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Theoretical and Experimental Researches on Repetition Rate of Pulses with Passive or Self-Q-switched Laser*

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Abstract: Coupled rate equaiton for passive or self-Q-switched solid-state lasers is studied in physics and mathematics methods. The formula about relationship to repetition rate of the pulses and optical parameters of the lasers has been derived, by means of the adiabatic elimination approximation (AEA) and the Hopf bifurcation condition. For the type of Cr:Nd:YAG saturable absorber Q-switched lasers, the theoretical calculations are agreement with the experimental results.

Key words: passive or self-Q-switch; repetition rate of pulses; adiabatic elimination approximation (AEA)

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被动与自调Q激光脉冲重复频率的理论和实验

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摘要: 对被动或自调Q激光器的速率方程运用绝热近似方法和Hopf分岔条件进行分析, 导出光学参量与脉冲重复频率之间的解析表达式。针对Cr:Nd:YAG调Q激光器的实验数据进行了计算, 所得出的理论曲线与实验结果相吻合。

关键词: 被动或自调Q; 脉冲重复频率; 绝热近似

1 Introduction

The tetravalent chromium doped garnets have attracted a great deal of attentions and triggered a series of experimental investigations in recent years, such as passive Q-switch for Nd:YAG, Nd:YALO (YAP), or self-Q-switch of Cr:Nd:YAG laser[1][2]. Because of large absorption cross section, moderate excited-state lifetime, which is much shorter than Nd-ion's, high doped-ion concentration, good thermal conductivity, chemical stability, Cr:YAG or Cr:Nd:YAG is considered to be an ideal saturable absorber as passive or self-Q-switch for high power, high repetition rate, all-solid-state compact lasers, and the repetition rate range of multi-pulses have been achieved from two till hundreds of kHz. However, it is important to give theoretical explanation to the experimental results of repetition rate or periodicity, and to establish the relationship of multi-pulses between different optical parameters. By means of the adiabatic elimination approximation (AEA) method, the formulation of repetitive rate or periodicity has been obtained. It

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helps to understand the operation property of passive or self-Q-switch laser.

2 Analysis

Because of the transversal relaxation time is much shorter than the longitudinal relaxation time, the most of solid-state lasers belong to the category of class-B. the polarization's rate equation can be adiabatically eliminated. According to the AEA, two rate equations are those for the spatial average fundamental mode intensity I and the population inversion density Δn[3]. In regards to the passive or self-Q-switched YAG laser, the rate equation of the absorber population density must be considered. The total absorber population density is

$$n_{a0} \approx n_{a1} + n_{a2} \tag{1}$$

where n_{a1}, n_{a2} are populations density of ground and excited states, respectively, for the two levels approximation. The resonance absorber coefficient is defined as

$$\chi_a = \sigma_a (n_{a1} - n_{a2}) \tag{2}$$

where σ_a is absorption cross-section of saturable absorber.

The coupled rate equation for I, Δn and χ_a can be written as

$$\frac{dI}{dt} = (\mu\upsilon\sigma\Delta n - \mu\upsilon\frac{l_a}{l}\chi_a - \frac{1}{\tau_c})I \tag{3a}$$

$$\frac{d\Delta n}{dt} = -\beta\frac{\sigma}{h\nu}\Delta n I + \frac{\Delta n_i - \Delta n}{\tau_u} \tag{3b}$$

$$\frac{d\chi_a}{dt} = -2\frac{\sigma_a}{h\nu}\chi_a I + \frac{\chi_{a0} - \chi_a}{\tau_a} \tag{3c}$$

where σ is the stimulated emission cross-section of lasing medium, the μ is filling factor[4]

$$\mu = \frac{ln}{ln + l_a n_a + [L - (l + l_a)]} \tag{4}$$

where L is the geometrical lengths of resonator, l, l_a and n, n_a are geometrical lengths and refractive indices of lasing, and absorber media, respectively. υ is the velocity of light in medium. τ_c is the photon lifetime inside resonator

