第一章 傅里叶分析

- 几种常用的函数
  - 阶跃函数
  - 符号函数
  - 矩形函数
  - 三角形函数
  - Sinc函数
  - 高斯函数
  - 园域函数
- 脉冲函数
  - 定义及其性质
  - 疏函数定义及其性质

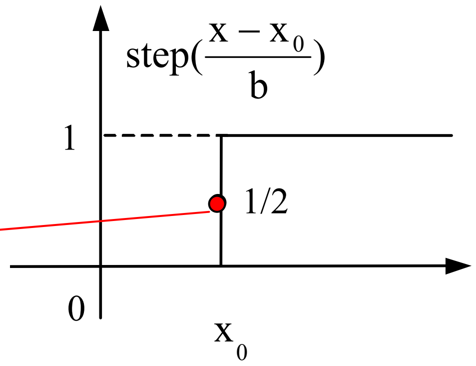
卷积的定义及其性质
  - 定义
  - 物理意义
  - 性质

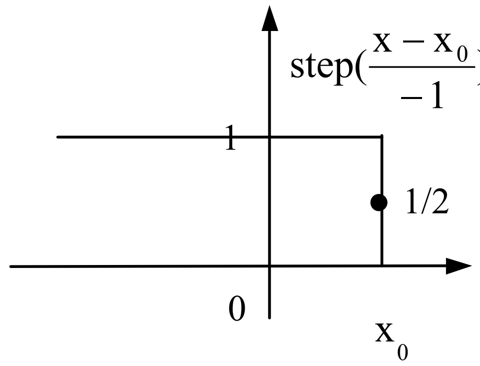
- 相关的定义及性质
  - 定义
  - 物理意义
  - 性质
  - 卷积的MATLAB实现
- 空间频率
- 傅里叶变换定义及其存在条件
  - 定义及存在条件
  - 广义傅里叶变换
  - 虚、实、奇、偶函数FT的性质
  - 可分离函数的FT
  - FT-BESSAL变换
  - 周期函数的变换
- 几种常用函数的傅里叶变换
- 几种常见图形的FT
- FT的数值实现

几种常用的函数

– 阶跃函数

$$\mathrm{step}(x)=\begin{cases}1, & x>0\\ \frac{1}{2}, & x=0\\ 0, & x<0\end{cases}$$

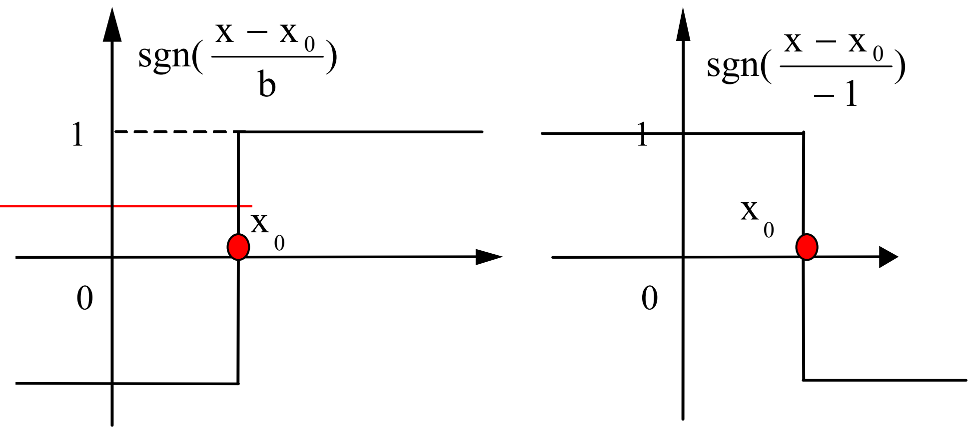





```
function y=step(x)
%STEP step function.
%
%   step(x)   =  0   if x <  0
%                =  1/2 if x == 0
%                =  1   if x > 0
y = (x>0).*1;
y(find(x==0)) = 0.5;
```

– 符号函数

$$\operatorname{sgn}(x)=\begin{cases}1, & x>0\\ 0, & x=0\\ -1, & x<0\end{cases}$$



%SIGN   Signum function.

%   For each element of X, SIGN(X) returns 1 if the element

%   is greater than zero, 0 if it equals zero and -1 if it is

%   less than zero. For the nonzero elements of complex X,

%   SIGN(X) = X ./ ABS(X).

%   Copyright 1984-2002 The MathWorks, Inc.

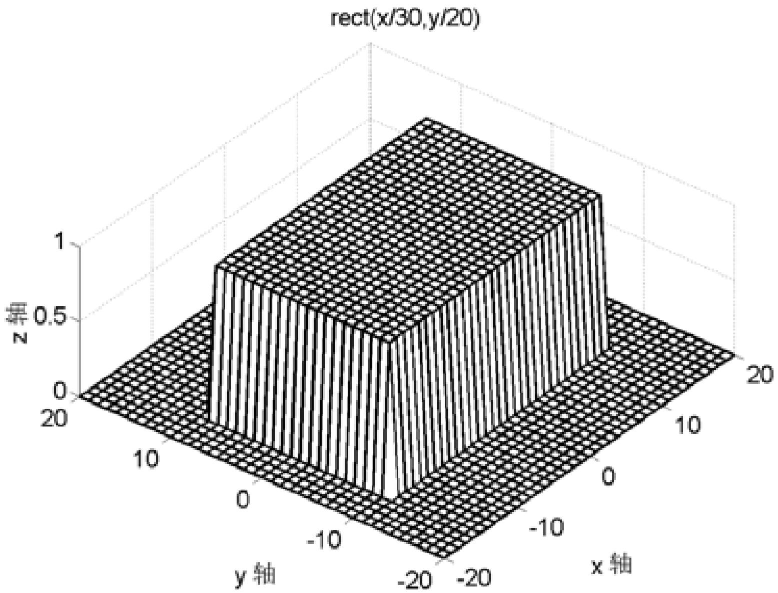
% $Revision: 5.8 $ $Date: 2002/04/08 20:11:24 $

%   Built-in function.

– 矩形函数

$$\mathrm{rect}(\frac{x}{a}) = \begin{cases} 1, & |x| \le \frac{a}{2} \\ 0, & \text{其它} \end{cases}$$

```
function y = rect(t,Tw)
%RECT Sampled aperiodic rectangle generator.
%   RECT(T) generates samples of a continuous, aperiodic,
%   unity-height rectangle at the points specified in array T, centered
%   about T=0.  By default, the rectangle has width 1.
%   RECT(T,W) generates a rectangle of width W.

error(nargchk(1,2,nargin));
if nargin<2, Tw=1;   end

% Returns unity in interval [-Tw/2,+Tw/2)
t = abs(t);
y = (t<Tw/2)*1.0 ;
y(find(t==Tw/2)) = 0.5;
```



脉冲函数与阶跃函数的微分

$$\delta(x) = \lim_{b \to 0} \frac{1}{b} \mathrm{rect}\left(\frac{x}{b}\right)$$

$$\mathrm{rect}\left(\frac{x}{b}\right) = \mathrm{step}\left(x + \frac{b}{2}\right) - \mathrm{step}\left(x - \frac{b}{2}\right)$$

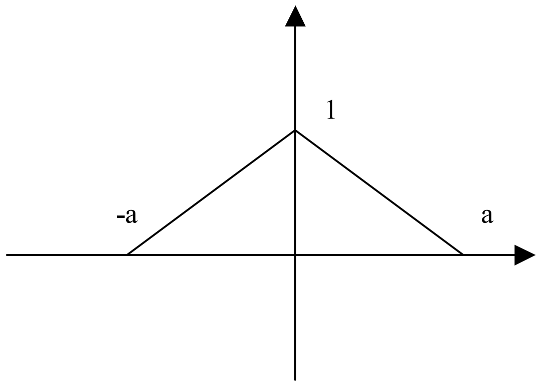
$$\lim_{b \to 0} \frac{1}{b} \mathrm{rect}\left(\frac{x}{b}\right) = \lim_{b \to 0} \frac{\mathrm{step}(x+b) - \mathrm{step}(x)}{b}$$

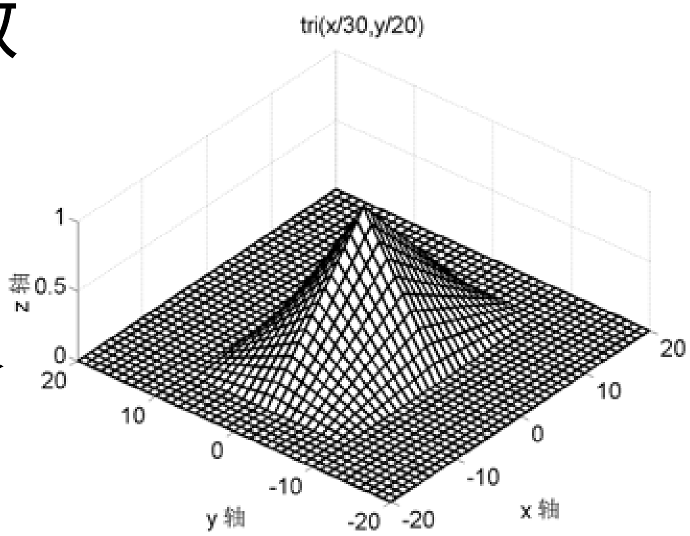
$$\boxed{\delta(x) = \frac{d}{dx} \mathrm{step}(x)}$$

几种常用的函数

– 三角形函数

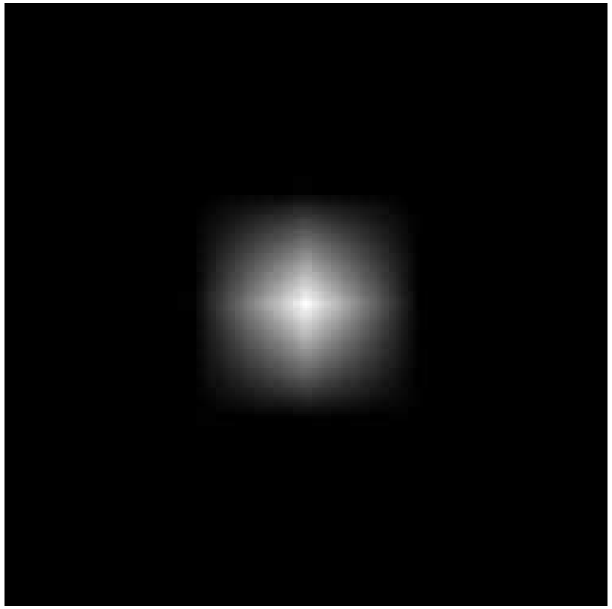
$$\mathrm{tri}\left(\frac{x}{a}\right)=\begin{cases}1-\dfrac{|x|}{a}, & |x|\le a\\ 0, & \text{其它}\end{cases}$$

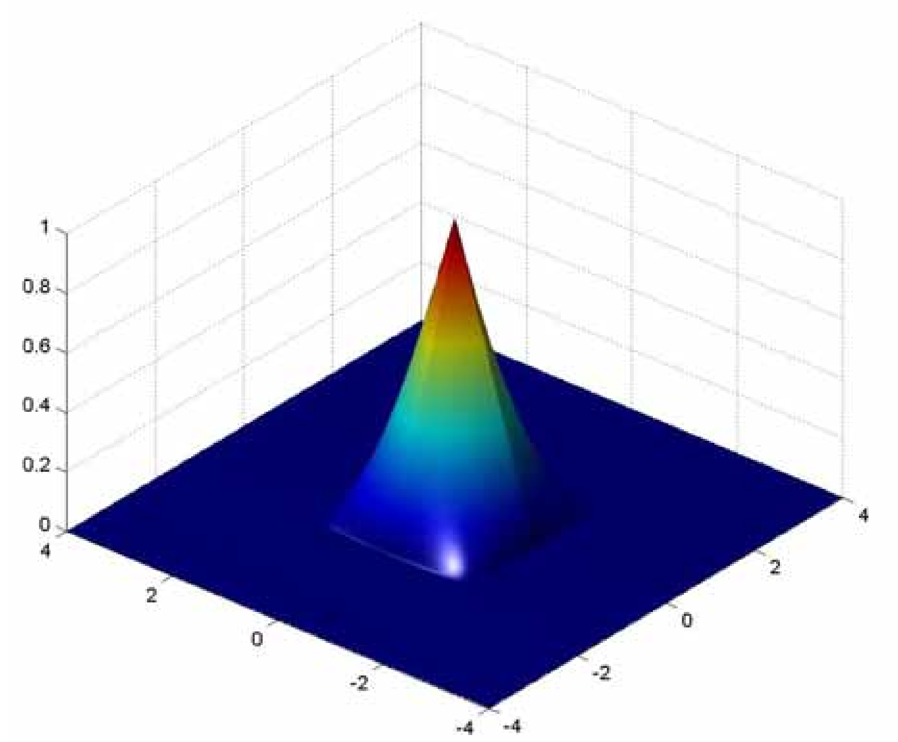





```
function y = tri(t)
%TRI Sampled aperiodic triangle generator.
%   TRI(T) generates samples of a continuous, aperiodic,
%   unity-height triangle at the points specified in array T, centered
%   about T=0.
% Returns non-zero in interval [-1,+1)
t = abs(t);
y = zeros(size(t));
idx = find(t<1.0);
y(idx)=1.0-t(idx);
```

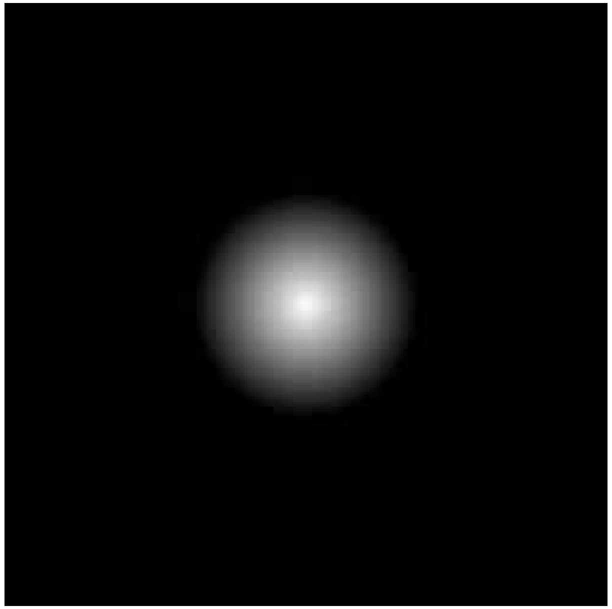
2D 三角函数 DrawTriangle_5.m

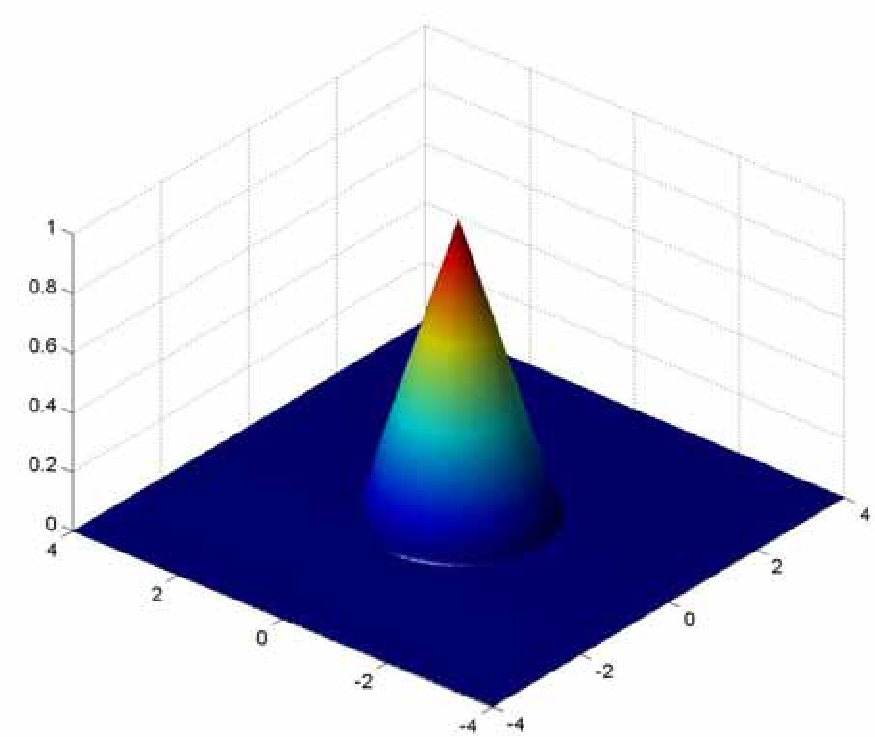





```
>> [x,y] = meshgrid(linspace(-4,4,251));
>> z = tripuls(x/3).*tripuls(y/3);
>> imwrite(z,‘tri.jpg');
>> mysurf(x,y,z);
```

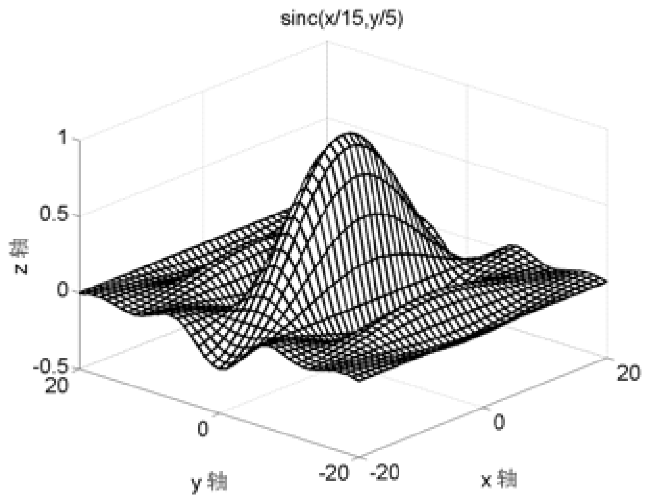
2D 锥形函数 DrawCone_6.m






```
>> [x,y] = meshgrid(linspace(-4,4,251));
>> r = sqrt(x.*x+y.*y);
>> z = tripuls(r/3);
>> imwrite(z,‘cone.jpg');
>> mysurf(x,y,z);
```

