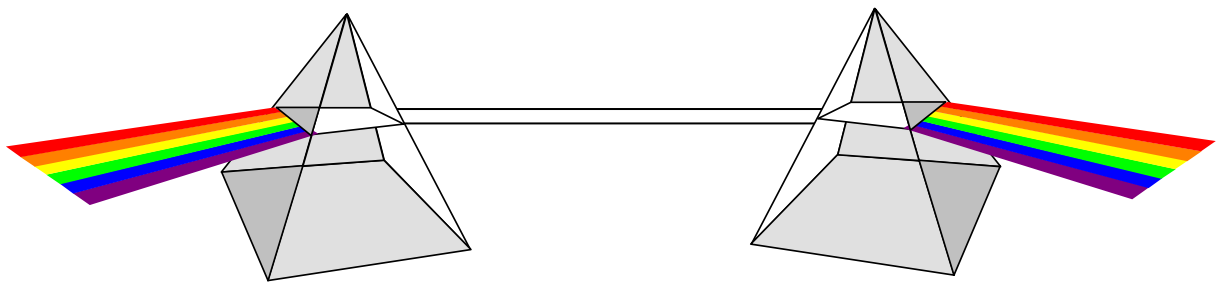


Simulation of Optical Communication Systems

Linear Impairments of Signal Propagation



IMPRESSUM

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Wireline Communications Group
Institute for Communications Engineering (LNT)
Technische Universität München
Theresienstraße 90
D-80333 München
Germany
Phone: +49 89 23472 (Dipl.-Ing. Stephan Hellerbrand)

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Theory

In the previous Lab sessions, the transmitters and the receivers used in optical communication systems have been introduced. Different modulation types have been described as well as ways to generate and detect signals using these modulation schemes. However, the most important component of an optical communication system has not been treated, yet. This component is the transmission medium: Optical Fiber. In this lab session, the linear propagation of signals within an optical fiber will be explained. In addition to that, the transmission impairments that arise from the propagation characteristics will be shown. Only single mode fibers¹ will be treated as these are the standard transmission medium in high-speed transmission links. For more information about the topics of this chapter please refer to [1]-[4].

1. Linear Propagation Equation

An optical fiber can be viewed as a linear system when either the nonlinear susceptibility is negligible or when the signal power is small enough. For the description of the transmission characteristics assume an optical signal with electrical field $\underline{E}(z, t)$.

This field can be split into its spectral components using the Fourier-Transform

$$\underline{E}(z=0, \omega) = \int_{-\infty}^{+\infty} \underline{E}(z=0, t) e^{-j\omega t} dt. \quad (1)$$

The linear propagation of the spectral components of the signal can be described by the transfer function

$$H(z, \omega) = e^{-\frac{\alpha(\omega)}{2} \cdot z} \cdot e^{-j\beta(\omega) \cdot z}. \quad (2)$$

The first term corresponds to the attenuation of the signal power and the second term incorporates the propagation constant $\beta(\omega)$. The resulting signal at the fiber length z in the frequency domain therefore is

$$\begin{aligned} \underline{E}(z, f) &= \underline{E}(z=0, f) H(z, \omega) \\ &= \underline{E}(z=0, f) e^{-\frac{\alpha(\omega)}{2} \cdot z} \cdot e^{-j\beta(\omega) \cdot z} \end{aligned} \quad (3)$$

The corresponding signal in the time domain is found using the inverse Fourier-Transform,

$$\underline{E}(z, t) = \int_{-\infty}^{+\infty} \underline{E}(z, \omega) e^{+j\omega t} d\omega. \quad (4)$$

¹ The singlemode character of a fiber actually depends on the frequency of the electrical field. The term singlemode-fiber implies that the fiber is designed for being operated at frequencies where the singlemode condition is fulfilled.

1.1. Signal Attenuation

In Eq. 3, the attenuation of the optical signal was described using the parameter $\alpha(\omega)$. This single parameter incorporates all the absorption effects that occur in an optical fiber. The main absorption mechanisms are **Infrared-Absorption**, **Ultraviolet-Absorption** and **Rayleigh-Scattering**. All these mechanisms are frequency dependent as highlighted in Figure 1.

Infrared Absorption is caused by the interaction of the light with the molecules of the fiber material. Infrared light excites oscillations of the silica molecules. Thereby some of the light power is lost. Similarly, for Ultraviolet-Absorption, electron oscillations in the fiber material are excited by the light and cause a loss of power.

The Rayleigh-Scattering stems from another process. Since the structure of the fiber material is irregular (fused silica) there are slight local variation of the physical density of the fiber. Due to these density variations, a small amount of the propagating light is scattered into random directions. The scattered light portions constitute a loss to the original propagating field. In the most relevant frequency window 400-1650 nm this is the dominant loss mechanism.

In addition to these types of loss, there are other peaks of the total attenuation that are caused by resonances of certain impurities in the fiber material, e.g. by the oscillation of OH⁻ ions, which are left in the fiber material after manufacturing (the so called **water-peak** at 1380nm).

Although the attenuation varies with the light frequency, for a single modulated channel the variation is very small and therefore the attenuation is taken to be constant within the channel. The power of the optical signal is then attenuated according to

$$P(z,t) = P(z=0,t) \cdot e^{-\alpha(\omega) \cdot z} \quad (5)$$

Question

Explain the factor $\frac{1}{2}$ in the exponent of the first term in Eq.3.

The factor $\frac{1}{2}$ stems from the fact that amplitudes are described by this equation.