$$\tau_c = \frac{1}{\mu\upsilon(\eta_1 + \frac{l_a}{l}\eta_a + \eta_2)} \tag{5}$$

where η_1 and η_a are unfavorable loss coefficients, η_2 is the output loss which can be written as

$$\eta_2 = \frac{1}{2L}\ln(\frac{1}{R_1 R_2}) \tag{6}$$

where R_1 and R_2 are the reflectivity of two mirrors. The τ_u and τ_a are the relaxation times of the excited-state in lasing and absorber media, respectively, Δn_i is the pumping parameter, and β equals 1 or 2 for four or three level lasing media. Coupled rate equations (3a, b, c) are complex nonlinear differential equations, the accurate analytical solution is too difficult to derive now. As is well known, in the passive or self-Q-switched lasers, not only the relaxation time τ_u of lasing medium should be longer, but also the relaxation time τ_a of the saturable absorber should be shorter, in favor of enough accumulation for the inverse population and fast time for the Q-switch. Therefore τ_u ought to be much longer than τ_a. For the Cr:YAG crystal, τ_u is about two magnitude times than τ_a, while, $\sigma_a \approx 8.8 \times 10^{-19}$ cm^2, is the same magnitude as σ[2], the condition of AEA,

$$\tau_a / \tau_u \ll \beta\sigma / 2\sigma_a \tag{7}$$

is always met. The absorber coefficient χ_a is a fast relaxation parameter, the steady-state solution of equation (3c) can be substituted into equation (3a), consequently[5]. Two dimension coupled rate equations of an autonomous system is obtained.

$$\frac{dI}{dt} = [\mu\upsilon\sigma\Delta n - \mu\upsilon l_a\frac{\chi_{a0}/l}{1 + I/I_{b0}} - \frac{1}{\tau_c}]I \tag{8a}$$

$$\frac{d\Delta n}{dt} = -\beta\frac{\sigma}{h\nu}\Delta n I + \frac{\Delta n_i - \Delta n}{\tau_u} \tag{8b}$$

where I_{b0} is corresponding to the saturation intensity of saturable absorber.

If the gain exceed loss, the stationary point of (8a, b) is about at $(I_2, \Delta n_{tm})$

$$I_2 = (\frac{\Delta n_i}{\Delta n_{tm}} - 1)\frac{h\upsilon}{\beta\sigma\tau_u} = B I_s,\ \sigma\Delta n_{tm} = \frac{l_a}{l}\chi_{a0} + \frac{1}{\mu\upsilon\tau_c} \tag{9}$$

where I_s is corresponding to the saturation intensity of lasing medium; B is the dimensionless positive number and relative to pump intensity and loss. By using Hopf bifurcation method, one can define B represent the bifurcation parameter of the system. As soon as $B > B_0 > 0$ is realized, the system will show the periodical oscillation from the steady-state, and B_0 is the critical bifurcation value. The limited cycles, that rotate around the stationary point $(I_2, \Delta n_{tm})$ on phase-plan of I and Δn, represent a series of pulses emerging from the Q-switched laser. In consideration of equations (8a, b) and (9) as well as $\tau_a/\tau_u \ll \beta\sigma/2\sigma_a$, Jacobian matrix near the stationary point $(I_2, \Delta n_{tm})$ is represented as

$$J_2 = \begin{bmatrix} \mu\upsilon \frac{l_a}{l}\chi_{a0} B \frac{I_s}{I_{b0}} & \mu\upsilon\sigma I_s B \\ -\frac{\beta\sigma}{h\nu}\Delta n_{tm} & -\frac{B+1}{\tau_u} \end{bmatrix} \tag{10}$$