– Sinc函数

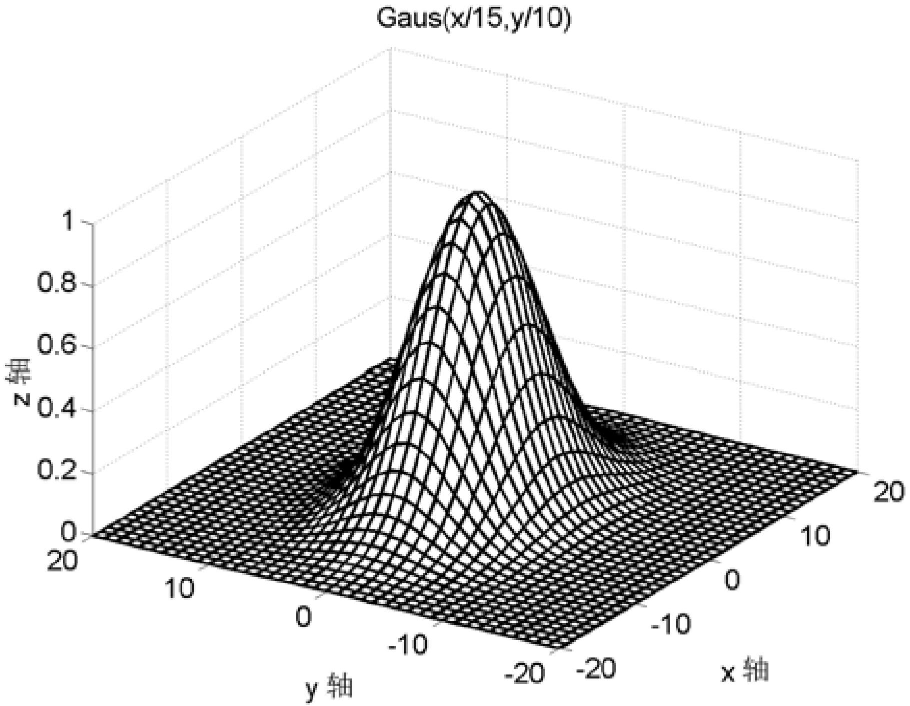



```
function y=sinc(x)
%SINC Sin(pi*x)/(pi*x) function.
%   SINC(X) returns a matrix whose
%elements are the sinc of the elements  of X, i.e.
%        y = sin(pi*x)/(pi*x)    if x ~= 0
%          = 1                   if x == 0
%   where x is an element of the input matrix and y is the resultant
%   output element.
%   See also SQUARE, SIN, COS, CHIRP, DIRIC, GAUSPULS,
    PULSTRAN, RECTPULS,
%   and TRIPULS.
%   Author(s): T. Krauss, 1-14-93
%   Copyright (c) 1988-98 by The MathWorks, Inc.
%  $Revision: 1.2 $  $Date: 1998/07/13 19:02:12 $
y=ones(size(x));
i=find(x);
y(i)=sin(pi*x(i))./(pi*x(i));
```

$$\sin c(\frac{x}{a}) = \frac{\sin(\frac{\pi x}{a})}{\frac{\pi x}{a}}$$

– 高斯函数

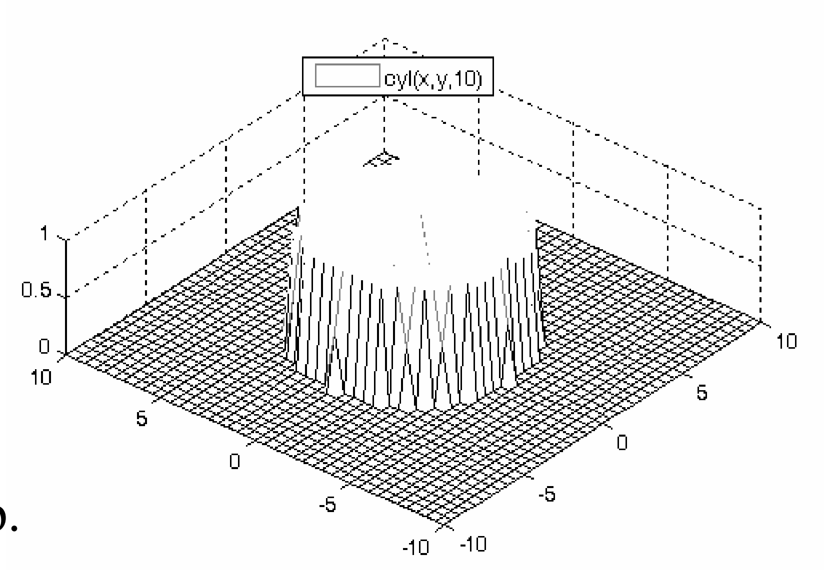
$$\mathrm{Gaus}\left(\frac{x}{a}\right) = \exp\left[-\pi\left(\frac{x}{a}\right)^2\right]$$




```
function y=gaus(x)
%   GAUS exp(-pi*N*x.*x) function.
%   By default, the GAUS d is 1.
y=exp(-pi*x.*x);
```

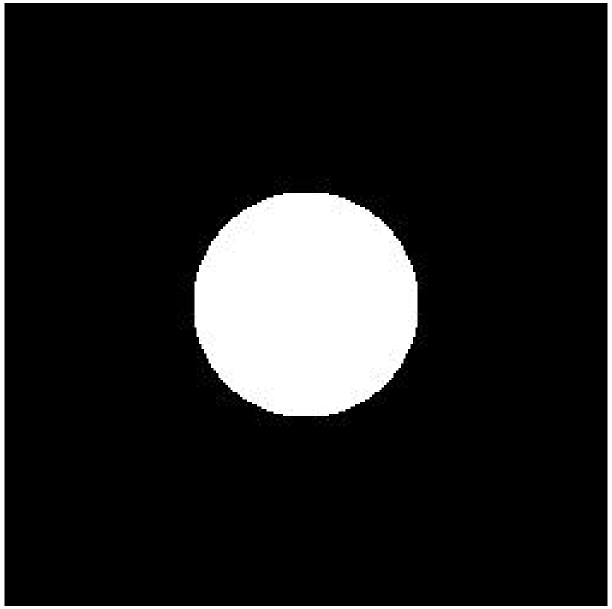
– 园域函数

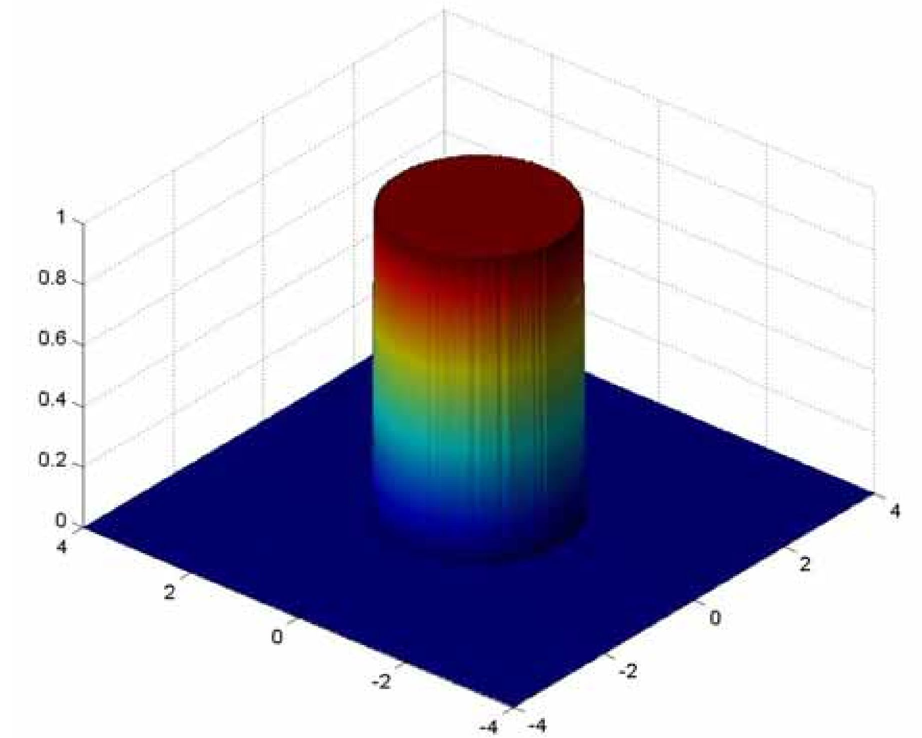
$$circ\left(\frac{\sqrt{x^2+y^2}}{r_0}\right)=\begin{cases}1, \sqrt{x^2+y^2}\le r_0 \\ 0, \qquad\qquad 其它\end{cases}$$




```
function z = cyl(x,y,d)
%CYL Sampled aperiodic cylindrical step.
%   CYL(X,Y) generates samples of a continuous, aperiodic,
%   unity-height cylinder at the points specified in array (X,Y)
%   about X=Y=0.  By default, the cylinder has diameter 1.

%   CYL(X,Y) is a cylinder of height 1 and diameter 1
%   CYL(X,Y,D) is a cylinder of diameter d
error(nargchk(2,3,nargin));
if nargin<3, d=1; end
r = sqrt(x.*x+y.*y);
z = (r<d/2)*1.0;
z(find(r==d/2))=0.5;
```

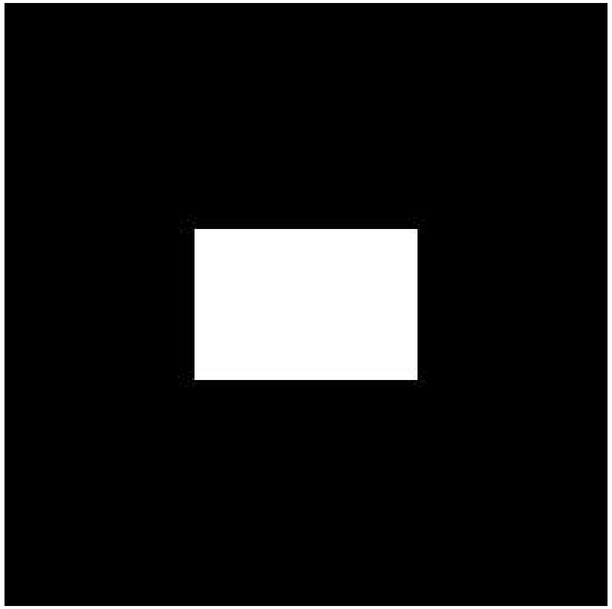
1、园域函数 DrawCyl_1.m

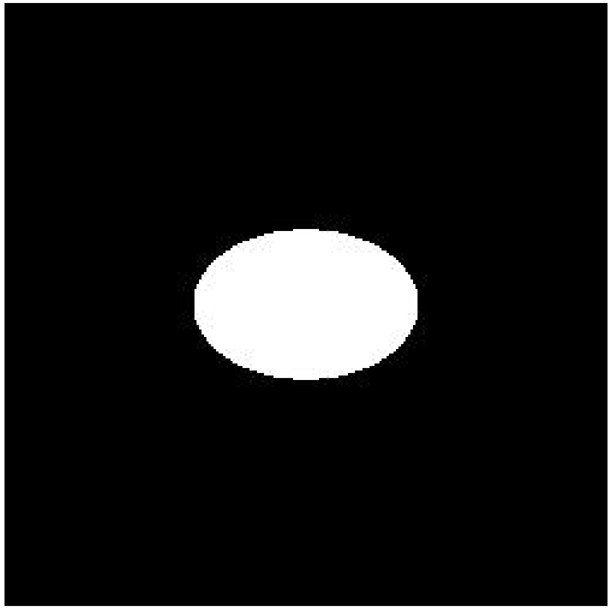





```
>> [x,y] = meshgrid(linspace(-4,4,251));
>> r = sqrt(x.*x+y.*y); ??
>> z = cyl(x/3,y/3);
>> mysurf(x,y,z);
>> imwrite(z,‘cyl3.jpg');
```

函数的变形



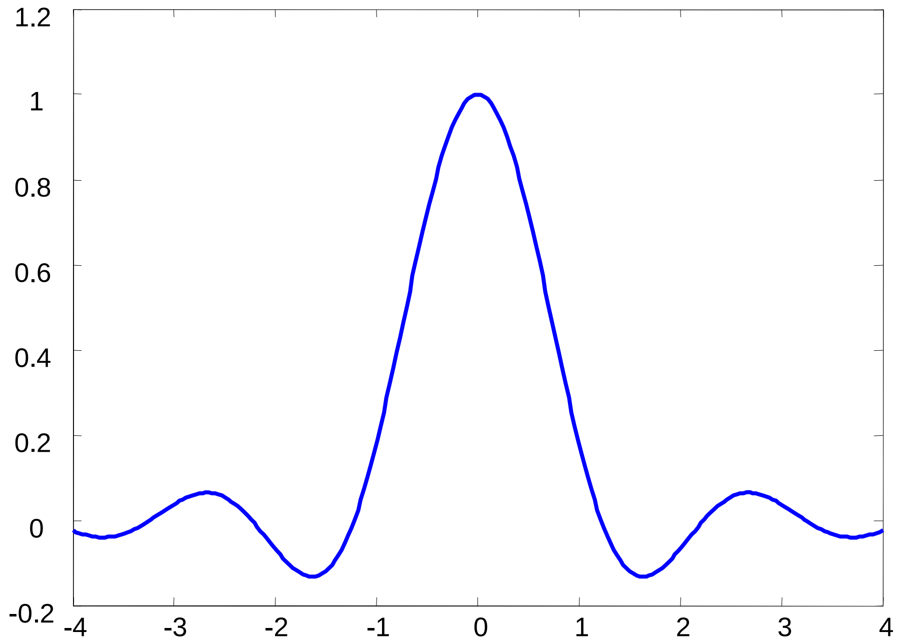


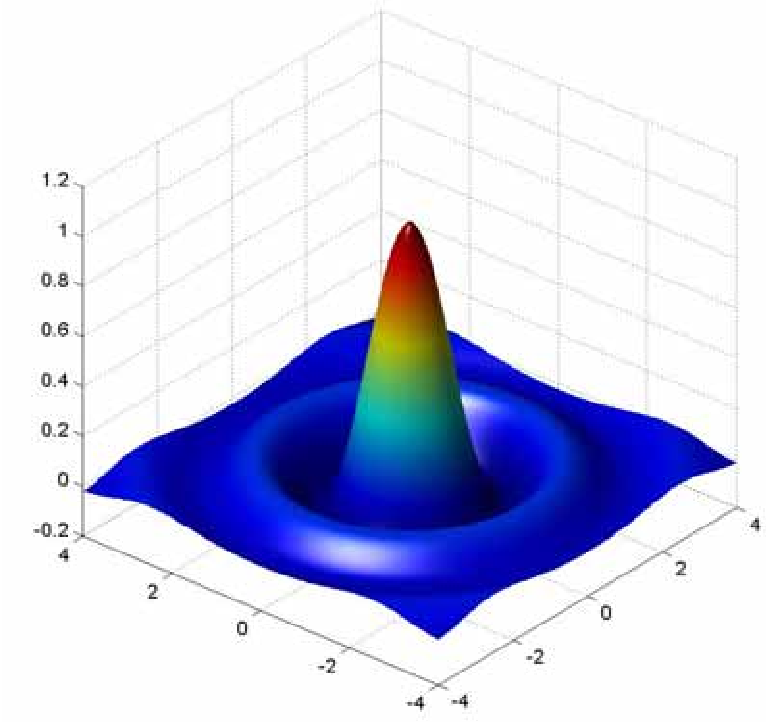
```
>> z = rect(x/3).*rect(y/2);
>> imwrite(z,'ana_rect.jpg');
```

```
>> z = cyl(x/3,y/2);
>> imwrite(z,'ana_cyl.jpg');
```