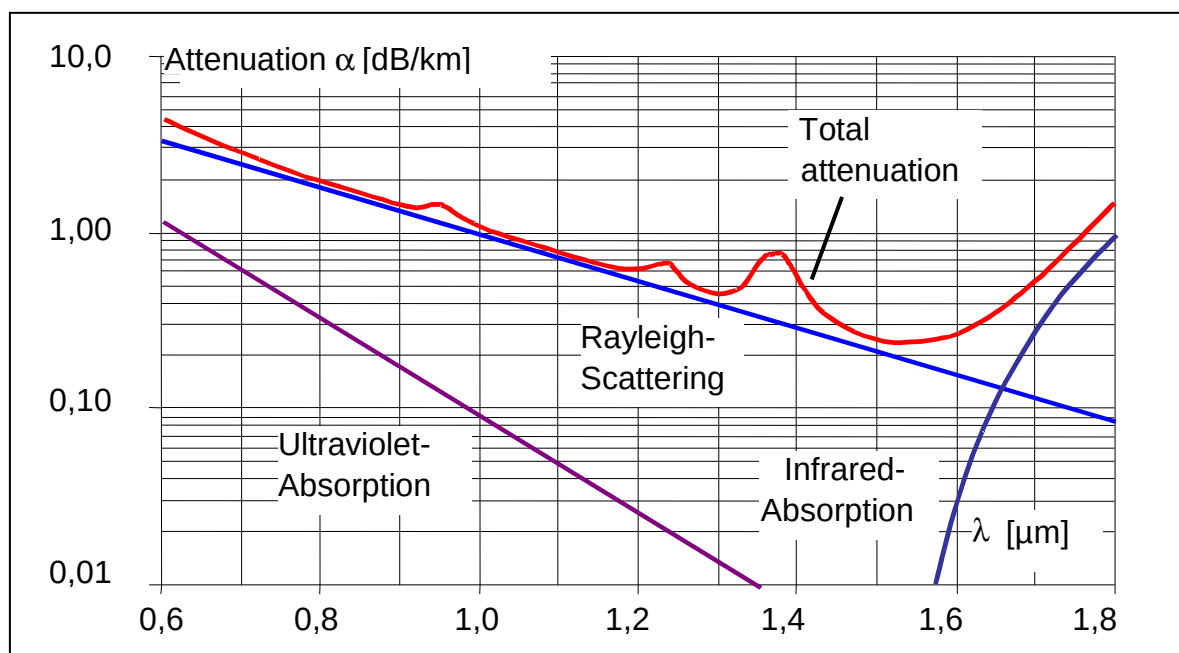


Figure 1 – Frequency dependence of the main fiber attenuation mechanisms and total fiber attenuation

In addition to the fiber induced loss, there are also various other sources of loss, e.g. connectors. Usually, these are modeled by a constant factor.

1.2. Chromatic Dispersion

The second term of Eq. 3 expresses the propagation of the individual frequency components. The signal propagation constant $\beta(\omega)$ also varies with the frequency². In order to evaluate the transfer function, it has proven useful to develop $\beta(\omega)$ into a Taylor-Series around the carrier frequency,

$$\begin{aligned}\beta(\omega) &= \beta(\omega_c) + \left. \frac{\partial \beta(\omega)}{\partial \omega} \right|_{\omega_c} (\omega - \omega_c) + \frac{1}{2} \left. \frac{\partial^2 \beta(\omega)}{\partial \omega^2} \right|_{\omega_c} (\omega - \omega_c)^2 + \frac{1}{6} \left. \frac{\partial^3 \beta(\omega)}{\partial \omega^3} \right|_{\omega_c} (\omega - \omega_c)^3 + \dots \\ &= \beta_0 + \beta_1 (\omega - \omega_c) + \frac{\beta_2}{2} (\omega - \omega_c)^2 + \frac{\beta_3}{6} (\omega - \omega_c)^3 + \dots\end{aligned}\quad (6)$$

If the derivatives of the propagation constant with order $N \geq 2$ are negligible, then the resulting time domain signal is

$$E(z, t) = E(z=0, t - \tau_0) \cdot e^{-\frac{\alpha(\omega_c)}{2} \cdot z} \cdot e^{j\phi_0} \quad (7)$$

where

$$\phi_0 = (\beta_1 \omega_c - \beta_0)z \quad (8)$$

is a constant phase delay and the time shift of the input signal is

$$\tau_0 = \frac{\beta_1 z}{2\pi}. \quad (9)$$

Hence, there is no change in the shape of the input waveform and consequently no intersymbol-interference between neighboring pulses during data transmission.

However, if the terms of order $N \geq 2$ cannot be neglected, there will be a change in the shape of the output signal compared to the fiber input. These terms impose a frequency dependence on the group delay τ as opposed to the case of constant group delay τ_0 when those higher order terms are 0. The coefficients of the Taylor-Series can be determined from characteristic parameters of an optical fiber. The relationships between these parameters and the coefficients are listed in Figure 2 up to 3rd order. Usually, only the coefficients up to third order are considered. In that case, the linear transfer function of the fiber in the frequency domain can be written

$$H(z, \omega) = e^{-\frac{\alpha(\omega_c)}{2} \cdot z} \cdot e^{-j \left[\beta_0 + \beta_1 (\omega - \omega_c) + \frac{\beta_2}{2} (\omega - \omega_c)^2 + \frac{\beta_3}{6} (\omega - \omega_c)^3 \right] \cdot z}. \quad (10)$$

The signal in an optical communication system is a band pass signal, i.e. the bandwidth much smaller than the carrier frequency. Instead of using the absolute frequency ω it is therefore also possible to use the slowly varying envelope $\hat{E}(z, t)$ of the signal

$$\underline{E}(z, t) = \hat{E}(z, t) e^{j(\omega_c \cdot t - \beta_0 z)} \quad (11)$$

² The author doubts the usefulness of the term „propagation constant“ in this case ☺. However, this is the term that is used in the literature.

to express the propagation of the linear signal. This corresponds to the treatment of signal propagation in the complex baseband. Note, that the constant phase is excluded from $\hat{E}(z,t)$. In addition to that, the normalized group delay, which is included by β_1 is not necessary for description of the linear distortion of the propagating waveform as it leads to a constant time delay that only depends on the fiber z-position. This fact can be included by using a time frame that moves at the same speed as the signal, i.e. with the definition

$$t' = t - \frac{\beta_1 z}{2\pi}. \quad (12)$$

Using the baseband signal and the normalized time, the transfer function can be expressed as

$$H(z, \Delta\omega) = e^{-\frac{\alpha(\omega_c)}{2} \cdot z} \cdot e^{-j(\frac{\beta_2}{2} \Delta\omega^2 + \frac{\beta_3}{6} \Delta\omega^3) \cdot z}, \quad (13)$$

where $\Delta\omega = \omega - \omega_c$.

