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Kazuaki Sakoda

Optical Properties of Photonic Crystals

Second Edition

With 107 Figures and 29 Tables



Springer

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To Fumi and Midori

Preface

During the three years since the first edition of this book was published, the study of photonic crystals has made steady progress. The propagation of electromagnetic waves in various types of photonic crystal has been investigated thoroughly by many researchers so that the basis for practical applications is now firmly established. On the other hand, the quantum-optical aspects of photonic crystals remain less-well investigated although they may bring about completely new quantum-electrodynamical effects, due to the anomalous photon density of states realized in such crystals.

One of the most peculiar and impressive examples of such phenomena is superfluorescence, which was first studied by Prof. Sajeev John and Prof. Tran Quang. The temporal profile of superfluorescent pulses expected for photonic crystals is completely different from that in free space or uniform materials. Moreover, fractional relaxation of population inversion, which originates from the Rabi splitting of electronic excited states in the presence of photonic bandgaps, was predicted.

In the second edition of this book, I have added a new chapter giving a detailed description of this interesting phenomenon. By way of introduction, I explain the standard mean-field theory of superfluorescence in uniform materials and the pseudo-spin density operator as an analytical tool. Thereafter I follow the mean-field theory presented by the two above-mentioned researchers. I also describe some numerical calculations on superfluorescence assuming a quite realistic specimen, work originally performed by Professor Joseph W. Haus and myself. I hope that the reader will enjoy this taste of the emerging new quantum optics.

Tsukuba
August 2004

Kazuaki Sakoda

Preface to the First Edition

The interaction between the radiation field and matter is the most fundamental source of dynamics in nature. It brings about the absorption and emission of photons, elastic and inelastic light scattering, the radiative lifetime of electronic excited states, and so on. The huge amount of energy carried from the sun by photons is the source of all activities of creatures on the earth. The absorption of photons by chlorophylls and the successive electronic excitation initiate a series of chemical reactions that are known as photosynthesis, which support all life on the earth. Radiative energy is also the main source of all meteorological phenomena.

The fundamentals of the radiation field and its interaction with matter were clarified by classical electromagnetism and quantum electrodynamics. These theories, we believe, explain all electromagnetic phenomena. They not only provide a firm basis for contemporary physics but also generate a vast range of technological applications. These include television, radar, optical and microwave telecommunications, lasers, light-emitting diodes, solar cells, etc.

Now, the interaction between the radiation field and matter is so fundamental that it may seem universal and invariant. But in fact it is *controllable*. This discovery has been the great motivating force of intensive investigations in optical physics during the last three decades. In this book, I will show how it is controlled using *photonic crystals*, a remarkable invention realized by the combination of optical physics and contemporary microfabrication techniques. I will also show how the controlled radiation field alters the optical properties of atoms and molecules embedded in the photonic crystals and what kinds of new phenomena and new physics are expected to manifest themselves.

This book was written to serve as a comprehensive textbook covering the optical properties of photonic crystals. It deals not only with the properties of the radiation field inside the photonic crystals but also their peculiar optical response to external fields. Only an elementary knowledge of electromagnetism, quantum mechanics, solid-state physics, and complex analysis is required of the reader. Therefore, undergraduate students in physics, applied physics, optics, electronics, and electrical engineering in the final year should be able to read this book without difficulty. Since the main recent devel-

opments, such as the enhancement of stimulated emission, second harmonic generation, and quadrature-phase squeezing, are also treated in a detailed and understandable manner, this book also provides important ideas for graduate students and researchers in this field.

I would like to thank Professor Eiichi Hanamura and Professor Kuon Inoue who gave me a wonderful introduction to this exciting field. I would also like to thank Professor Kazuo Ohtaka and Professor Joseph W. Haus who gave me many suggestions on important problems. I am grateful to Professor Sajeev John for giving me the opportunity to visit his laboratory in Toronto University in the summer of 1998. During that period, I learned much about the quantum optics of photonic crystals. I am also grateful to Dr. Kurt Busch who helped me to confirm that the group theory worked well for the fcc structure. I acknowledge Professor Takao Koda for his continuous encouragement since I was an undergraduate student of Tokyo University. I deeply acknowledge Professor Toshimitsu Asakura and Dr. Claus Ascheron for giving me the opportunity to write this book. I am grateful to many of my graduate students for their efforts in numerical calculations. In particular, Ms. Hitomi Shiroma-Hirata, Ms. Noriko Kawai, and Mr. Takunori Ito did many of the calculations that are shown in this book. I am also grateful to Dr. Tetsuyuki Ochiai who made the main contribution to the study of photonic crystal slabs presented in Chapter 8.

Sapporo
February 2001

Kazuaki Sakoda

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