Somb 函数

DrawSymbol_2.m





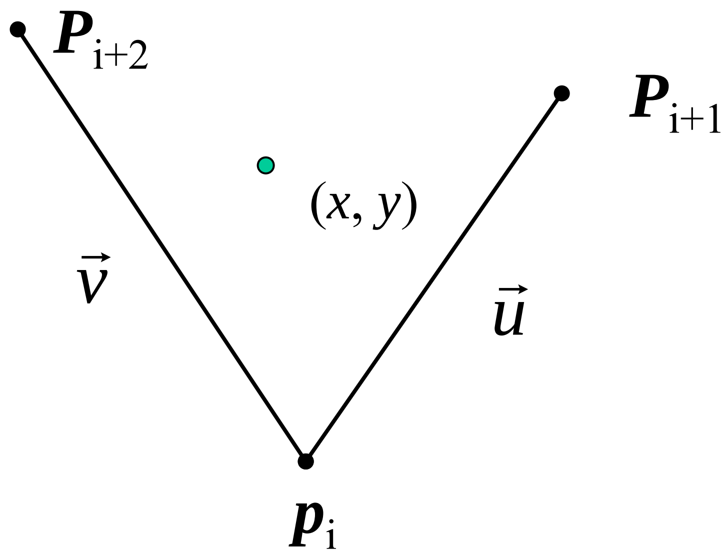

```
>> [x,y] = meshgrid(linspace(-4,4,251));
>> r = sqrt(x.*x+y.*y);
>> z = somb(r);
>> mysurf(x,y,z);
>> r = linspace(-4,4,251);
>> z = somb(r);
>> plot(r,z);
```

$$\mathrm{somb}(r) = \frac{2\,\mathrm{J}_1(\pi r)}{\pi r}$$

多边形函数

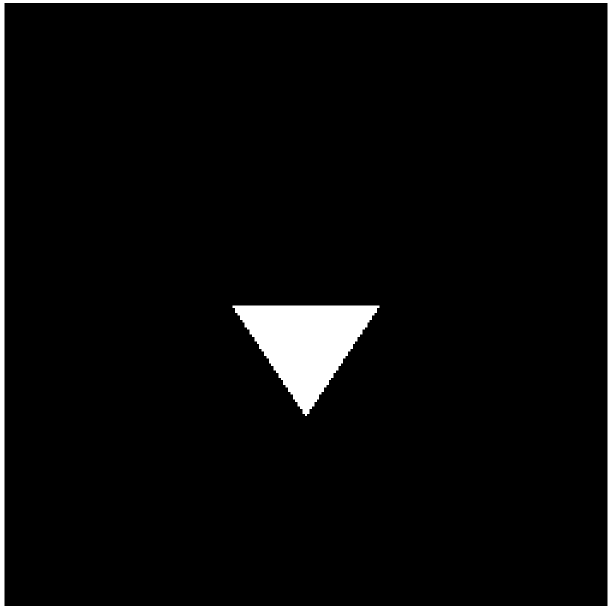
DrawPolyGonTriangle_7.m

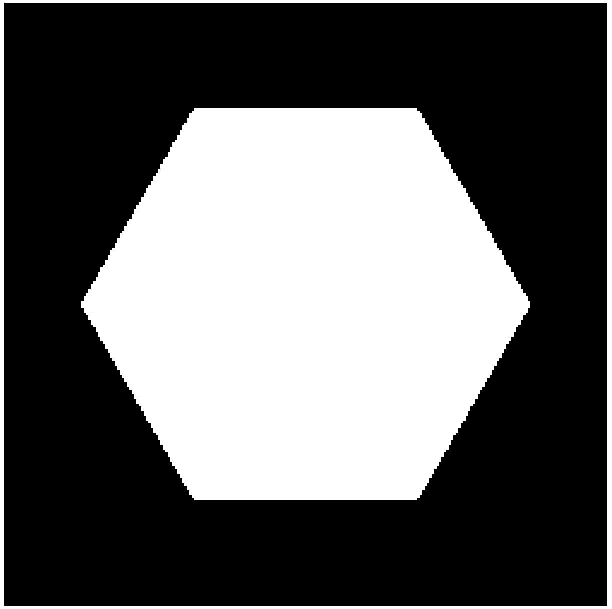
```
function z = polygon(x,y,p)


if nargin<3
    p = [ -1 0; 1 0; 0 1.5 ];
end

n = size(p,1);
z = ones(size(x));
for i=1:n,
    u = p(mod(i,n)+1,:) - p(i,:);
    v = p(mod(i+1,n)+1,:) - p(i,:);
if ((u(1)*v(2)-u(2)*v(1))<0)
    u = -u;
end
s = u(1)*(y-p(i,2))-u(2)*(x-p(i,1));
z = z.*(s>0);
end
```



判断一个点是否处在P矢量围成的范围内

多边形举例



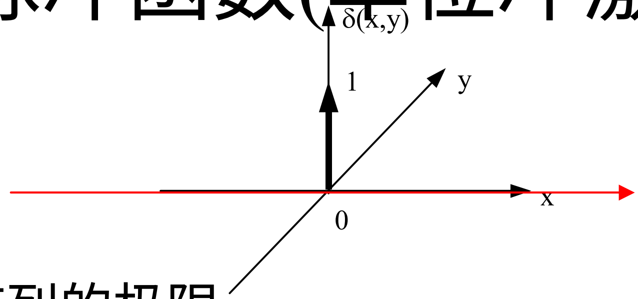


```
>> [x y] = meshgrid(linspace(-4,4,251));
>> z = polygon(x,y);
>> imshow(z);%??
>> imwrite(z,'triangle.bmp');
```

```
>> theta = (pi/3)*(0:5);
>> p = [cos(theta') sin(theta')];
>> z = polygon(x,y,3*p);
>> imshow(z);
>> imwrite(z,'hexagon.bmp');
```

脉冲函数(单位冲激函数)

– 定义与性质



$$\begin{cases} \delta(x,y)=0, \quad x\neq 0 \text{或} y\neq 0 \\ \iint_{-\infty}^{\infty} \delta(x,y)dxdy=1 \end{cases}$$

– 可以表示成序列的极限

$$\delta(x)=\lim_{N\to\infty} N\mathrm{rect}(Nx)$$

$$\delta(x)=\lim_{N\to\infty} N\exp(-N^2\pi x^2)$$

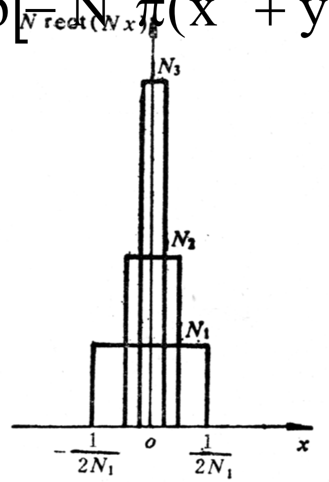
$$\delta(x,y)=\lim_{N\to\infty} N^2\mathrm{rect}(Nx)\mathrm{rect}(Ny)$$

$$\delta(x,y)=\lim_{N\to\infty} N^2\exp\left[-N^2\pi(x^2+y^2)\right]$$

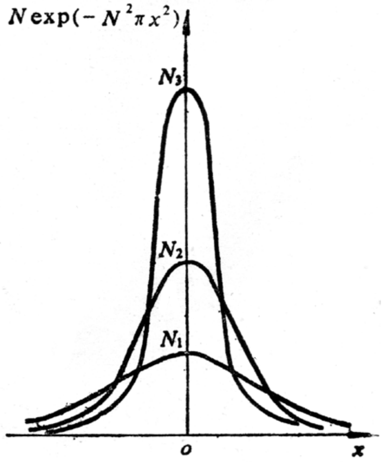
$$\delta(x,y)=\lim_{N\to\infty} N^2\mathrm{sinc}(Nx)\mathrm{sinc}(Ny)$$

$$\delta(x,y)=\lim_{N\to\infty} \frac{N^2}{\pi}\mathrm{circ}(N\sqrt{x^2+y^2})$$

$$\delta(x,y)=\lim_{N\to\infty} N\frac{J_1(2\pi N\sqrt{x^2+y^2})}{\sqrt{x^2+y^2}}$$



a)



b)

脉冲函数性质

– 性质

$$\iint_{-\infty}^{\infty} \delta(x-x_0, y-y_0)\phi(x,y)dxdy = \phi(x_0, y_0)$$

筛选性质（得到某点函数值）

比例变化性质（范围减少，式左边为1/|ab|，右边也应减少）

$$\delta(ax, by) = \frac{1}{|ab|}\delta(x, y)$$

与普通函数乘积的性质（相当于对该点取样

$$h(x,y)\delta(x-x_0, y-y_0) = h(x_0, y_0)\delta(x-x_0, y-y_0)$$

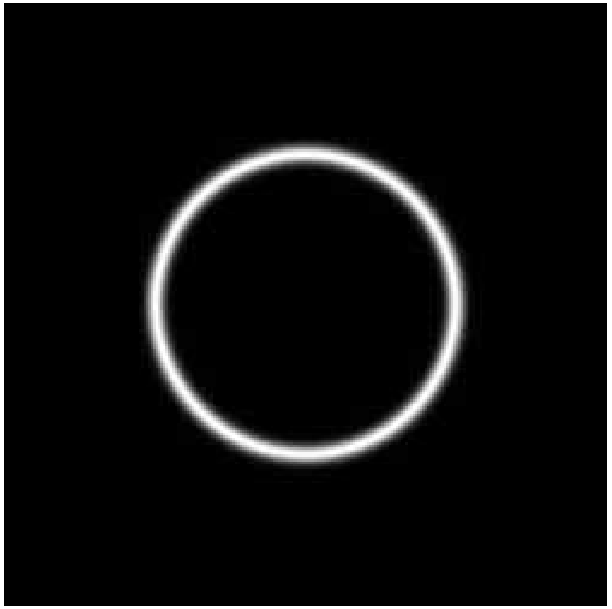
x=linspace(-4,4,521);

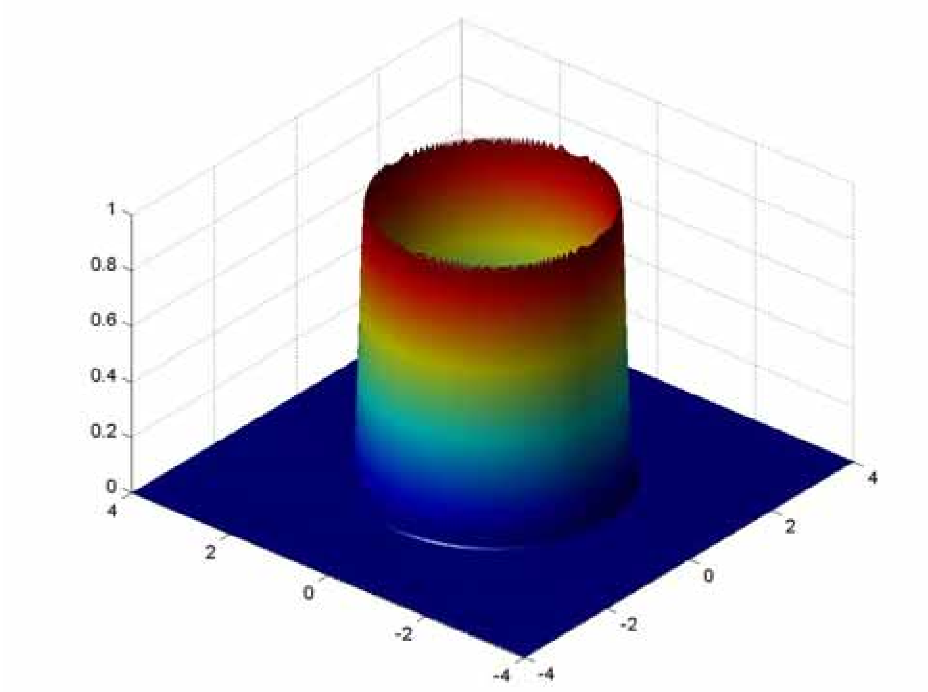
z=100*gaus(10000*x);

plot(x,z/100);%注意除100的意义(wcy wcy j o b)

环输入

DrawRing_3.m






```
>> [x,y] = meshgrid(linspace(-4,4,251));
>> r = sqrt(x.*x+y.*y);
>> z = gaus((r-2)/0.2);
>> imwrite(z,'ring.jpg');
>> colormap(jet);
>> mysurf(x,y,z);
```

$$\delta(r - r_0)$$

vs.

$$\delta(\vec{r} - \vec{r}_0)$$

复合函数

$$f\left(g\left(x\right)\right)= f\circ g\left(x\right)$$

$$f\left(ax+b\right)= f\circ l\left(x\right)$$

这里 $l\left(x\right)=ax+b$

带参数的函数 (或操作数)

$$\mathrm{linear}[a,b]\{x\}=ax+b$$

$$f\left(ax+b\right)= f\circ \mathrm{linear}\left[a,b\right]\{x\}$$

疏函数(Comb=疏子)

– 定义和性质

$$\sum_{n=-\infty}^{\infty}\delta(x-n)=\mathrm{comb}(x)$$

间隔为1的无穷多脉冲序列

$$\sum_{n=-\infty}^{\infty}\delta(x-n\tau)=\frac{1}{\tau}\sum_{n=-\infty}^{\infty}\delta\left(\frac{x}{\tau}-n\right)=\frac{1}{\tau}\mathrm{comb}\left(\frac{x}{\tau}\right)$$

间隔为τ的无穷多脉冲序列

$$f(x)\bullet\frac{1}{\tau}\mathrm{comb}\left(\frac{x}{\tau}\right)=\sum_{n=-\infty}^{\infty}f(n\tau)\delta(x-n\tau)$$

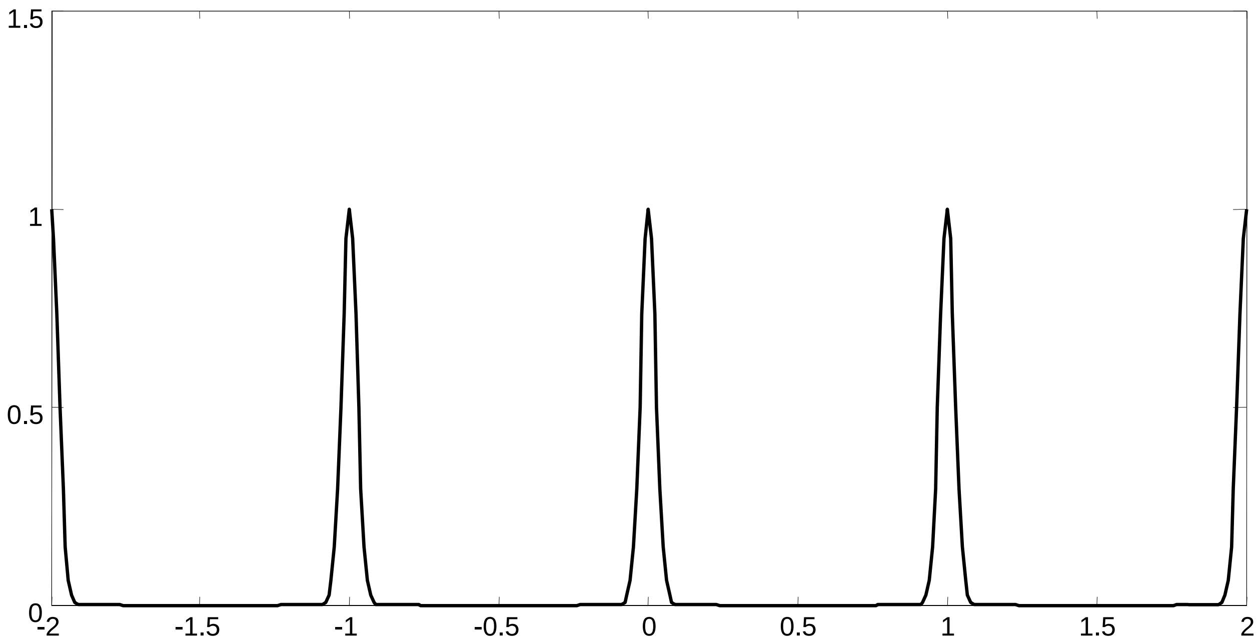
等间隔取样

疏函数的MATLAB实现

```
function y =comb(x,p)
% comb function
% P 周期
error(nargchk(1,2,nargin));
if nargin<2, p=1;   end
y = 1000*gaus(1000000*sin(pi*x/p));
```

在实际应用时，x 要踩在取无穷大值的点上，　否则显示会造成遗漏情况

复合函数举例 DrawComb_4.m




```
>> x = linspace(-2,2,401);
>> y = gaus(sin(pi*x)/0.2);
>> plot(x,y,'k','LineWidth',1.5);
>> axis([-2 2 0 1.5]);
```

用gaus 函数来模拟comb函数

•卷积

– 定义和性质

$$g(x,y)=\iint_{-\infty}^{\infty} f(\xi,\eta)h(x-\xi,y-\eta)d\xi d\eta = f(x,y)*h(x,y)$$