The signal transmission is then described by

$$\hat{E}(z, t') = \int_{-\infty}^{+\infty} \underline{E}(z=0, \Delta\omega) \cdot e^{-\frac{\alpha(\omega_c)}{2} z - j(\frac{\beta_2}{2} \Delta\omega^2 + \frac{\beta_3}{6} \Delta\omega^3) \cdot z} e^{+j\Delta\omega \cdot t'} d\Delta\omega. \quad (14)$$

Therefore, the only parameters that are necessary to describe the linear signal distortion are the parameters β_2 and β_3 or equivalently the fiber chromatic dispersion D and the dispersion slope S. In the *OptiSystem* simulation software, you can use either of these parameter pairs to adjust the dispersion characteristics of a fiber simulation component.

modelling parameters	corresponding measured quantities	units	
$\beta_1 = \frac{\partial}{\partial \omega} \beta(\omega) = \bar{\tau}_{Gr}(\omega);$	$\bar{\tau}_{Gr}(\lambda)$	$\frac{ns}{km}$	normalised group delay
$\beta_2 = \frac{\partial^2}{\partial \omega^2} \beta(\omega) = -D(\lambda) \cdot \frac{\lambda^2}{2\pi \cdot c}$	$D(\lambda) = \frac{d}{d\lambda} \bar{\tau}_{Gr}(\lambda)$	$\frac{ps}{km \cdot nm}$	$D(\lambda)$: chromatic dispersion
$\beta_3 = \frac{\partial^3}{\partial \omega^3} \beta(\omega) = \left[\frac{D(\lambda)}{\lambda} + S(\lambda) \right] \frac{\lambda^4}{(2\pi \cdot c)^2}$	$S(\lambda) = \frac{d}{d\lambda} D(\lambda)$	$\frac{ps}{km \cdot nm^2}$	$S(\lambda)$: dispersion slope

Figure 2 - Characteristic dispersion fiber parameters and relationship to Taylor-series coefficients of the propagation constant

Especially the parameter D of the fiber allows for an intuitive interpretation of the effect of dispersion. For $D = 1 ps/(km \cdot nm)$ and a signal that occupies a spectrum of width 2nm this implies that the time delay between the highest frequency component and the lowest frequency component increases by 2 ps for each kilometer. You can look at Figure 3 to find out about the chromatic dispersion D of several commercially available fibers.

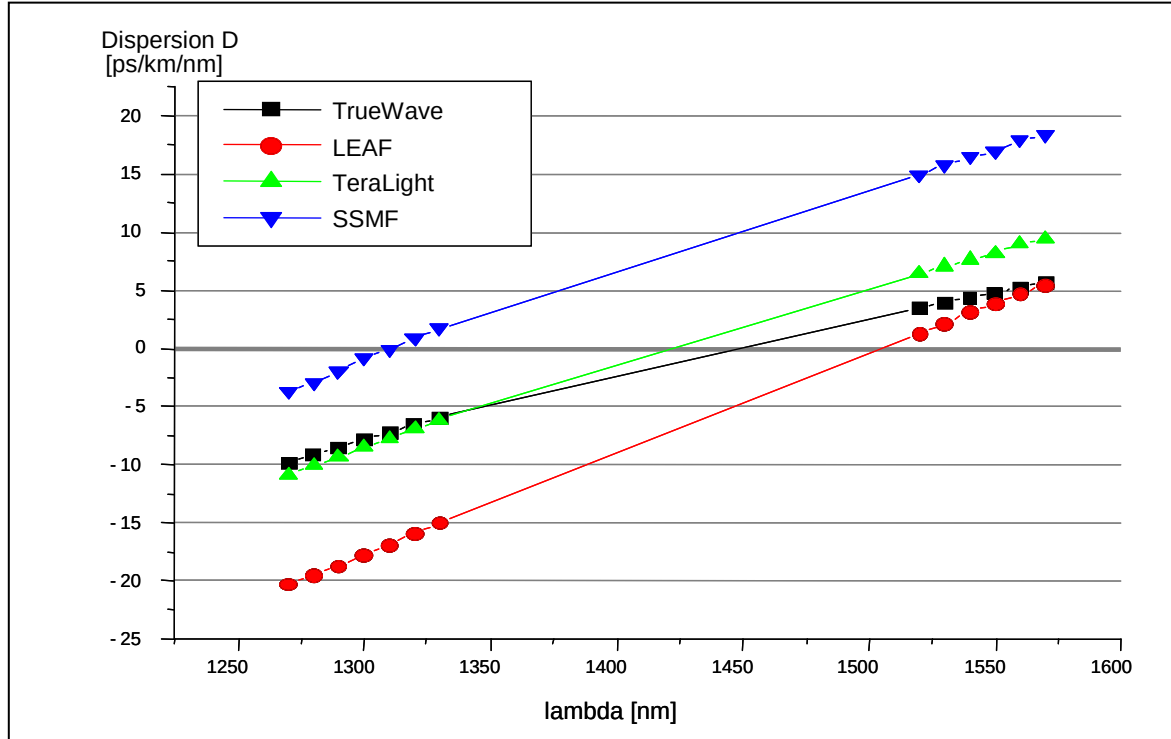


Figure 3 - Chromatic dispersion D for selected commercially available fibers

Question

Assume you are transmitting a 10 Gbit/s signal over a Standard Singlemode Fiber (SSMF) at 1530 nm. What is the delay of the highest and lowest frequency component after 100 km. Assume that the bandwidth is twice the bit-rate and that the dispersion D for the entire spectrum of the signal is constant. Take the value of D as shown in Figure 3.

First, convert the wavelength into a frequency: 1530 nm correspond to 196078 GHz. With the assumption that the bandwidth is about twice the bit rate you have to add +10 GHz and -10 GHz to this frequency. Convert these frequencies back into wavelengths and you obtain 1529.93 nm and 1530.08 nm. Using the difference in wavelength and the value $D=15\text{ps/nm/km}$, you can find the maximum delay between the highest and the lowest frequency component of the pulse after 100 km is

$$D \cdot \Delta\lambda \cdot L = 15 \text{ ps/nm/km} \cdot 0.15 \text{ nm} \cdot 100 \text{ km} = 225 \text{ ps}$$

1.3. Effect of chromatic dispersion

The effect of the chromatic dispersion shall be shown using the example of a Gaussian pulse that is transmitted over an optical fiber (this and more also can be found in [2]). The input signal of the fiber in this case is

$$\hat{E}(z=0, t) = E_0 \exp \left[-\frac{1+iC}{2} \left(\frac{t}{T_0} \right)^2 \right]. \quad (15)$$

The parameter T_0 describes the width of the Gaussian pulse. The full-width at half maximum is

$$T_{FWHM} = 2(\ln 2)^{1/2} T_0. \quad (16)$$

The chirp of the pulse is incorporated by the factor C . The chirp implies that there is a variation of the frequency that goes along with the variation of the pulse amplitude. The instantaneous change of the frequency can be found by taking the derivative of the phase of the pulse

$$\delta\omega(t) = \frac{\partial\phi}{\partial t} = -\frac{C}{T_0^2} t. \quad (17)$$

The spectrum of a chirped pulse is broader than that of an unchirped pulse. It can be found using the Fourier transform:

$$\underline{\hat{E}}(z=0, \Delta\omega) = E_0 \sqrt{\frac{2\pi T_0^2}{1+iC}} \exp\left[-\frac{\Delta\omega^2 T_0^2}{2(1+iC)}\right]. \quad (18)$$