The condition of Hopf bifurcation demands that eigenvalues must be a pair of conjugate complex numbers

$$\lambda_{1,2} = \alpha(B) \pm i\omega(B) \tag{11}$$

and

$$\alpha(B) = \frac{1}{2}\left(\mu\upsilon\frac{l_a}{l}\chi_{a0}B\frac{I_s}{I_{b0}} - \frac{B+1}{\tau_u}\right) \tag{12}$$

$$\omega(B) = \frac{1}{\tau_u}\sqrt{\mu\upsilon\sigma\Delta n_{tm}\tau_u B - \frac{1}{4}\left[B\left(1+\mu\upsilon\frac{l_a}{l}\chi_{a0}\frac{I_s}{I_{b0}}\tau_u\right)+1\right]^2} \tag{13}$$

where α and ω are amplitude and angle frequency of the limit cycle individually.

Let α equal to zero, the critical bifurcation value B_0 can be determined as

$$B_0 = \frac{1}{\mu\upsilon\frac{l_a}{l}\chi_{a0}\frac{I_s}{I_{b0}}\tau_u - 1}, \quad \Delta n_0 = (1+B_0)\Delta n_{tm} \tag{14}$$

where Δn_0 is called the threshold condition of producing Q-switch pulses. Only if, $\Delta n_i \geq \Delta n_0$ or $B \geq B_0$, is satisfied by the pump, a series of Q-switch pulses will be generated. The repetition rate F equals $\omega/2\pi$. Under consideration of equations (13) and (14), one can find

$$F(B) = \frac{1}{2\pi\tau_u}\sqrt{\mu\upsilon\sigma\Delta n_{tm}\tau_u B - \frac{1}{4}\left[B\left(2+\frac{1}{B_0}\right)+1\right]^2} \tag{15}$$

specially

$$F(B_0) = \frac{1}{2\pi\tau_u}\sqrt{\mu\upsilon\sigma\Delta n_{tm}\tau_u B_0 - (1+B_0)^2} \tag{16}$$

where $F(B_0)$ indicates the prime or critical repetitive rate for the beginning of Q-switch pulses generation.

3 Discussion

The formulation of determining the repetition rate passive or self-Q-switched laser has been derived. The condition of $F(B_0) > 0$ and $B_0 > 0$, provides the restriction of small signal transmission T_0 and gain coefficient $\sigma\Delta n_{tm}$ to the passive or self-Q-switch. That is

$$T_0 = \exp(-l_a\chi_{a0}) < \exp\left(-\frac{l\beta\sigma}{\mu\upsilon\tau_a\sigma_a}\right) \tag{17}$$

$$\sigma\Delta n_{tm} > \frac{(1+B_0)^2}{\mu\upsilon\tau_u B_0} \tag{18}$$

In the case of flashlamp cw-pumped passive Q-switched Nd: YAG laser[2], L = 82.0 cm, l = 10.4 cm, l = 0.2 cm, R = 80 %, T_0 = 81 %, These correspond to $B_0 \approx 10^{-3}$, $\sigma\Delta n_{tm} \times 5.5 \times 10^{-2}$/cm, and $F(B_0) \approx 2.7$ kHz. If the experimental data are introduced into equations (9), (17), etc, it is consistent with the experimental result in Ref. 2. The diagram of the repetitive rate F *vs* the relative pump intensity B is shown in Fig. 1.

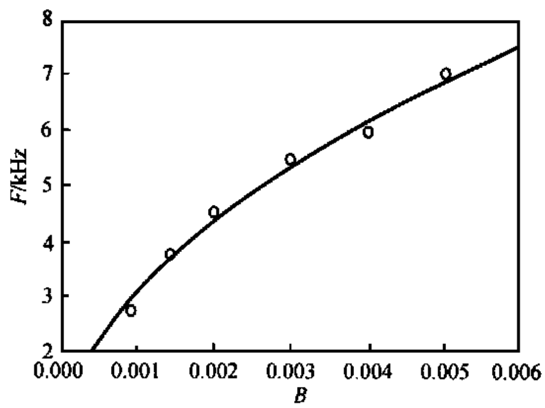

Fig. 1　Relation between F and B. Solid line—theoretical calculation using equation (15), open circles—experimental measurements[2]

In the case of LD pumped self-Q-switched microchip laser, experimental setup is shown in Fig. 2.