– 卷积的四个过程

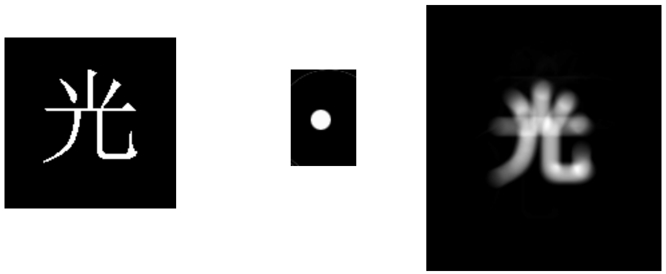
折叠、位移、相乘、积分

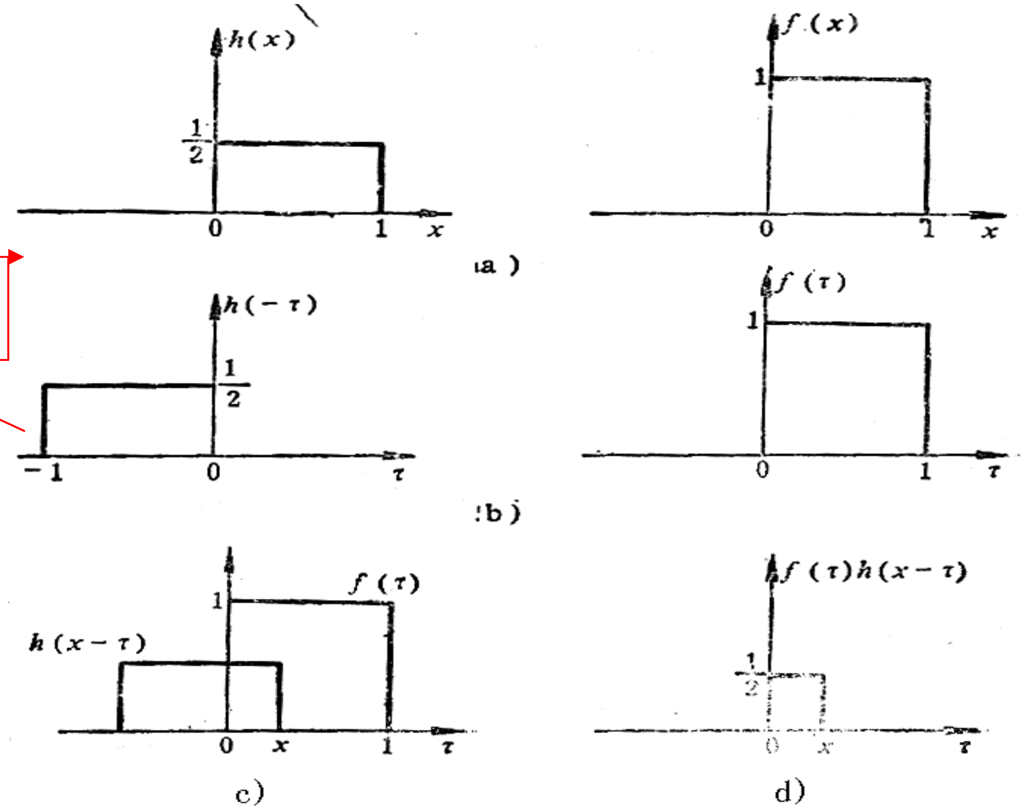
折叠

$\boxed{\text{位移、相乘、积分}}$

– 卷积的两个效应

展宽、平滑





a) 函数 f(x)和 h(x)　b) 折叠　c) 位移　d) 相乘

•卷积

– 卷积运算定理

$$f(x,y)*h(x,y)=h(x,y)*f(x,y) \longrightarrow$$ 交换律

$$[v(x,y)+w(x,y)]*h(x,y)=v(x,y)*h(x,y)+w(x,y)*h(x,y) \longrightarrow$$ 分配律

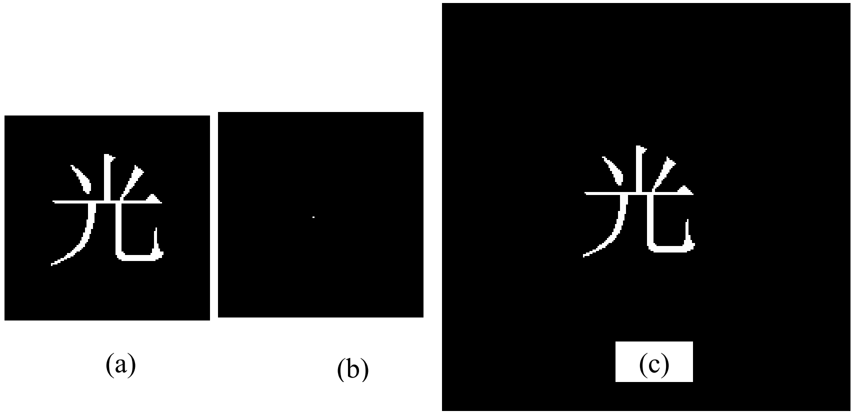
$$[v(x,y)*w(x,y)]*h(x,y)=v(x,y)*[w(x,y)*h(x,y)] \longrightarrow$$ 结合律

– 包含脉冲函数的卷积

$$f(x,y)*\delta(x,y)=\iint_{-\infty}^{\infty} f(\xi,\eta)\delta(x-\xi,y-\eta)d\xi d\eta$$

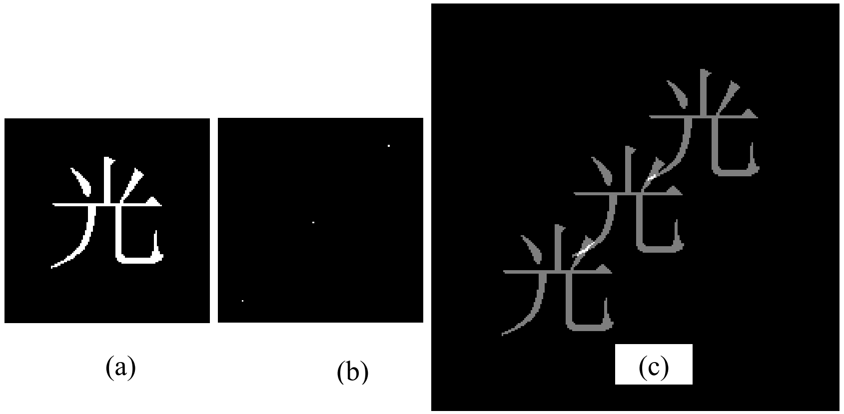
$$f(x,y)*\delta(x,y)=\iint_{-\infty}^{\infty} f(\xi,\eta)\delta(\xi-x,\eta-y)d\xi d\eta=f(x,y)$$

$$f(x,y)*\delta(x-x_0,y-y_0)=f(x-x_0,y-y_0)$$



(a) (b) (c)

第三个卷积的结果是把函数平移到脉冲函数所在的位置

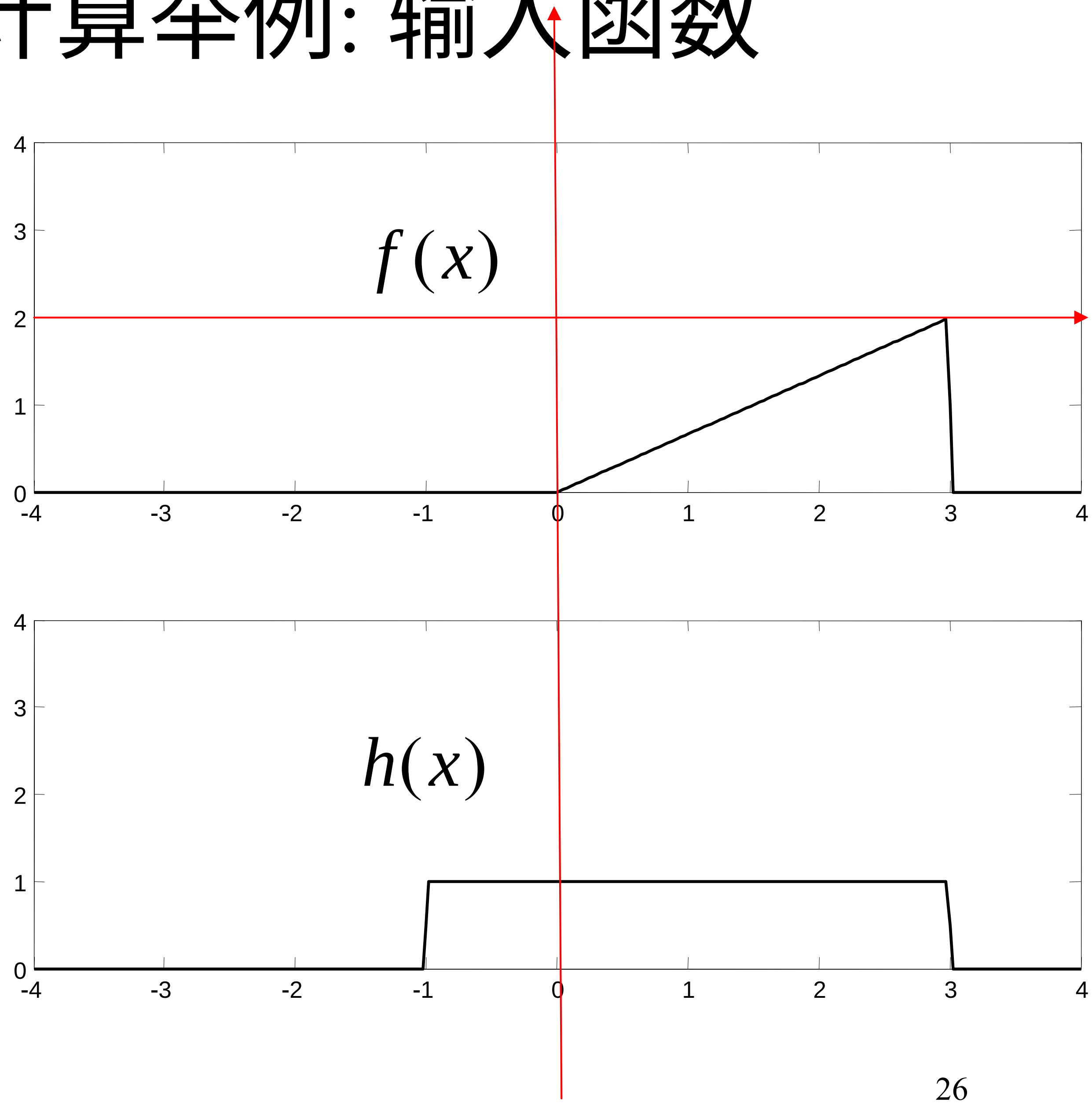


(a) (b) (c)

卷积计算举例: 输入函数

```
function y = f(x)
y=zeros(size(x));
idx = find(x>0 & x<3);
y(idx) = 2*x(idx)/3;
idx = find(x==3);
y(idx)=1;
```

```
function y = h(x)
y = rect(x-1,4);
```



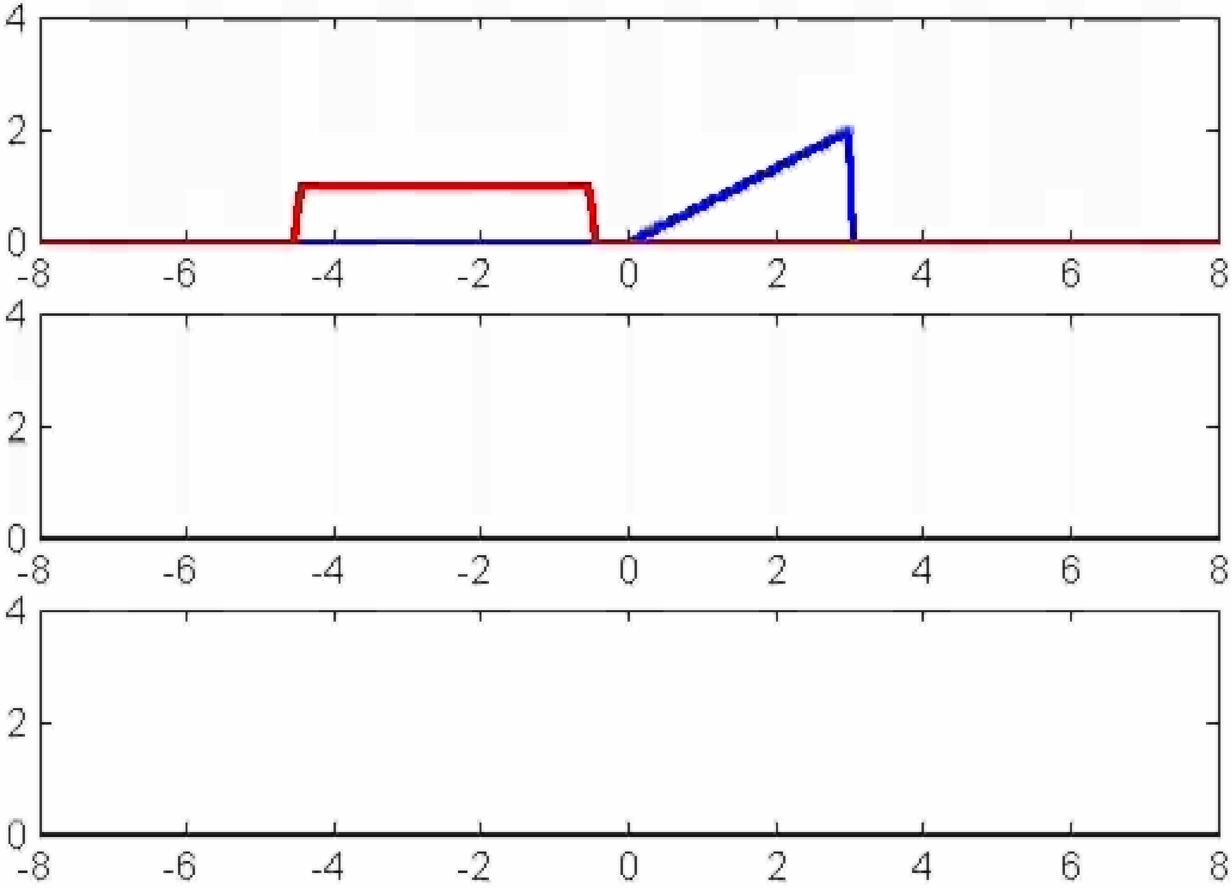
卷积演示

$h(x\text{-}\alpha)$

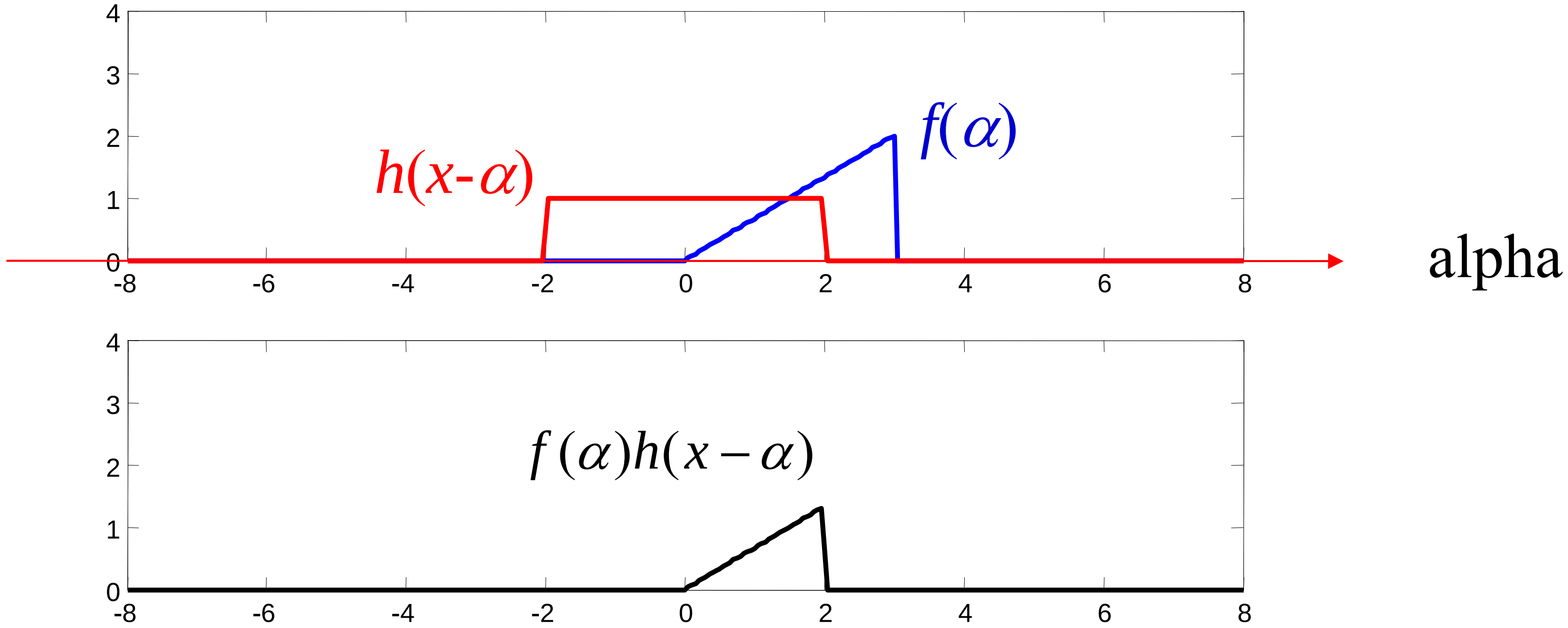
$f(\alpha)$

$f(\alpha)h(x-\alpha)$

$\int_{-\infty}^{\infty} f(\alpha)h(x-\alpha)d\alpha$



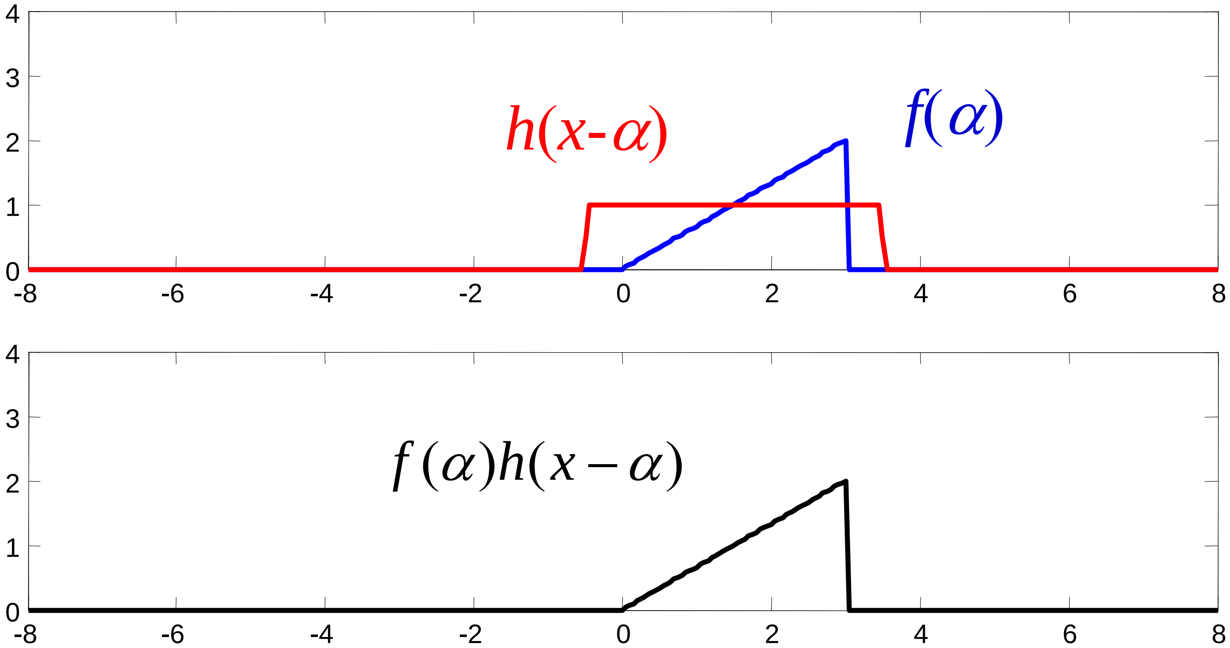
第一区间 ($x = 1$)



$$\int_{-\infty}^{\infty} f(\alpha)h(x-\alpha)d\alpha$$

$$\int_0^{x+1} \frac{2}{3}\alpha d\alpha = \frac{1}{3}\alpha^2\Big|_0^{x+1} = \frac{1}{3}(x+1)^2$$