Now, the fiber transfer function can be used to determine the signal after linear propagation through the fiber. Assume that there is no fiber loss, then the signal after propagation is

$$\underline{\hat{E}}(z, t') = \int_{-\infty}^{+\infty} \underline{\hat{E}}(z=0, \Delta\omega) \cdot e^{-j(\frac{\beta_2}{2}\Delta\omega^2 + \frac{\beta_3}{6}\Delta\omega^3) \cdot z} e^{+j\Delta\omega t} d\Delta\omega. \quad (19)$$

This integral can be solved in closed form and for $\beta_3=0$ the result is

$$\underline{\hat{E}}(z, t) = \frac{E_0}{\sqrt{Q(z)}} \exp\left[-\frac{(1+iC)}{2Q(z)} \left(\frac{t}{T_0}\right)^2\right]. \quad (20)$$

where

$$Q(z) = 1 + (C - i) \frac{\beta_2}{T_0^2} z. \quad (21)$$

From this result it can be seen that the Gaussian shape of the pulse is not changed but the width and the chirp are different from those of the initial pulse. The width T_1 is defined similarly as T_0 and describes the width of the pulse after propagation. The ratio of these widths is called **Broadening Factor** and results in

$$\frac{T_1}{T_0} = \sqrt{\left(1 + \frac{C\beta_2 z}{T_0^2}\right)^2 + \left(\frac{\beta_2 z}{T_0^2}\right)^2}. \quad (22)$$

It can be seen that for a chirp $C=0$ the pulse broadens continuously with increasing distance the signal has traveled. For a positive value of the chirp parameter, the pulse spreads even more quickly. However, for a negative chirp parameter, the pulse compresses before it eventually starts to broaden over distance. This is illustrated in Figure 4. In this graph, the broadening is shown versus the propagated distance normalized with respect to the **dispersion length**,

$$L_D = \frac{T_0^2}{\beta_2}. \quad (23)$$

The dispersion length is the length, at which an unchirped Gaussian pulse has expanded by a factor $\sqrt{2}$ compared to its initial width.

The broadening of the pulse leads to intersymbol-interference if it occurs to the extent that neighboring pulses overlap. The interference significantly degrades the transmission quality, i.e. it leads to an increasing probability of bit errors.

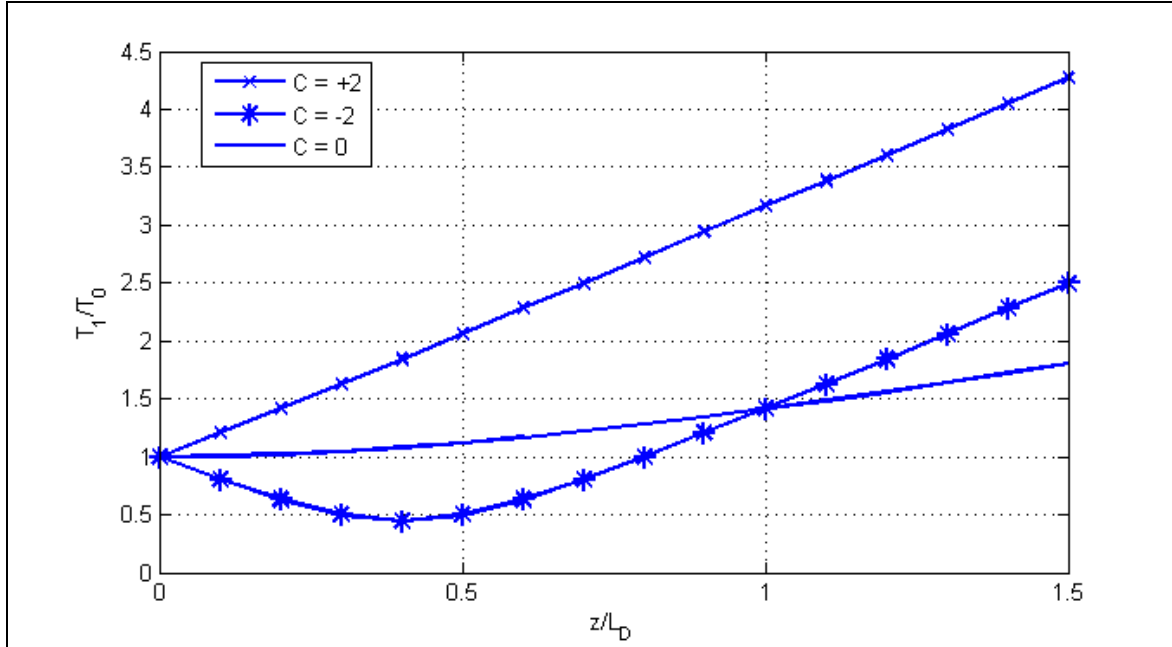


Figure 4 – Broadening of chirped Gaussian pulse over transmission distance for different chirp parameters.

1.4. Dispersion Compensating Fiber

It has just been explained that the dispersion causes a distortion of the initially transmitted signal that manifests itself as intersymbol-interference and the corresponding deterioration of transmission quality. The most widely used technique to compensate for the effect of the fiber dispersion is to use another piece of fiber with length L_2 having a dispersion parameter D_2 of opposite sign (if the first section has L_1 and dispersion D_1). This fiber is called a **dispersion compensating fiber (DCF)**. For the linear signal propagation, the signal after both sections is

$$\hat{E}(L_1 + L_2, t') = \int_{-\infty}^{+\infty} \underline{E}(z=0, \Delta\omega) \cdot e^{-\frac{\alpha(\omega_c)}{2}(L_1+L_2) - j\left(\frac{\beta_{21}}{2}\Delta\omega^2 L_1 + \frac{\beta_{31}}{6}\Delta\omega^3 L_1 + \frac{\beta_{22}}{2}\Delta\omega^2 L_2 + \frac{\beta_{32}}{6}\Delta\omega^3 L_2\right)} e^{+j\Delta\omega t} d\Delta\omega. \quad (24)$$

The second index j of β_{ij} indicates the fiber segment that the coefficient belongs to. From Eq. 24 it can be seen that the dispersion is completely compensated if the following conditions are met:

$$\beta_{21}L_1 + \beta_{22}L_2 = 0 \quad (25)$$

and

$$\beta_{31}L_1 + \beta_{32}L_2 = 0. \quad (26)$$

These conditions are equivalently expressed using the measured quantities D and S:

$$D_1 L_1 + D_2 L_2 = 0 \quad (27)$$

and

$$S_1 L_1 + S_2 L_2 = 0. \quad (28)$$

In long haul optical communication systems, long segments of transmission fiber are usually followed by spools of DCF and the signal is re-amplified using optical amplifiers. In this way, communication links can be operated successfully over thousands of kilometers. For more information about dispersion compensation you can read [2] or [3].