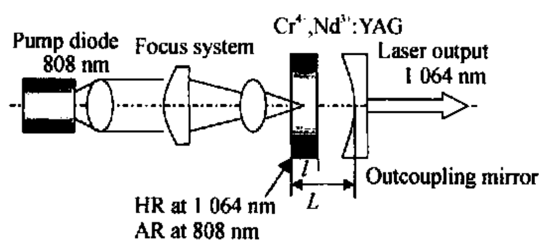

Fig. 2　Experimental setup of the diode pumped self-Q-switched microchip laser

Here, L = 3.5 cm, $l = l_a$ = 0.2 cm, R = 98.0 %, T_0 = 93.8 %, these correspond to $B_0 \approx 10^{-4}$. $\sigma\Delta n_{tm} \approx 33 \times 10^{-2}$/cm, and $F(B_0) \approx 1.6$ kHz. The diagram of the repetition rate F *vs* the relative pump intensity is shown in Fig. 3.

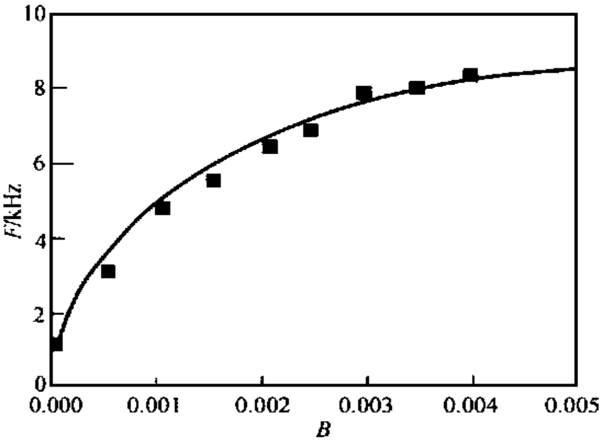

Fig. 3 Relation between F and B. Solid line —theoretical calculation using equation, full squares—experimental measurements[3]

In comparison with Fig. 1, the $F(B_0)$ in Fig. 3 is smaller, and the increase rate of F with the relative pump intensity B is slower near the 8 kHz. From Figs. 1, 3, the following conclusions can be drawn. The values of B_0 and $\sigma\Delta n_{tm}$ decide the primary and maximum repetitive frequencies $F(B_0)$ and F_{max} of passive of self-Q-switched lasers in general. The higher reflection R of the output mirror is, the less is F_{max} attainable to, and vice versa. For the small relative pump intensity, the repetitive rate F increases with the pump power in an approximate linear relation, just as many articles have reported. But when F is close to the F_{max}, its increase rate with pumping is obviously slow. These seem to be effect of the gain saturation in the periodical state.

The analytical formula of repetitive rate has been obtained for passive or self-Q-switched lasers. It is able to provide the distinct relationship between the parameters concerned, to predict the primary and maximum repetitive frequencies $F(B_0)$ and F_{max} as well as the operation state, and output power. For example, one can estimate the optical parameters from the determined F by equation (17). If the passive or self-Q-switched laser is operated near F_{max}, not only the laser output power is more, but also the stability of Q-switch pulses is better, because the $dF/dB \approx 0$ in this area. The theoretical studies on the passive or self Q-switched laser are of benefit for its optimizing design, enhancing stability, increasing output power and extending applications.

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Biography:

Zhu Cghanghong was born in 1957. He is Associate Professor working at laser technology in Huazhong University of Science & Technology (HUST), China. From July to Dec. 1999, as a visiting scientist has researched diode-pumped solid-state self-Q-switched laser and "green problem of SHG" in Technology University of Berlin, Germany.