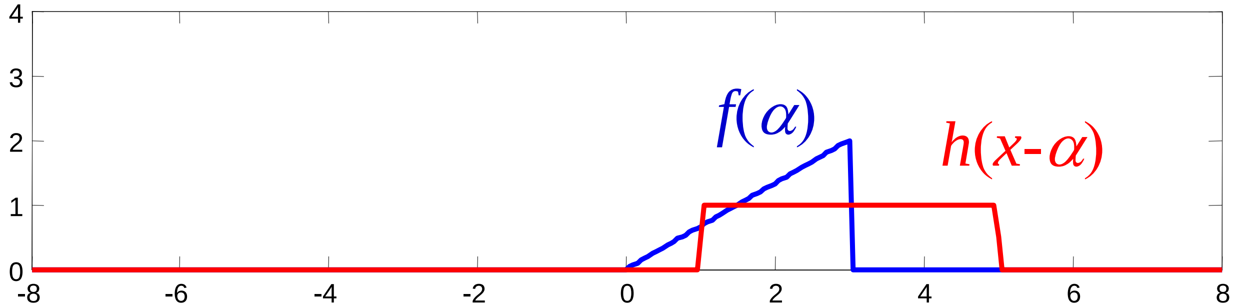
第二区间 $(x = 2.5)$

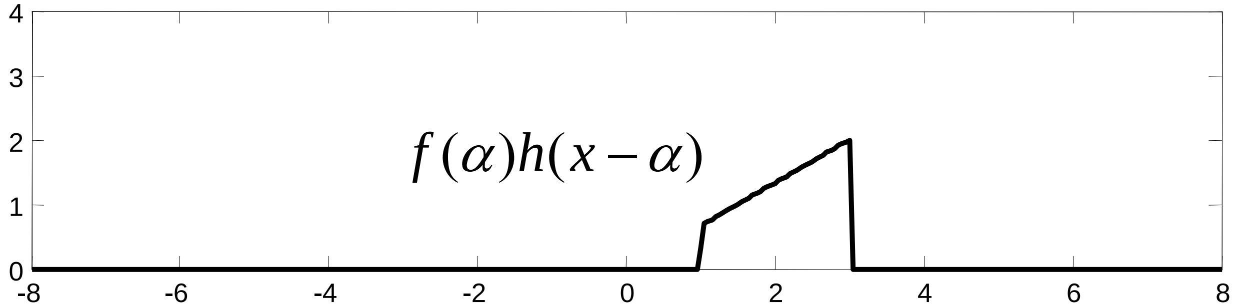


$$\int_{-\infty}^{\infty} f(\alpha)h(x-\alpha)d\alpha$$

$$\int_0^3 \frac{2}{3}\alpha d\alpha = \frac{1}{3}\alpha^2 \Big|_0^3 = 3$$

第3区间 $(x = 4)$





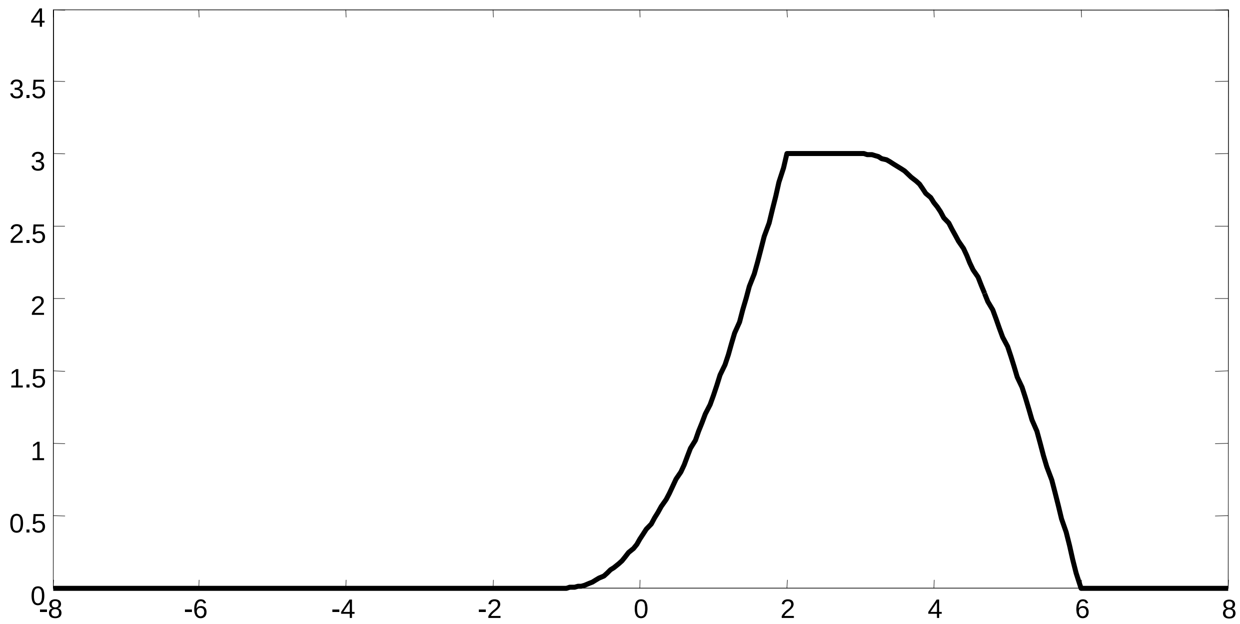
$$\int_{-\infty}^{\infty} f(\alpha)h(x-\alpha)d\alpha$$

$$\int_{x-3}^{3} \frac{2}{3}\alpha d\alpha = \frac{1}{3}\alpha^2\Big|_{x-3}^{3} = 3 - \frac{1}{3}(x-3)^2$$

最后结果

DrawfhConv_1.m

```
function y = g(x)

y = zeros(size(x));

%region 1
idx = find(x>-1 & x<2);
if ~isempty(idx)
    t=x(idx)+1;
    y(idx)=t.*t/3;
end

%region 2
idx = find(x>=2 & x<=3);
if ~isempty(idx)
    y(idx)=3;
end

%region 3
idx = find(x>3 & x<6);
if ~isempty(idx)
    t = x(idx)-3;
    y(idx)=3-t.*t/3;
end
```




```
>> plot(x,g(x))
>> plot(x,g(x),'k','LineWidth',2);
>> axis([-8 8 0 4]);
```

晕

Matlab conv函数

w = conv(u,v)

定义

假使t m = length(u)， n = length(v). w 为一 m+n-1长度的矢量，其第K个元素为

$$w(k) = \sum_{j} u(j)v(k+1-j)$$

这里对j的和是对所有的u(j)和 v(k+1-j)进行, 即j = max(1,k+1-n): min(k,m).

Matlab conv函数

w(1) = u(1)*v(1)

w(2) = u(1)*v(2)+u(2)*v(1)　　　　(u：12，u：21)

w(3) = u(1)*v(3)+u(2)*v(2)+u(3)*v(1)　　　　（u：123，v：321）

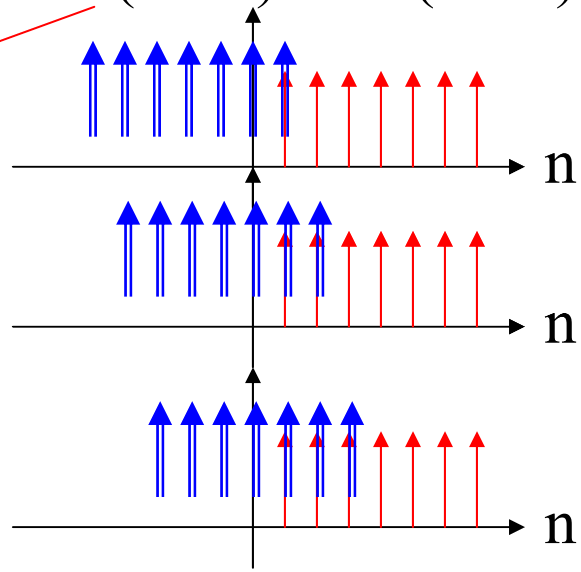
w(n) = u(1)*v(n)+u(2)*v(n-1)+ ... +u(n)*v(1)　　　　**u:123…n**

v:n…321

w(2*n-1) = u(n)*v(n)

(7<-1)v　　u(1->7)

注意反转



w(1)

w(2)

w(3)

Matlab `convn` 函数

N-D 卷积

```
C = convn(A,B)
C = convn(A,B,'shape')
```

`C = convn(A,B)` 计算阵列**A** and **B**的 N-D卷积. 结果的大小为 **`size(A)+size(B)-1`**.

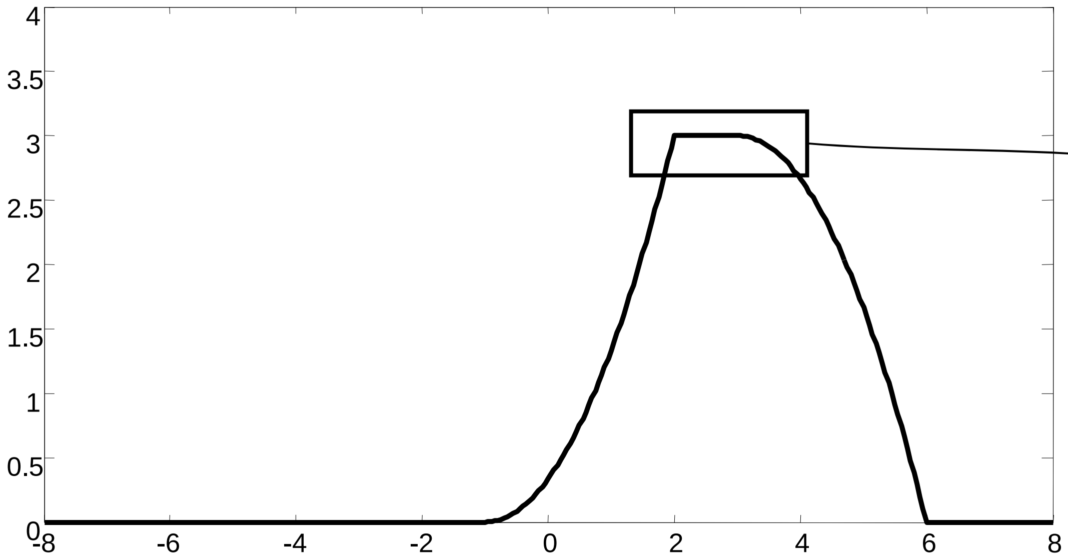
`C = convn(A,B,'`*shape*`')` 返回 N-D 卷积， 阵列大小由 ***shape*** 参数确定 :

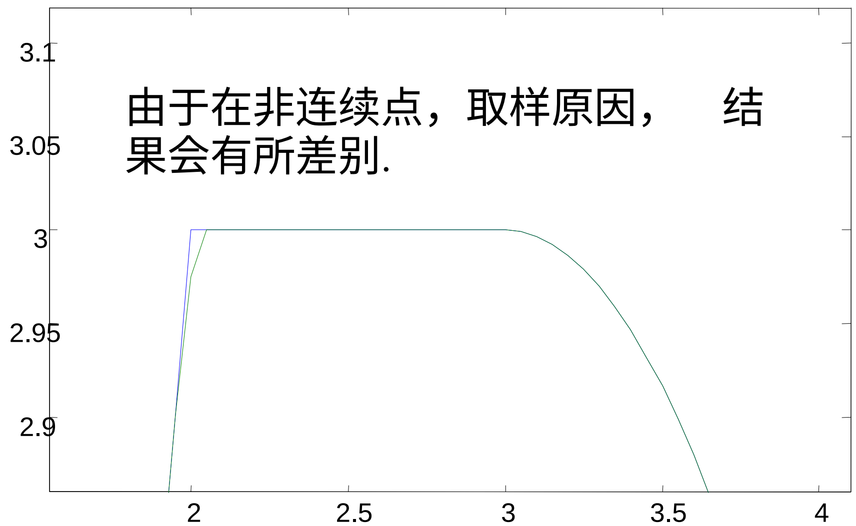
- **`'full'`** 缺省尺寸 (default).
- **`'same'`** 尺寸同 **A**.
- **`'valid'`** 未补0的尺寸,其尺寸为 max(size(A)-size(B) + 1, 0)

  size(A) >= size(B)

convn vs. 准确函数比较

DrawfhConvn_2.m



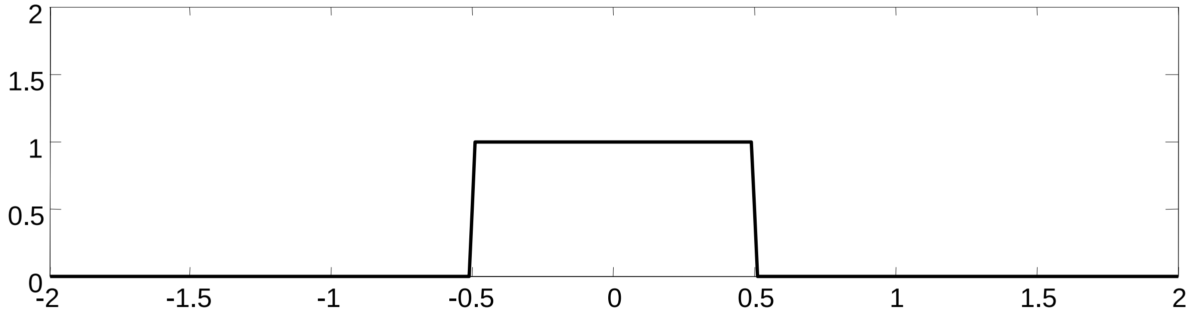


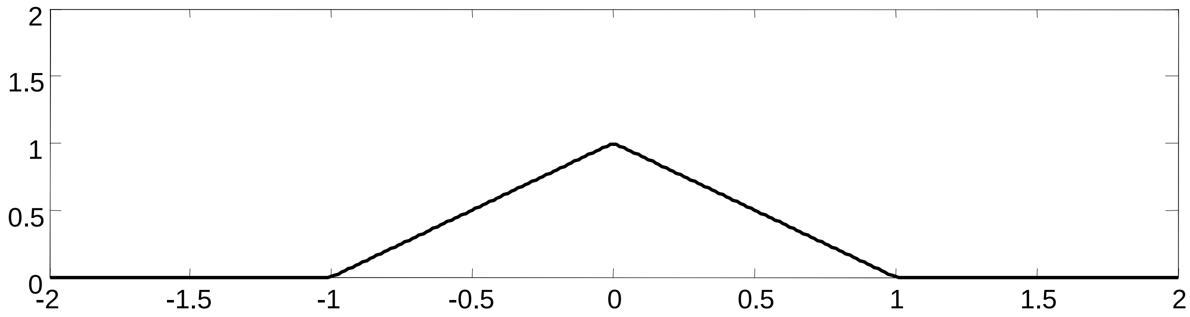
$$f(x)*h(x)=\mathrm{convn}(f(x),h(x),\text{'same'})\Delta x$$

```
x = linspace(-8,8,321);
dx = x(2)-x(1);
g1 = g(x);
g2 = convn(f(x),h(x),'same')*dx;
plot(x,g1,x,g2);
```

DrawRectRectConv_3.m

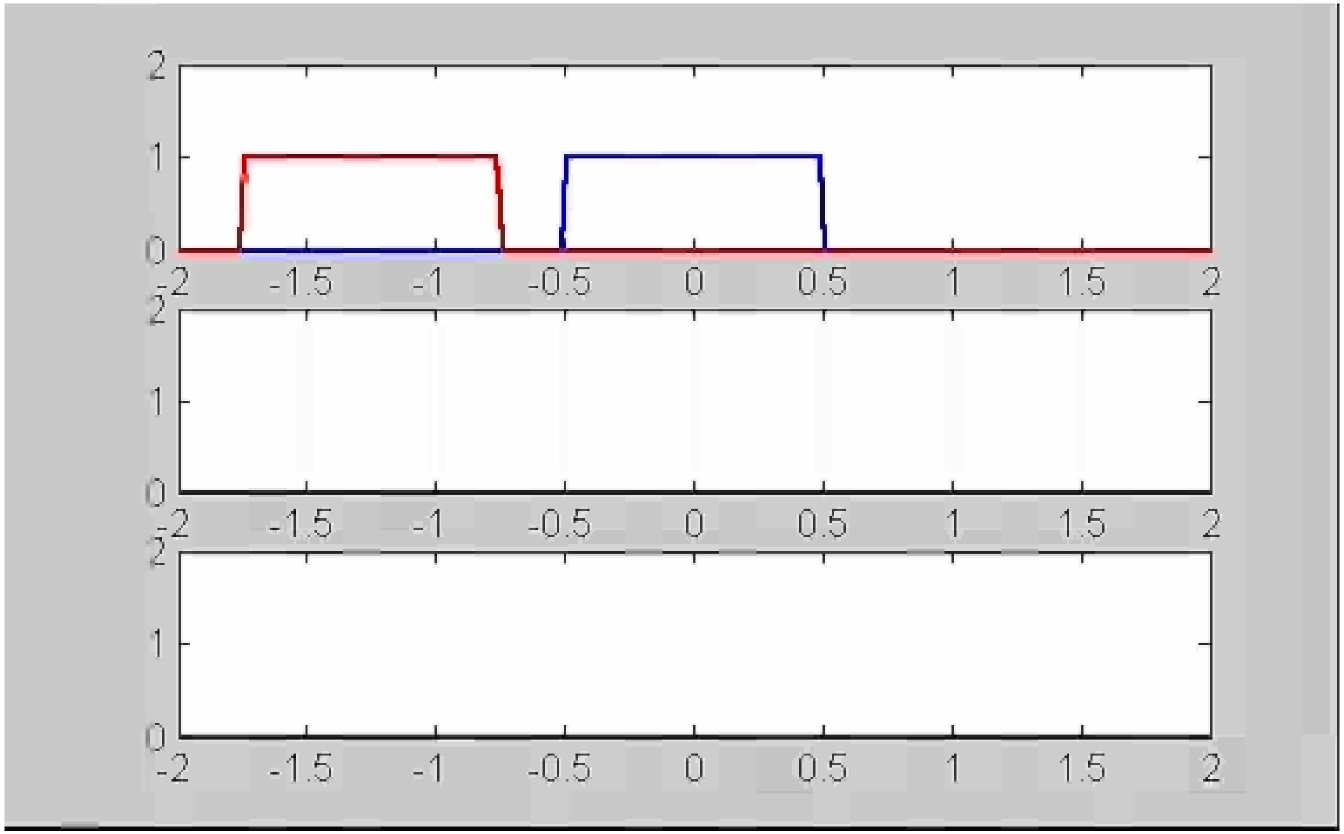
$\mathrm{rect}(x) * \mathrm{rect}(x) = \mathrm{tri}(x)$






```
>> x = linspace(-2,2,321);
>> dx=x(2)-x(1);
>> subplot(2,1,1)
>> plot(x,rect(x),'k','LineWidth',1.5);
>> axis([-2 2 0 2]);
>> g = convn(rect(x),rect(x),'same')*dx;
>> subplot(2,1,2);
>> plot(x,g,'k','LineWidth',1.5);
>> axis([-2 2 0 2]);
```

rect(x)*rect(x)