2. Polarization Dispersion

So far, single mode fibers have been assumed to literally support only one single mode. However, this is not completely true. One field distribution that is a solution of the propagation equation has field-vectors parallel to the fiber x-Axis. However, an absolutely equivalent solution is the distribution orthogonal to the previous one with field vectors parallel to the y-Axis. This is depicted in Figure 5.

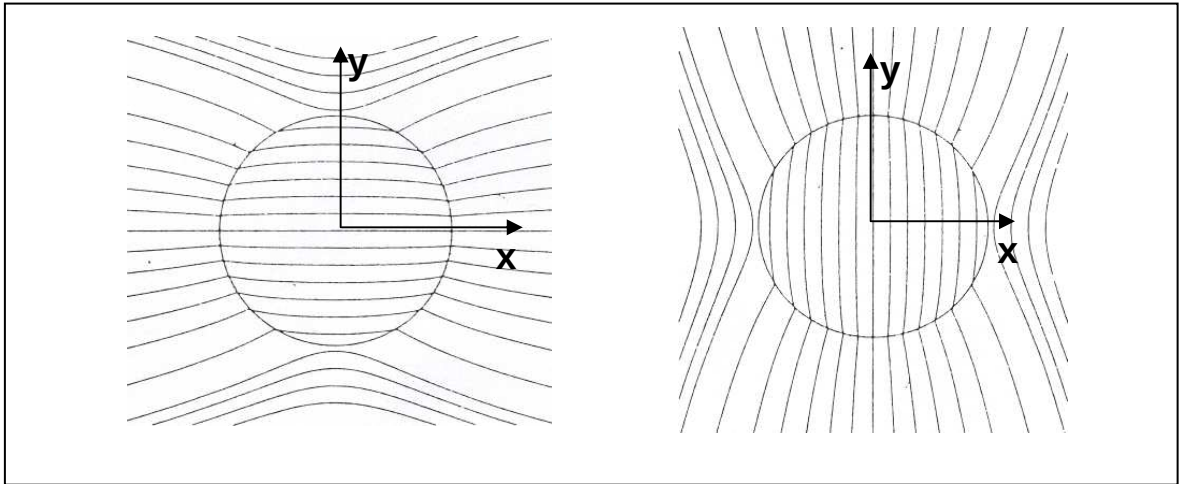


Figure 5 -Two orthogonal fundamental mode of a singlemode fiber

The total field in the fiber is a superposition of the fields of these two orthogonal modes. For a perfectly cylinder-symmetrical fiber the propagation and attenuation are constant in both fiber modes and therefore the electrical fields of both modes can jointly be described by the **Jones-Vector**

$$\underline{\vec{E}}(z, t) = \begin{pmatrix} \hat{E}_x(z=0) \\ \hat{E}_y(z=0) \end{pmatrix} \cdot e^{-\frac{\alpha(\omega_c)}{2} \cdot z} \cdot e^{-j\beta(\omega_c) \cdot z} \cdot e^{j\omega_c \cdot t} \quad (29)$$

The two modes are called **degenerate** because they propagate through the fiber in exactly the same way. Therefore, it is sufficient to consider only one mode in an ideal fiber.

2.1. Light Polarization

The polarization state of the light wave is defined as the time-dependent curve of the end-point of the electrical field-vector at a fixed position z in the fiber. It can be observed by looking at a distinct infinitely thin slice of the fiber at position z and watching the vector that is spanned by the real parts of the signals in the orthogonal modes. The real parts of the electrical fields at position z are

$$\text{Re}\{\tilde{\underline{E}}(z,t)\} = \text{Re}\left\{\begin{pmatrix} \left|\hat{\underline{E}}_x(z=0)\right| \cdot e^{j\delta_x(z=0)} \\ \left|\hat{\underline{E}}_y(z=0)\right| \cdot e^{j\delta_y(z=0)} \end{pmatrix} \cdot e^{j(\omega_c \cdot t - \beta(\omega_c)z)}\right\} \cdot e^{-\frac{\alpha(\omega_c)}{2}z}. \quad (30)$$

The signals in the two orthogonal modes in general have different amplitudes $\hat{\underline{E}}_{x/y}(z=0)$ and phases $\delta_{x/y}(z=0)$.

For the case of **linear polarization**, the curve is just a line. For **circular polarization**, the shape of the curve is a circle. All other polarization states can be described as a mixture of these two cases and yield **elliptical polarization**. The elliptical polarization can be described with respect to the parameters of the corresponding ellipse. These parameters are the **ellipticity**,

$$\varepsilon = \pm \arctan\left(\frac{E_{\min}}{E_{\max}}\right). \quad (31)$$

and the **inclination** angle ϕ ,

$$\begin{aligned} |\hat{\underline{E}}_x| \geq |\hat{\underline{E}}_y| : \phi &= \frac{1}{2} \cdot \arctan\left(\frac{2 \cdot |\hat{\underline{E}}_x| \cdot |\hat{\underline{E}}_y| \cdot \cos(\delta_y - \delta_x)}{|\hat{\underline{E}}_x|^2 - |\hat{\underline{E}}_y|^2}\right), \\ |\hat{\underline{E}}_x| < |\hat{\underline{E}}_y| \quad \text{and} \quad \cos(\delta_y - \delta_x) &\geq 0 : \phi \rightarrow \phi + 90^\circ, \\ |\hat{\underline{E}}_x| < |\hat{\underline{E}}_y| \quad \text{and} \quad \cos(\delta_y - \delta_x) < 0 : \phi &\rightarrow \phi - 90^\circ. \end{aligned} \quad (32)$$

