224 cm^2

P36.42 2.18 mm away from the film

P36.44 (a) at $q = -34.7 \text{ cm}$
virtual, upright and diminished;
(b) at $q = -36.1 \text{ cm}$
virtual, upright and diminished

P36.46 $\frac{f}{1.41}$

P36.48 23.2 cm

P36.50 (a) at 4.17 cm; (b) 6.00

P36.52 2.14 cm

P36.54 (a) see the solution; (b) $h' = -\frac{hf}{p}$;
(c) -1.07 mm

P36.56 3.38 min

P36.58 if $M < 1$, $f = \frac{-Md}{(1-M)^2}$,
if $M > 1$, $f = \frac{Md}{(M-1)^2}$

P36.60 (a) inside the rod, 47.1 cm from the
second surface;
(b) virtual, inverted, and enlarged

P36.62 25.3 cm to right of mirror,
virtual, upright, enlarged 8.05 times

P36.64 place the lenses 9.00 cm apart and let light
pass through the diverging lens first.
1.75 times

P36.66 (a) 1.40 kW/m^2 ; (b) 6.91 mW/m^2 ;
(c) 0.164 cm ; (d) 58.1 W/m^2

P36.68 11.7 cm

P36.70 (a) 0.334 m or larger;
(b) $\frac{R_a}{R} = 0.0255$ or larger

P36.72 (a) 1.99;
(b) 10.0 cm to the left of the lens; -2.50 ;
(c) inverted

P36.74 (a) 13.3 cm; (b) -5.90 ; (c) inverted

P36.76 see the solution;
real, inverted, and actual size

P36.78 see the solution