•相关

用于描述两输入之间相似性的量度

– 定义和性质

$$r_{fg}(x,y)=\iint_{-\infty}^{\infty} f(\xi,\eta)g^*(\xi-x,\eta-y)d\xi d\eta = f(x,y)\otimes g(x,y)$$

– 相关的四个过程

取复共轭、位移、相乘、积分

– 自相关

$$r_{ff}(x,y)=\iint_{-\infty}^{\infty} f(\xi,\eta)f^*(\xi-x,\eta-y)d\xi d\eta = f(x,y)\otimes f(x,y)$$

– 自相关性质

$$|r_{ff}(x,y)|\le r_{ff}(0,0)$$

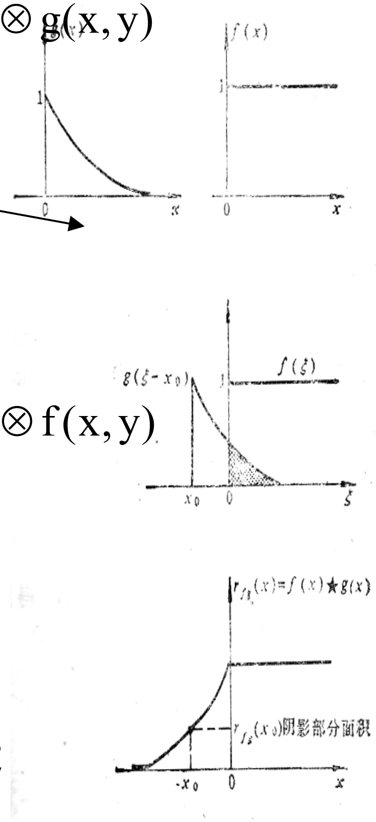
完全重合时，达到完全匹配

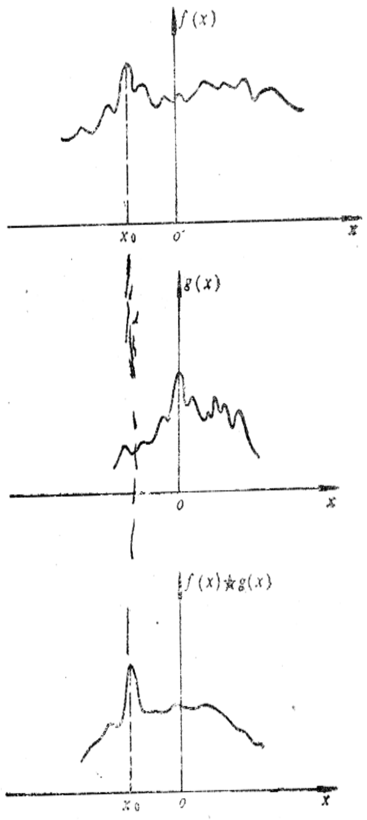
$$r_{ff}(x,y)=r_{ff}^*(-x,-y)$$

复函数f(x,y)，相关是厄米的

$$r_{ff}(x,y)=r_{ff}(-x,-y)$$

实函数f(x,y)，相关是偶函数





互相关运算不满足交换律

$$r_{fg}(x,y)=r_{gf}^*(-x,-y)$$

$$r_{fg}(x,y)\ne r_{fg}(x,y)$$

•相关

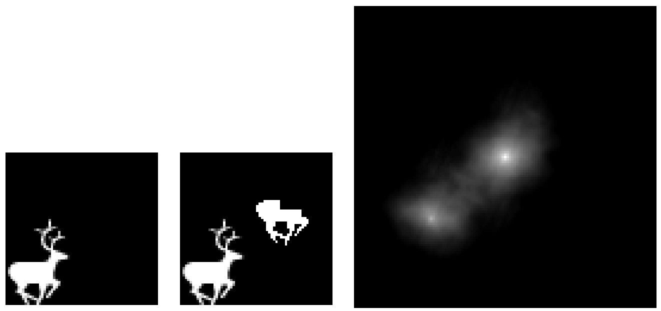
– 相关与卷积的关系

$$f(x)\otimes g(x)=\int_{-\infty}^{\infty} f(\xi)g^*(\xi-x)d\xi=\int_{-\infty}^{\infty} f(\xi)g^*\left(\frac{x-\xi}{-1}\right)d\xi=f(x)*g^*(-x)$$

(a) (b) (b) (a) (b) (b)

f(x,y)与g(x,y)的相关　　g(x,y) 与f(x,y)的相关

– 相关的相似性量度



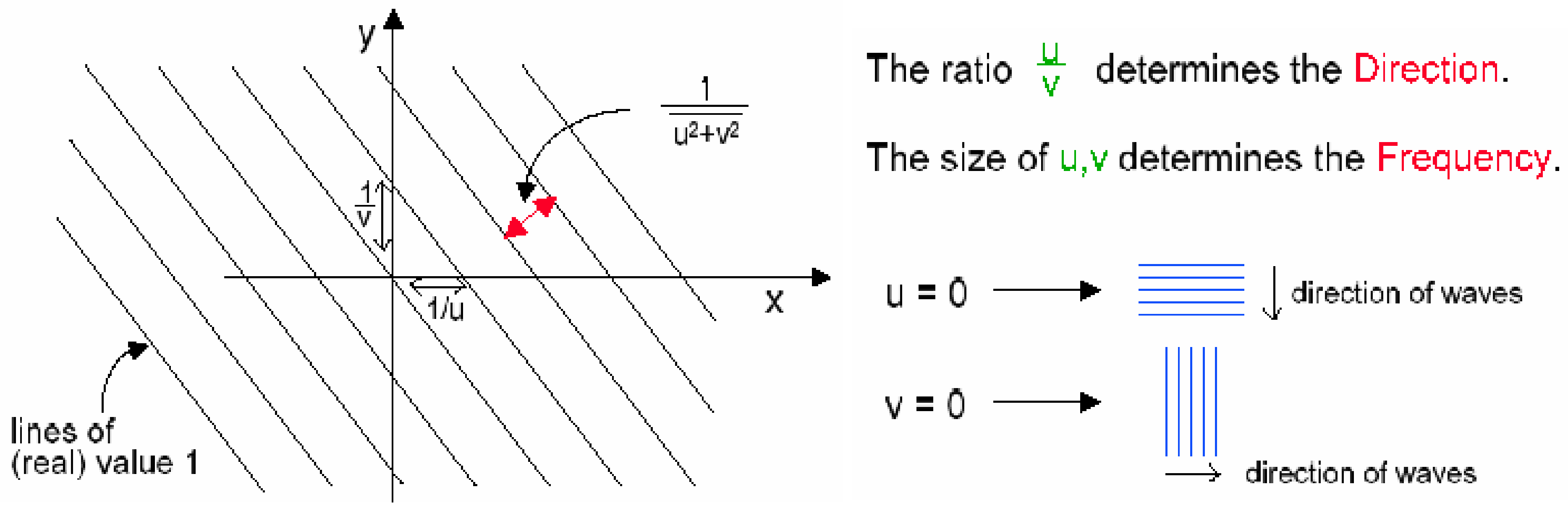
(a) (b) (c)

2D空间频率

物分布

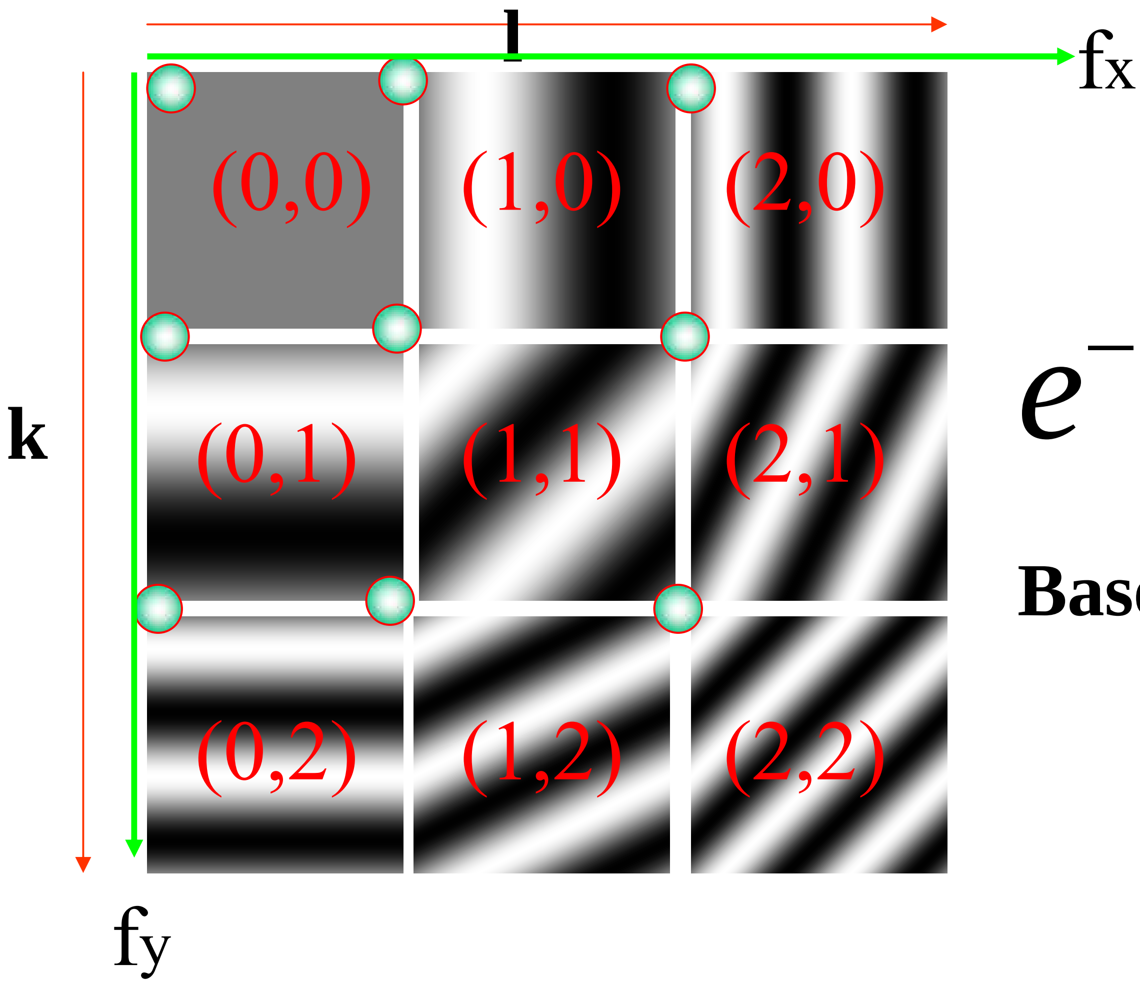
$$A\cos(x\cdot 2\pi i/N)\, B\cos(y\cdot 2\pi j/M) = A\cos(x\cdot 2\pi f_x)\, B\cos(y\cdot 2\pi f_y)$$

这里我们发现 $f_x = u = i/N,\quad f_y = v = j/M$



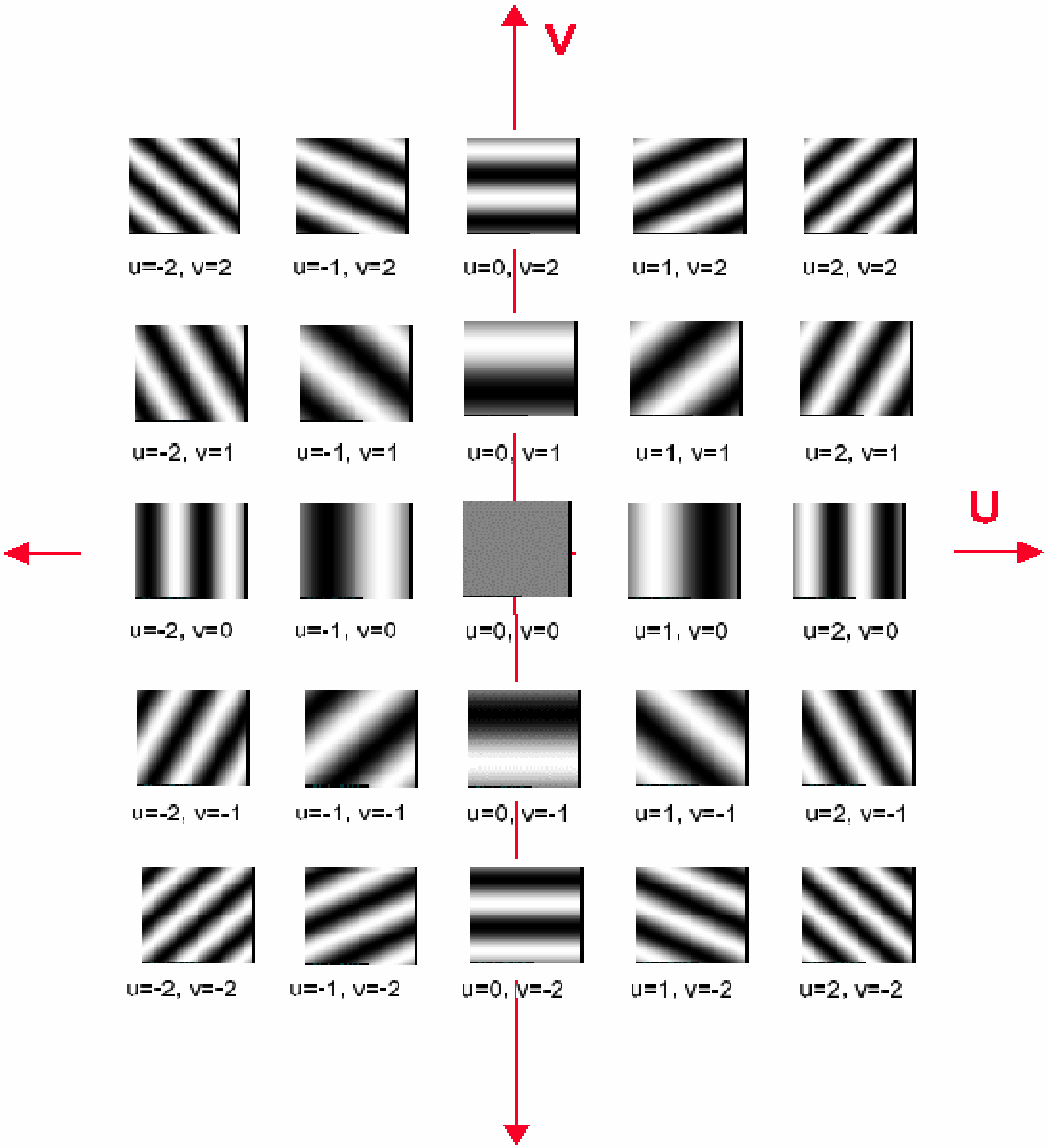
The spatial frequency of i=1and j=1

2D 傅里叶变换



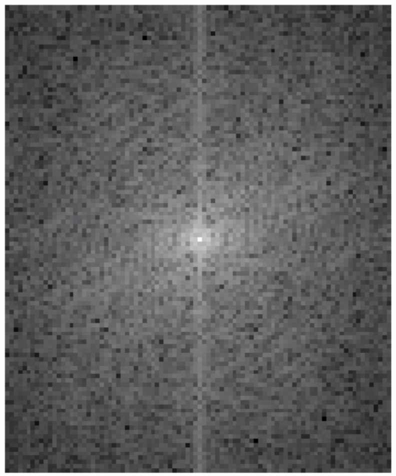
$$e^{-j2\pi(nk+ml)/N}$$

Base-functions are waves


v
u
u=-2, v=2
u=-1, v=2
u=0, v=2
u=1, v=2
u=2, v=2
u=-2, v=1
u=-1, v=1
u=0, v=1
u=1, v=1
u=2, v=1
u=-2, v=0
u=-1, v=0
u=0, v=0
u=1, v=0
u=2, v=0
u=-2, v=-1
u=-1, v=-1
u=0, v=-1
u=1, v=-1
u=2, v=-1
u=-2, v=-2
u=-1, v=-2
u=0, v=-2
u=1, v=-2
u=2, v=-2