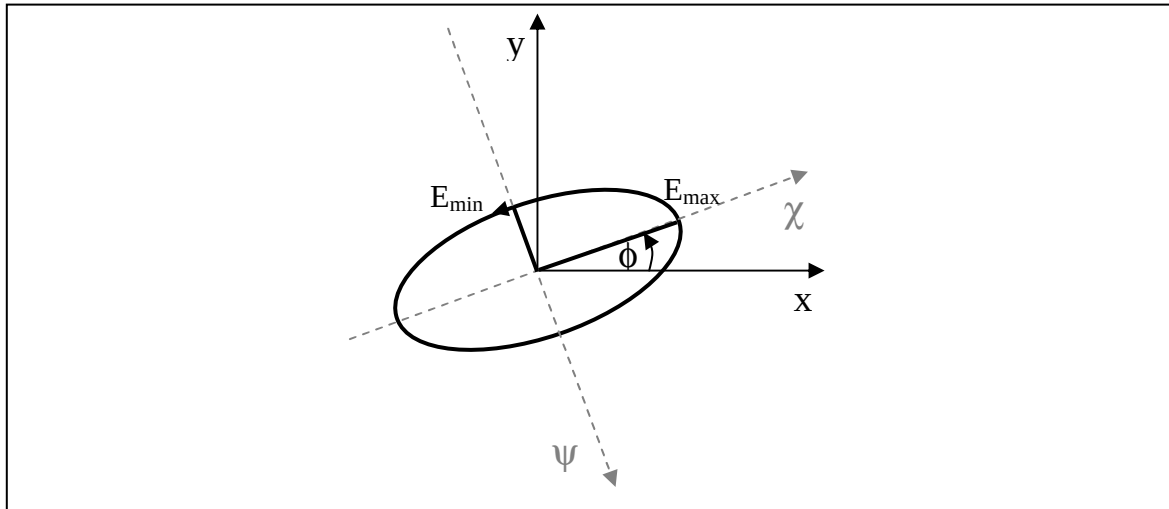


Figure 6 - Polarization ellipse

The semi-major and semi-minor axes of the ellipse are calculated as

$$E_{\max}^2 = \frac{1}{2} \cdot \left[|\hat{E}_x|^2 + |\hat{E}_y|^2 + \sqrt{(|\hat{E}_x|^2 + |\hat{E}_y|^2) - 4|\hat{E}_x|^2|\hat{E}_y|^2 \cdot \sin^2(\delta_y - \delta_x)} \right] \quad (33)$$

and

$$E_{\min}^2 = \frac{1}{2} \cdot \left[|\hat{E}_x|^2 + |\hat{E}_y|^2 - \sqrt{(|\hat{E}_x|^2 + |\hat{E}_y|^2) - 4|\hat{E}_x|^2|\hat{E}_y|^2 \cdot \sin^2(\delta_y - \delta_x)} \right]. \quad (34)$$

In OptiSystem, the output polarization of certain transmitter components can be selected using the **ellipticity** and the **inclination** (called **azimuth** in OptiSystem).

The parameters and the curve drawn by the real parts of the field vectors for various polarization states are shown in Figure 7.

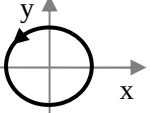
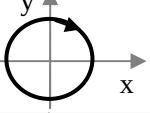
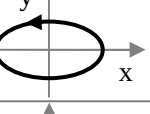
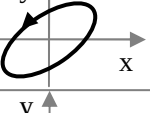
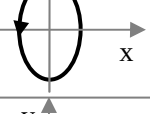
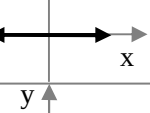
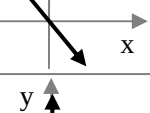
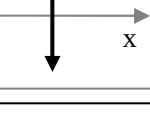
$\text{Re}\{\vec{E}(z,t)\}$	$\begin{pmatrix} \hat{E}_x \cdot e^{j\delta_x} \\ \hat{E}_y \cdot e^{j\delta_y} \end{pmatrix}$	Name	Ellipticity ϵ	Inclination /Azimuth	Grade of Pol. P
	$\begin{pmatrix} \hat{E} /\sqrt{2} \cdot e^{j\delta_0} \\ \hat{E} /\sqrt{2} \cdot e^{j(\delta_0-90^\circ)} \end{pmatrix}$	Left hand circular	45° anti-clockwise	-	0
	$\begin{pmatrix} \hat{E} /\sqrt{2} \cdot e^{j\delta_0} \\ \hat{E} /\sqrt{2} \cdot e^{j(\delta_0+90^\circ)} \end{pmatrix}$	Right hand circular	-45° clockwise	-	0
	$\begin{pmatrix} \sqrt{3}/2 \cdot \hat{E} \cdot e^{j\delta_0} \\ 1/2 \cdot \hat{E} \cdot e^{j(\delta_0-90^\circ)} \end{pmatrix}$	Left hand elliptic	30°	0°	0.5
	$\begin{pmatrix} \hat{E} /\sqrt{2} \cdot e^{j\delta_0} \\ \hat{E} /\sqrt{2} \cdot e^{j(\delta_0-45^\circ)} \end{pmatrix}$	Left hand elliptic	22.5°	45°	1/√2
	$\begin{pmatrix} 1/2 \cdot \hat{E} \cdot e^{j\delta_0} \\ \sqrt{3}/2 \cdot \hat{E} \cdot e^{j(\delta_0-90^\circ)} \end{pmatrix}$	Left hand elliptic	30°	90°	0.5
	$\begin{pmatrix} \hat{E} \cdot e^{j\delta_0} \\ 0 \end{pmatrix}$	Linear horizontal	0°	0°	1
	$\begin{pmatrix} - \hat{E} /\sqrt{2} \cdot e^{j\delta_0} \\ \hat{E} /\sqrt{2} \cdot e^{j\delta_0} \end{pmatrix}$	Linear -45°	0°	-45°	1
	$\begin{pmatrix} 0 \\ \hat{E} \cdot e^{j\delta_0} \end{pmatrix}$	Linear vertical	0°	90°	1

Figure 7 - Various types of polarization and the corresponding parameters

2.2. Poincaré Sphere

In addition to the polarization ellipsis, there is another useful representation of the polarization state of light. In this representation, all polarization states are mapped onto a sphere. This idea was developed by H. Poincaré in 1892, hence the sphere is called **Poincaré-Sphere**.

The coordinates in the coordinate system of the Poincaré-Sphere are the **Stokes-Parameters**³ s_1, s_2, s_3 of the signal. These can be found either by using the ellipticity ε and the inclination ϕ of the polarization ellipse or directly from the Jones-Vector:

$$\begin{aligned} S_1 &= |\hat{E}_x|^2 - |\hat{E}_y|^2 = \cos(2\varepsilon)\cos(2\phi), \\ S_2 &= 2 \cdot |\hat{E}_x| \cdot |\hat{E}_y| \cdot \cos(\delta_y - \delta_x) = \cos(2\varepsilon)\sin(2\phi), \\ S_3 &= 2 \cdot |\hat{E}_x| \cdot |\hat{E}_y| \cdot \sin(\delta_y - \delta_x) = \sin(2\varepsilon), \end{aligned} \quad (35)$$

Certain points in Stokes-space that correspond to the different types of polarization are highlighted in Figure 8. In addition to that, the Poincaré-Sphere representation of the polarization as provided by the OptiSystem **Polarization analyzer** is shown. On the Poincaré-Sphere orthogonal polarization states are on the opposite side of the sphere, i.e. the vector of Stokes-Parameters is multiplied by -1.

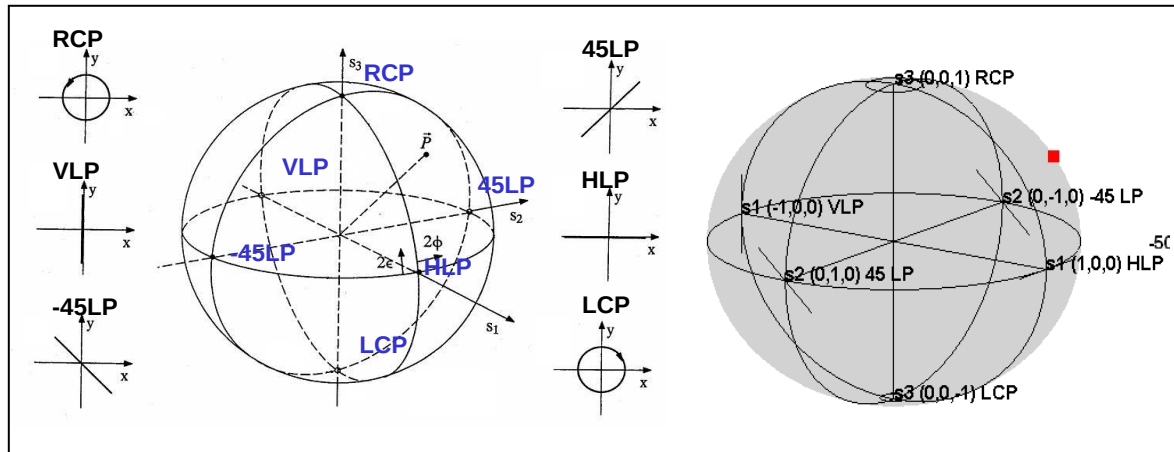


Figure 8 - Poincaré sphere representation of the polarization state in the lecture notes (left) and in polarization analyzer in OptiSystem (right)

For more information about light polarization and its representations you can refer to [4].

2.3. Fiber Birefringence

The fiber manufacturing process is not ideal. Therefore, real fibers show slight variations from a circular symmetry. These variations can also be caused by external effects, such as torsion, bending, pressure or externally applied electric or magnetic fields. This deviation from the ideal fiber geometry has the effect that the light polarization before and after transmission through the fiber will be different in general.

³ Generally, there is another Stokes-Parameter S_0 that represents the intensity of polarized **and** unpolarized light components. However, we assume that there is only polarized light in this case. This implies that the length of the vector of Stokes-Parameters is $\sqrt{S_1^2 + S_2^2 + S_3^2} = 1$. Therefore, polarization is a point **on** the Poincaré-Sphere

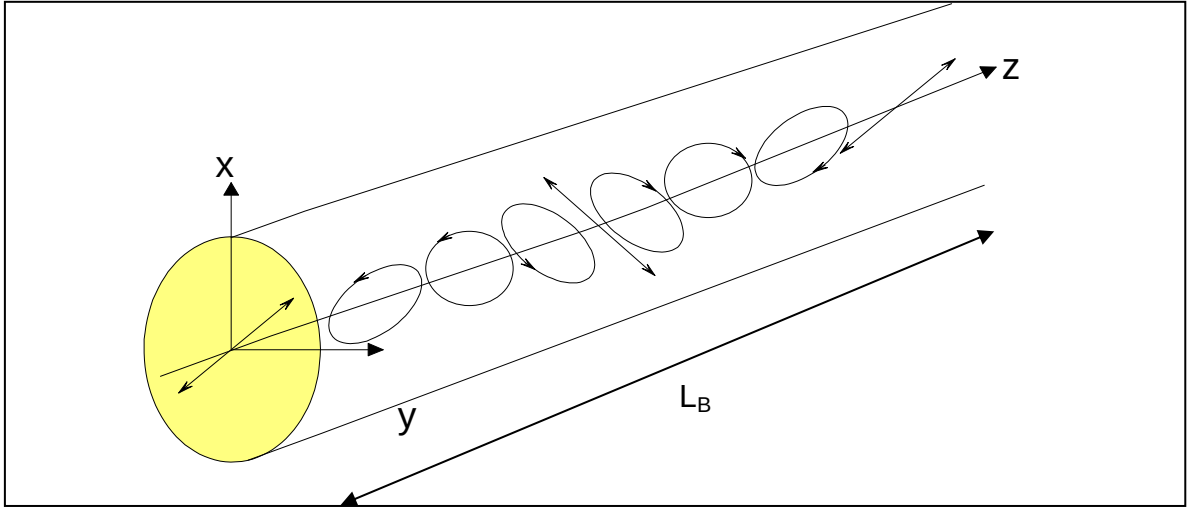


Figure 9 - Evolution of the polarization state in a fiber with slightly elliptical fiber-core

However, such a fiber still has two orthogonal **Eigenpolarizations**, $\vec{P}_y$ and $\vec{P}_x$, which remain unchanged after transmission. These Eigenpolarizations propagate at different phase-velocities β_y and β_x . Therefore, the polarization of light will change periodically over the fiber length if a signal that excites both Eigenpolarizations is fed into this optical fiber.

For example, in case of an elliptical fiber-core⁴ the Eigenpolarizations are linear. If you assume that the x- and y-coordinates are aligned with the axes of the elliptic fiber core, then the output of the fiber is found to be

$$\underline{\vec{E}}(z, t) = \begin{pmatrix} \hat{\underline{E}}_x(z=0) \cdot e^{-j\beta_x(\omega_c) \cdot z} \\ \hat{\underline{E}}_y(z=0) \cdot e^{-j\beta_y(\omega_c) \cdot z} \end{pmatrix} \cdot e^{-j\beta(\omega_c) \cdot z} \cdot e^{-\frac{\alpha(\omega_c)}{2} \cdot z} \cdot e^{j\omega_c \cdot t}. \quad (36)$$

The evolution of the polarization state for an input signal that equally excites both Eigenpolarizations is illustrated in Figure 9. After a certain length the initial polarization state is repeated. This length is called **beat length** L_B . The same polarization state as the initial one results when the phase difference between the two components is 2π . Consequently, the beat length can be calculated by

$$L_B = \frac{2\pi}{|\beta_x - \beta_y|}. \quad (37)$$

Question

Try to derive the expression in **Eq. 37** yourself!

The absolute phase difference between both modes has to be 2π to obtain the same polarization state as at the beginning of the fiber. Therefore

$$-|\beta_x(\omega_c) \cdot L_B - \beta_y(\omega_c) \cdot L_B| = 2\pi$$

Therefore the beat length is $L_B = \frac{2\pi}{|\beta_x - \beta_y|}.$

⁴ Note, that the elliptical fiber-core and the polarization ellipsis are completely different things. The first describes the geometry of the fiber, whereas the latter describes a polarization state.