空间频率

- 首先它是一种有效数据表示方式
- 可以模拟或去除噪声（以某种频率表示的）
- 物理过程在频率域可以得到更好的表示
- 一种图像分析的更好的方法
- 描述一副图像并不需用一个个像素分布表示出来
- 而是通过一系列的不同“空间频率”的 cosines函数来表述图像



傅里叶变换定理

定义

类似于傅里叶级数正交展开，函数看做复指数函数在整个连续频率区间上的积分和

$$g(x)=\int_{-\infty}^{\infty}G(f)e^{j2\pi fx}df \qquad \text{其中} \quad G(f)=\int_{-\infty}^{\infty}g(x)e^{-j2\pi fx}dx$$

G(f)称为g(x)的傅里叶变换，或频谱。若g(x)表示某空间域的物理量，G(f)则是该物理量在频率域的表示形式。

振幅与相位频谱

$$G(f)=A(f)e^{j\phi(f)}$$

$A(f)=|G(f)|$，是g(x)的振幅频谱；$\phi(f)$是g(x)的相位频谱。

傅里叶变换定理

– 存在条件

$$g(x)=\int_{-\infty}^{\infty}G(f)e^{j2\pi fx}df$$

$$G(f)=\int_{-\infty}^{\infty}g(x)e^{-j2\pi fx}dx$$

1.函数绝对可积
2.在任一有限区域里，g(x,y)必须只有有限个间断点和有限个极大和极小点
3. g(x,y)必须没有无穷大间断点

根据存在条件知：Delta函数、正、余弦函数、阶跃函数不存在FT

– 广义傅里叶变换

若函数可以看作是某个可变换函数所组成的序列的极限，对序列中每一函数进行变换，组成一个新的变换式序列，这个新序列的极限就是原来函数的广义傅里叶变换

$$g(x,y)=\lim_{\tau\to\infty}rect\left(\frac{x}{\tau}\right)rect\left(\frac{y}{\tau}\right)$$

$$\Im\{1\}=\delta(f_x,f_y)$$

$$\Im\left\{rect\left(\frac{x}{\tau}\right)rect\left(\frac{y}{\tau}\right)\right\}=\tau^2\sin c(\tau f_x)\sin c(\tau f_y)$$

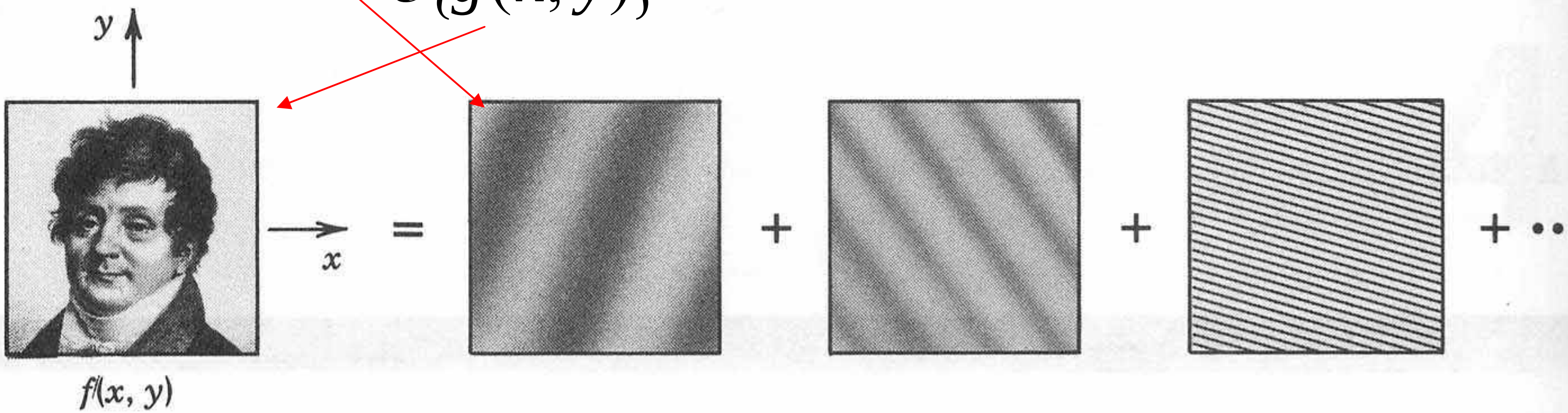
$$\Im\{g(x,y)\}=\lim_{\tau\to\infty}\tau^2\sin c(\tau f_x)\sin c(\tau f_y)=\delta(f_x,f_y)$$

2D FT的物理意义

$$g(x,y)=\iint_{-\infty}^{\infty} G(f_x,f_y)\exp\left[j2\pi(f_x x+f_y y)\right]df_x df_y=\mathfrak{I}^{-1}\left\{G(f_x,f_y)\right\}$$

$$G(f_x,f_y)=\iint_{-\infty}^{\infty} g(x,y)\exp\left[-j2\pi(f_x x+f_y y)\right]dxdy$$

$$=\mathfrak{I}\left\{g(x,y)\right\}$$



• 虚、实、奇、偶函数傅里叶变换的性质

$$G(f)=\int_{-\infty}^{\infty} g(x)e^{-j2\pi fx}dx=\int_{-\infty}^{\infty} g(x)\cos 2\pi fxdx - j\int_{-\infty}^{\infty} g(x)\sin 2\pi fxdx$$

令 $g(x)=g_r(x)+jg_i(x)$

$$G(f)=\left[\int_{-\infty}^{\infty} g_r(x)\cos 2\pi fxdx+\int_{-\infty}^{\infty} g_i(x)\sin 2\pi fxdx\right]$$
$$+j\left[\int_{-\infty}^{\infty} g_i(x)\cos 2\pi fxdx-\int_{-\infty}^{\infty} g_r(x)\sin 2\pi fxdx\right]$$
$$=R(f)+jI(f)$$

我们有(1)：g(x)是实函数　　$g_i(x)=0, g(x)=g_r(x)$

$$G(f)=\int_{-\infty}^{\infty} g(x)\cos 2\pi fxdx - j\int_{-\infty}^{\infty} g(x)\sin 2\pi fxdx$$

$R(f)=\int_{-\infty}^{\infty} g(x)\cos 2\pi fxdx$　　偶函数

$I(f)=-\int_{-\infty}^{\infty} g(x)\sin 2\pi fxdx$　　奇函数

$G(f)$ 是厄米型函数， 即 $G(f)=G*(-f)$

• 虚、实、奇、偶函数傅里叶变换的性质

(2)g(x)是实值偶函数

$$G(f)=\left[\int_{-\infty}^{\infty} g_r(x)\cos 2\pi fxdx+\int_{-\infty}^{\infty} g_i(x)\sin 2\pi fxdx\right]$$
$$+\ j\left[\int_{-\infty}^{\infty} g_i(x)\cos 2\pi fxdx-\int_{-\infty}^{\infty} g_r(x)\sin 2\pi fxdx\right]$$
$$=R(f)+jI(f)$$

$$G(f)=2\int_{0}^{\infty} g(x)\cos 2\pi fxdx$$

$$G(f)=G(-f)$$

所以$G(f)$是实值偶函数

(3)g(x)是实值奇函数

$$G(f)=-2j\int_{0}^{\infty} g(x)\sin 2\pi fxdx$$

$$G(-f)=-G(f)$$

所以$G(f)$是虚值奇函数

• 虚、实、奇、偶函数傅里叶变换的性质—总结

$$G(f)=\left[\int_{-\infty}^{\infty} g_r(x)\cos 2\pi fx dx+\int_{-\infty}^{\infty} g_i(x)\sin 2\pi fx dx\right]$$

$$+j\left[\int_{-\infty}^{\infty} g_i(x)\cos 2\pi fx dx-\int_{-\infty}^{\infty} g_r(x)\sin 2\pi fx dx\right]$$

$$=R(f)+jI(f)$$

空域 $g(x,y)$	频域 $G(f_x,f_y)$	空域 $g(x,y)$	频域 $G(f_x,f_y)$
实函数	厄米型函数	虚值偶函数	虚值偶函数
虚函数	反厄米型函数	虚值奇函数	实值奇函数
实值偶函数	实值偶函数	偶函数	偶函数
实值奇函数	虚值奇函数	奇函数	奇函数

注：

若实部为偶函数，虚部为奇函数，则函数是 厄米型函数

若实部为奇函数，虚部为偶函数，则函数是反 厄米型函数

傅里叶变换定理

1、线性定理

$$\Im\left\{\alpha g(x,y)+\beta h(x,y)\right\}=\alpha G(f_x,f_y)+\beta H(f_x,f_y)$$

两个函数和的富里叶变换等于它们各自变换的和

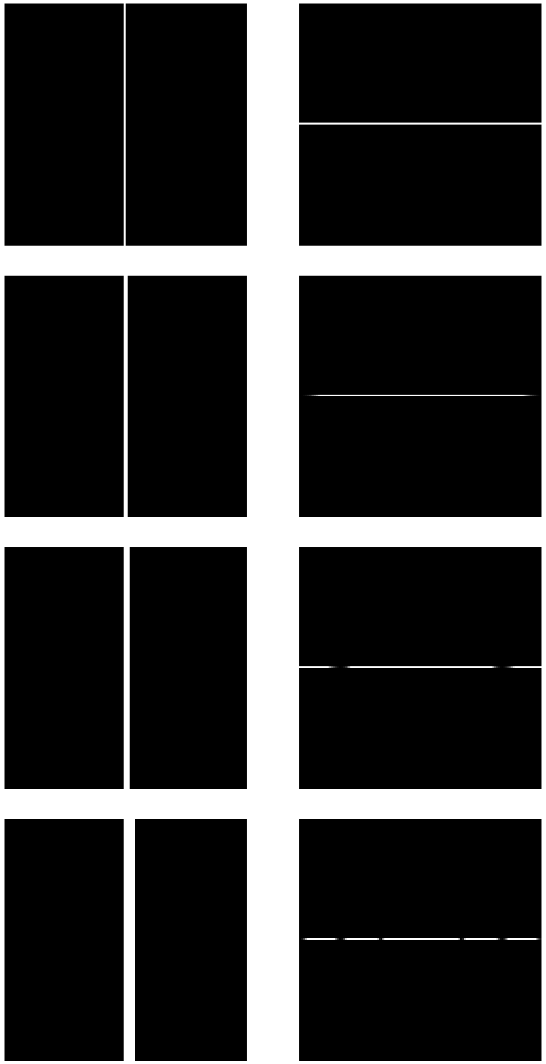
傅里叶变换定理

2.相似性定理

$$\Im\{g(ax,by)\}=\frac{1}{|ab|}G\left(\frac{f_x}{a},\frac{f_y}{b}\right)$$

空域的压缩在频域表现为展宽及能量的降低

空域的展宽在频域表现为压缩及能量的增加

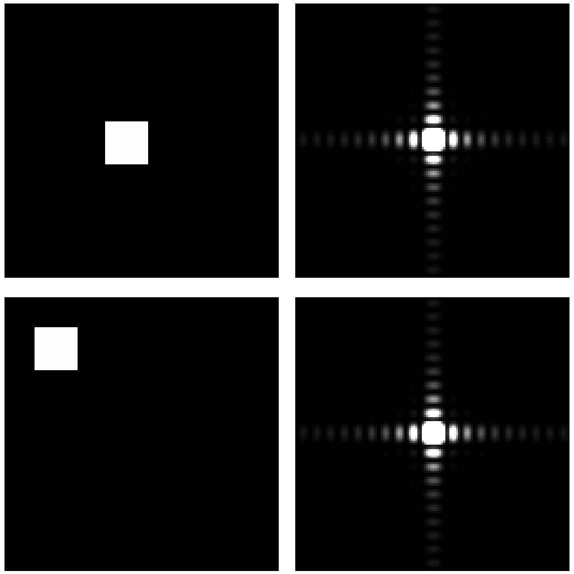


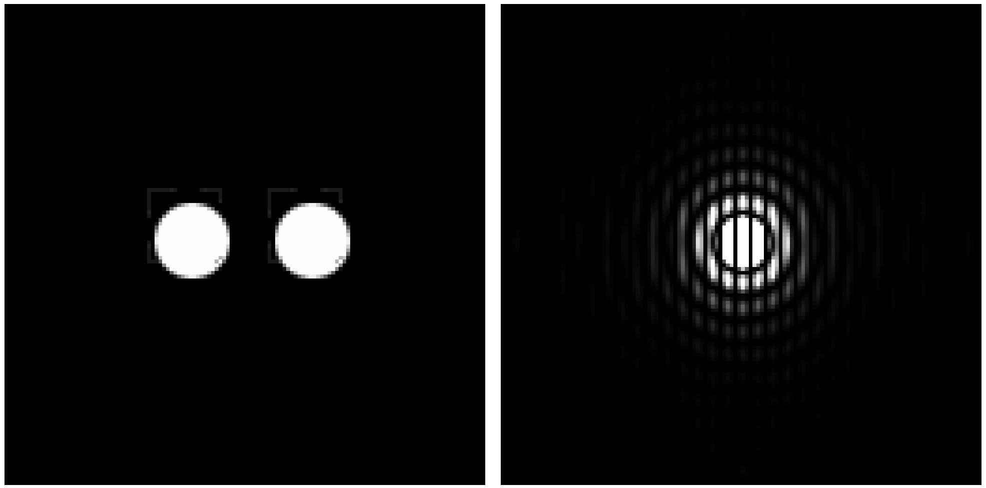
傅里叶变换定理

3.位移定理

$$\Im\{g(s-a, y-b)\} = G(f_x, f_y)\exp\left[-j2\pi(f_x a + f_y b)\right]$$

物空间的位置平移带来频域空间的相移



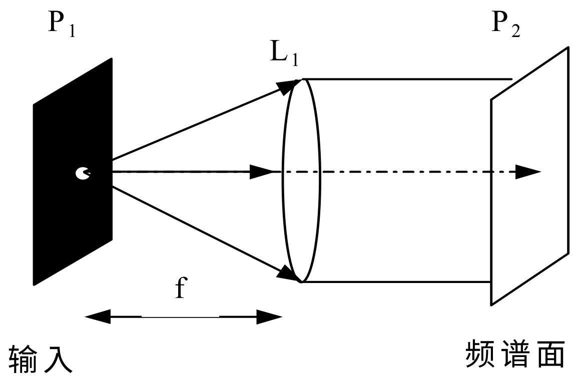


傅里叶变换定理

4.帕色伐(Parseval)定理

能量=
$$\iint_{-\infty}^{\infty} |g(x,y)|^2 dxdy = \iint_{-\infty}^{\infty} |G(f_x,f_y)|^2 df_x df_y$$

Transient Functions Are Space-limited， Therefore Have Finite Energy As Represented in Either Time or Frequency Domain



傅里叶变换定理

5.卷积定理 $$\Im\{g(x,y)*h(x,y)\}=G(f_x,f_y,)\bullet H(f_x,f_y)$$

$$g(x,y)h(x,y) \longleftrightarrow G(f_x,f_y)*H(f_x,f_y)$$

Proof:

$$\mathcal{F}[f*g] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-2\pi i k x} f(x')g(x-x')\,dx'\,dx$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \boxed{[e^{-2\pi i k x'}} f(x')\,dx'][e^{-2\pi i k (x-x')} g(x-x')\,dx]$$

$$= \left[\int_{-\infty}^{\infty} e^{-2\pi i k x'} f(x')\,dx'\right]\left[\int_{-\infty}^{\infty} e^{-2\pi i k x''} g(x'')\,dx''\right]$$

$$= \mathcal{F}[f]\mathcal{F}[g], \qquad x'' \equiv x - x'$$

Convolution in One Domain Corresponds to Multiplication in the Other Domain

Application??

傅里叶变换定理

6.自相关定理(维纳一辛钦定理)

$$\Im\{g(x,y)\otimes g(x,y)\} = \left|G(f_x,f_y)\right|^2$$

Correlation of Two Signals in the Spatial Domain Corresponds to Multiplication in the Frequency Domain by the Complex Conjugate of One Signal

Application ??