2.4. Polarization Mode Dispersion (PMD)

It has just been explained that the propagation-constants differ for birefringent-fibers. However, also the frequency derivatives of propagation constants differ. Hence, there are different normalized group delays $\bar{\tau}_{Gr,\psi}$ and $\bar{\tau}_{Gr,\chi}$ for the signal components of the two Eigenpolarizations. The difference between the normalized group delays is

$$\begin{aligned} d\tau_{Gr}(\omega_c) &= \left. \frac{\partial(\beta_\psi(\omega) - \beta_\chi(\omega))}{\partial\omega} \right|_{\omega_c} \\ &= \bar{\tau}_{Gr,\psi}(\omega_c) - \bar{\tau}_{Gr,\chi}(\omega_c) \end{aligned} \quad (38)$$

The delay that accumulates between the two modes after a fiber of length L is called the **Differential Group Delay (DGD)** and is mathematically expressed as

$$DGD = d\tau_{Gr}(\omega_c) \cdot L. \quad (39)$$

The effect of the DGD on the transmission of pulses of light is shown in Figure 10. For a perfect fiber without any difference in the propagation for both polarization modes, the pulse shape is preserved in the output of the photodetector. However, for a birefringent fiber, the pulse of the slower axis arrives at the fiber end with a delay equal to the DGD. Since the photodetector converts the power contained in both polarization modes into the output-current, the receiver pulse appears broadened. This is another source of intersymbol-interference and therefore leads to degradation of system performance.

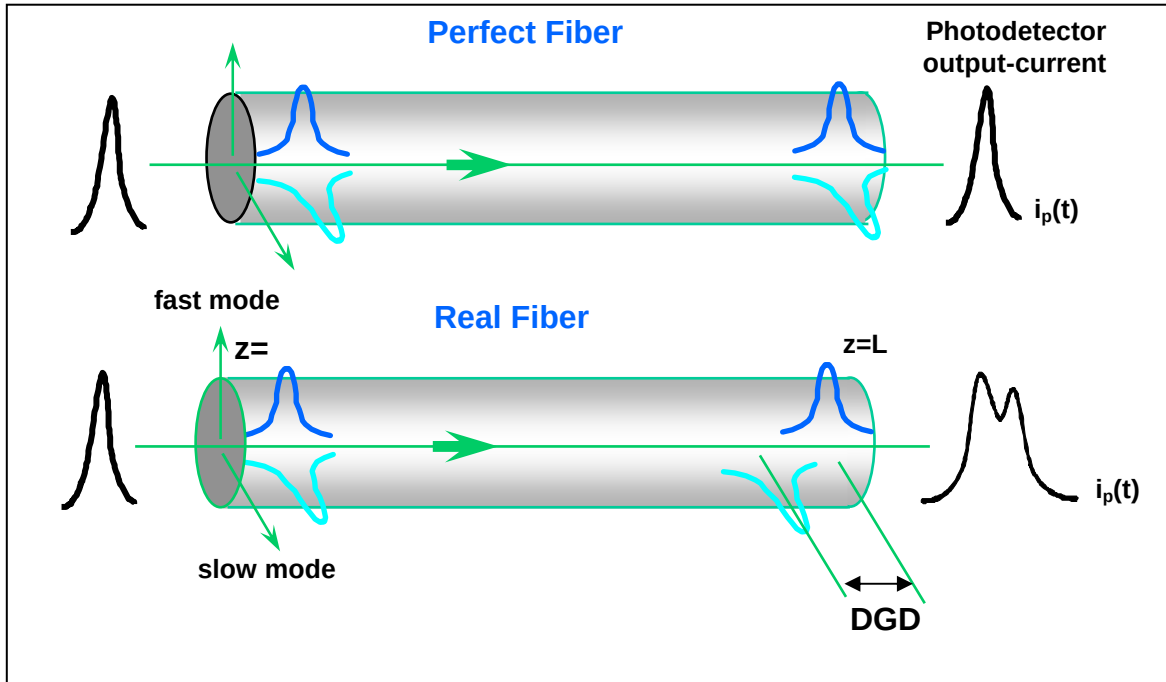


Figure 10 - Illustration of the effect of the Differential Group Delay on pulse propagation

Question

What can be done to avoid the pulse broadening for one piece of birefringent fiber?

Pulse broadening can be avoided by exciting only one of the two Eigenpolarizations.

For now, only fibers with one single type of birefringence have been considered. In long fibers, however, there may be many different external effects and locally varying fiber asymmetries. Long fibers are therefore commonly modeled as a sequence of birefringent sections with constant Eigenpolarizations and polarization dependent propagation constants within each individual section. The length of the sections is called correlation length, L_{corr} . The concatenation of sections is depicted in Figure 11.

For such a long fiber there are also polarization states that are comparable to the Eigenpolarizations of a single section. These polarization states are called **Principle States of Polarization (PSOP)**. If a pulse is launched into the fiber with a polarization state equal to either of the two PSOPs then it will experience the smallest possible broadening⁵. The **average** DGD for such a long fiber does not depend linearly on the total length of the fiber because of the statistically varying birefringence of the section. On average, the DGD shows a dependence on the length according to

$$DGD = PMD \cdot \sqrt{L} . \quad (40)$$

The polarization mode dispersion coefficient, PMD , is the average differential group delay of the total fiber normalized by the square root of the distance.

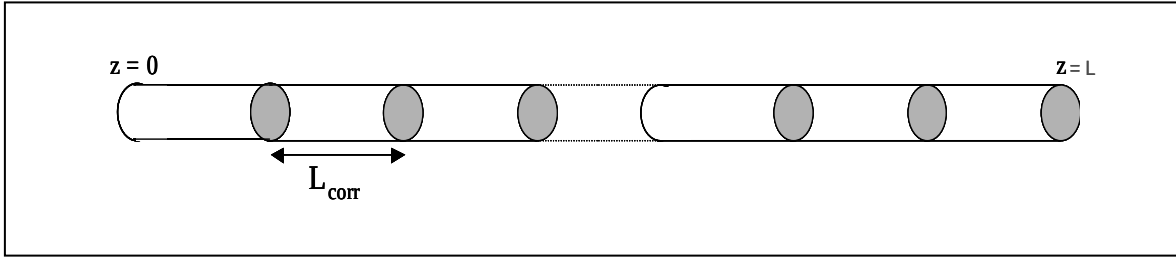


Figure 11 - Long fiber as sequence of birefringent sections

⁵ In contrast to the case of a single birefringent section, it will not experience zero broadening except if the spectrum is infinitely thin.

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