Relative Agreement between Two Functions as a Function of Misalignment

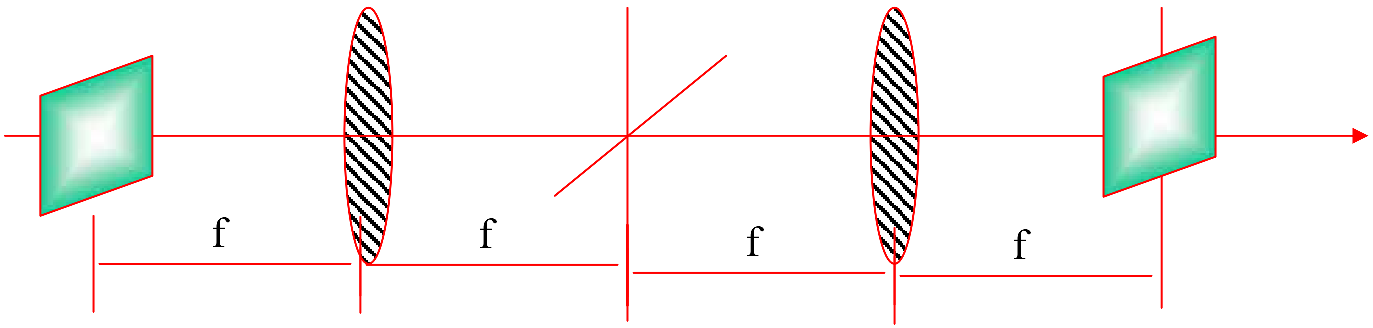
傅里叶变换定理

7.傅里叶积分定理

$$\Im\Im^{-1}\{g(x,y)\} = \Im^{-1}\Im\{g(x,y)\} = g(x,y)$$

对函数进行FT以及RFT， 重新得到原函数(条件：在g(x,y)的各个连续点上)

请证明：



傅里叶变换定理

8、可分离变量函数的FT

$$g(x,y)=g_x(x)\bullet g_y(y)$$

$$\Im\{g(x,y)\}=\iint_{-\infty}^{\infty} g(x,y)\exp\left[-j2\pi(f_x x+f_y y)\right]dxdy$$

$$=\int_{-\infty}^{\infty} g_x(x)\exp(-j2\pi f_x x)dx\bullet\int_{-\infty}^{\infty} g_y(y)\exp(-j2\pi f_y y)dy$$

$$=\Im_x\{g_x\}\bullet\Im_y\{g_y\}$$

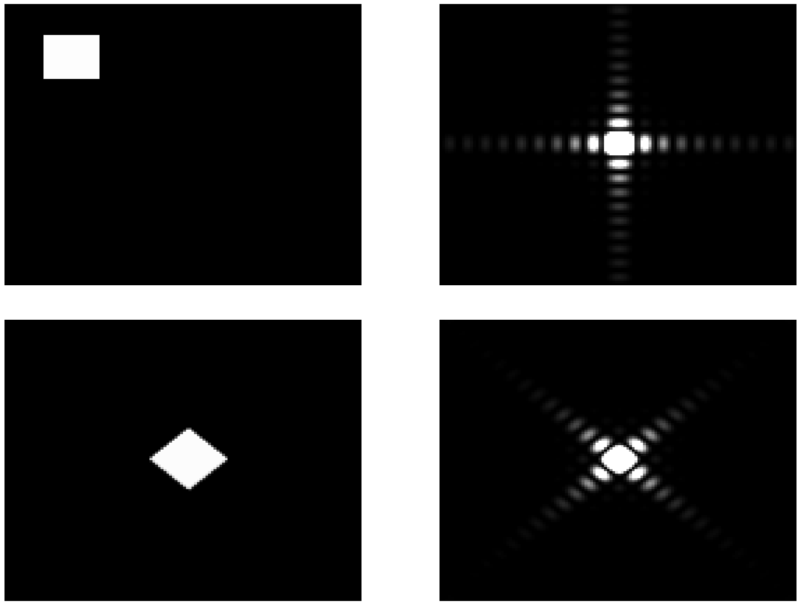
由物空间的可分离性，可简化谱空间的计算

傅里叶变换定理

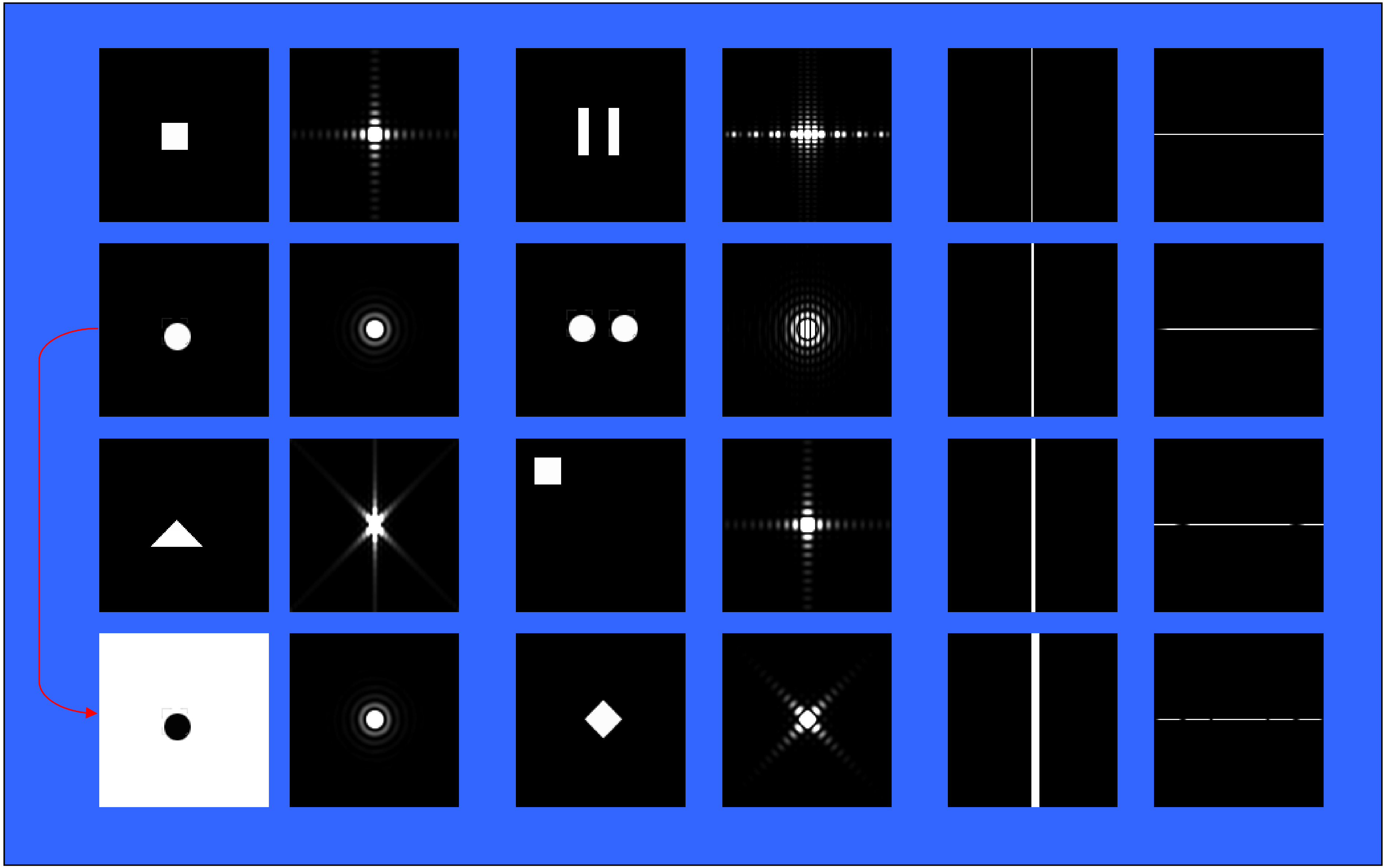
8、 Rotation Theorem

$$\mathrm{F}\{f(x\cos\theta+y\sin\theta,-x\sin\theta+y\cos\theta)\}=$$
$$F(u\cos\theta+v\sin\theta,-u\sin\theta+v\cos\theta)$$

由物空间的形状，可判断谱空间的基本分析

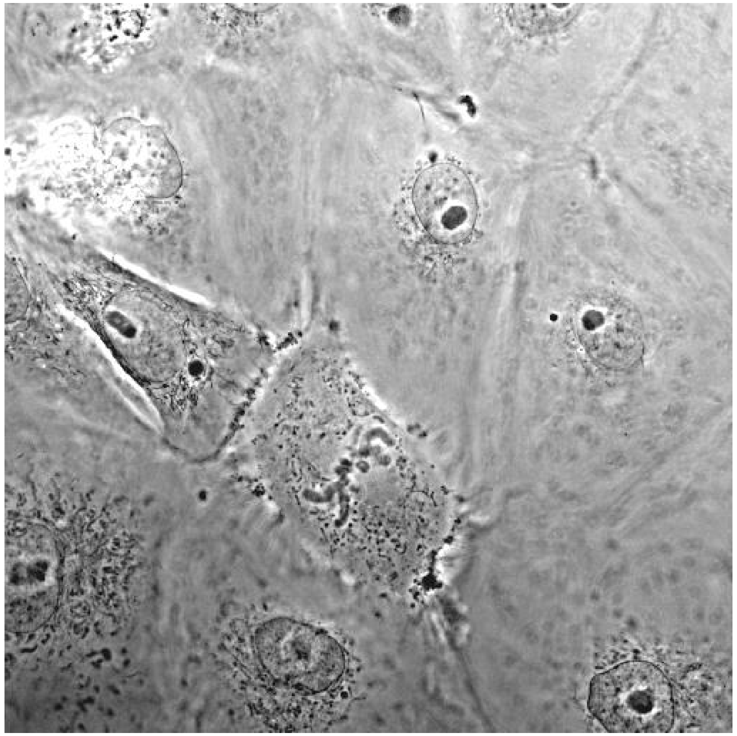


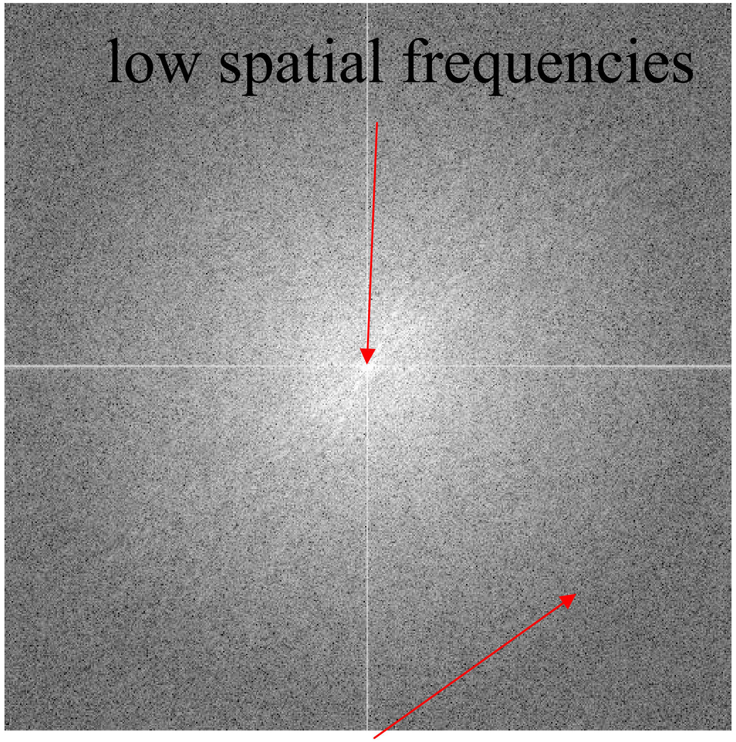
傅里叶变换定理——图解



傅里叶变换定理—图解

f(x,y) $\rightarrow$ $|F(f_x, f_y)|$





high spatial frequencies

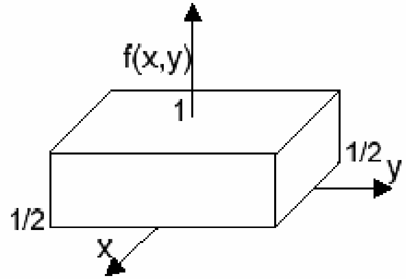
傅里叶变换定理—总结

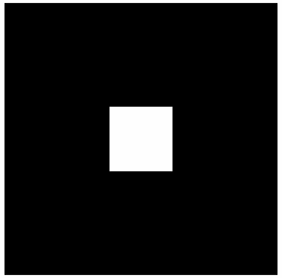
Table 10-3. Properties of the two-dimensional Fourier transform

Property	*Spatial Domain*	*Frequency Domain*
Addition theorem	$f(x, y) + g(x, y)$	$F(u, v) + G(u, v)$
Similarity theorem	$f(ax, by)$	$\frac{1}{\lvert ab\rvert}F\left(\frac{u}{a}, \frac{v}{b}\right)$
Shift theorem	$f(x - a, y - b)$	$e^{-j2\pi(au+bv)}F(u, v)$
Convolution theorem	$f(x, y) * g(x, y)$	$F(u, v)G(u, v)$
Separable product	$f(x)g(y)$	$F(u)G(v)$
Differentiation	$\left(\frac{\partial}{\partial x}\right)^m\left(\frac{\partial}{\partial y}\right)^n f(x, y)$	$(j2\pi u)^m(j2\pi v)^n F(u, v)$
Rotation	$f(x\cos\theta + y\sin\theta,$ $-x\sin\theta + y\cos\theta)$	$F(u\cos\theta + v\sin\theta,$ $-u\sin\theta + v\cos\theta)$
LaPlacian	$\nabla^2(x, y) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)f(x, y)$	$-4\pi^2(u^2 + v^2)F(u, v)$
Rayleigh's theorem	$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \lvert f(x, y)\rvert^2\,dx\,dy = \int_{+\infty}^{\infty}\int_{-\infty}^{\infty} \lvert F(u, v)\rvert^2\,du\,dv$	

一些函数的富里叶变换

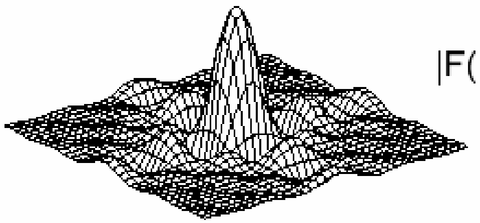
1、RECT函数



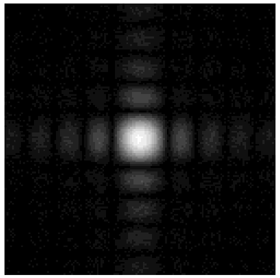


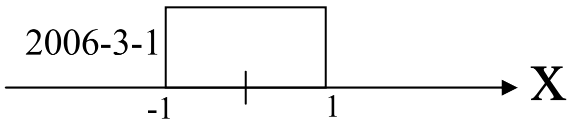
$$f(x,y) = \mathrm{rect}(x,y) = \begin{cases} 1 & |x| \quad 1/2,\ |y| \quad 1/2 \\ 0 & \text{otherwise} \end{cases}$$

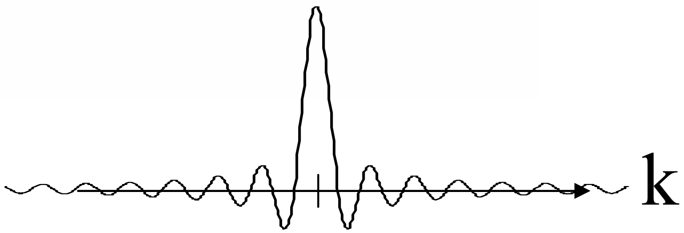
$$F(u,v) = \mathrm{sinc}(u) \quad \mathrm{sinc}(v) = \mathrm{sinc}(u,v)$$



|F(u,v)|







一些函数的富里叶变换

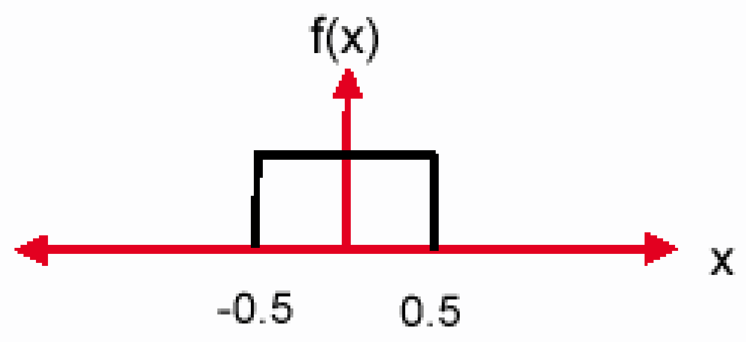
1、RECT 函数：进一步

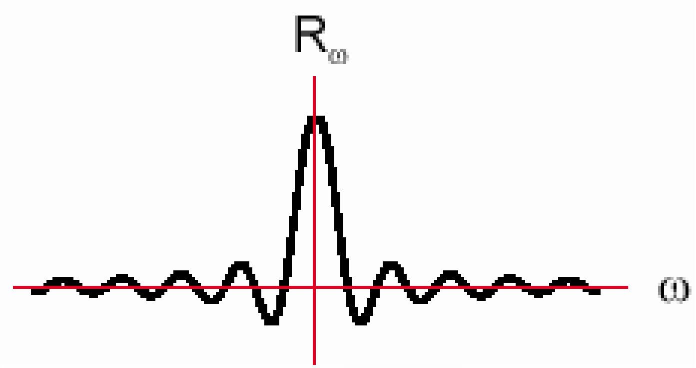
The Window Function (rect):

- Let $\mathrm{rect}_{1/2}(x) = \begin{cases} 1 & \text{if } |x| < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$

求指数函数的积分即可：

$$F(\omega) = \int_{-0.5}^{0.5} e^{-i2\pi\omega x}\,dx = \frac{\sin(\pi\omega)}{\pi\omega} = \mathrm{sinc}(\pi\omega)$$





一些函数的富里叶变换

2、Delta function

$$g(x,y)=\lim_{\tau\to\infty} rect\left(\frac{x}{\tau}\right)rect\left(\frac{y}{\tau}\right)=1$$

$$\Im\left\{rect\left(\frac{x}{\tau}\right)rect\left(\frac{y}{\tau}\right)\right\}=\tau^2\sin c(\tau f_x)\sin c(\tau f_y)$$

$$\Im\{g(x,y)\}=\lim_{\tau\to\infty}\tau^2\sin c(\tau f_x)\sin c(\tau f_y)=\delta(f_x,f_y)$$

$$\Im\{1\}=\delta(f_x,f_y)$$

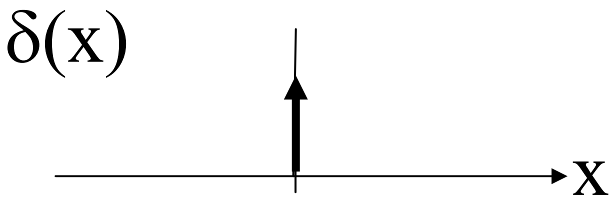
另一方面， 可利用Delta 函数的性质来计算

$$\iint_{-\infty}^{\infty}\delta(x-x_0,y-y_0)\phi(x,y)dxdy=\phi(x_0,y_0)$$

一些函数的富里叶变换

2、Delta function: further discussion

$$\iint_{-\infty}^{\infty} \delta(x,y) e^{-j2\pi(f_x x + f_y y)} dxdy = e^{-j2\pi(f_x * 0 + f_y * 0)} = 1$$

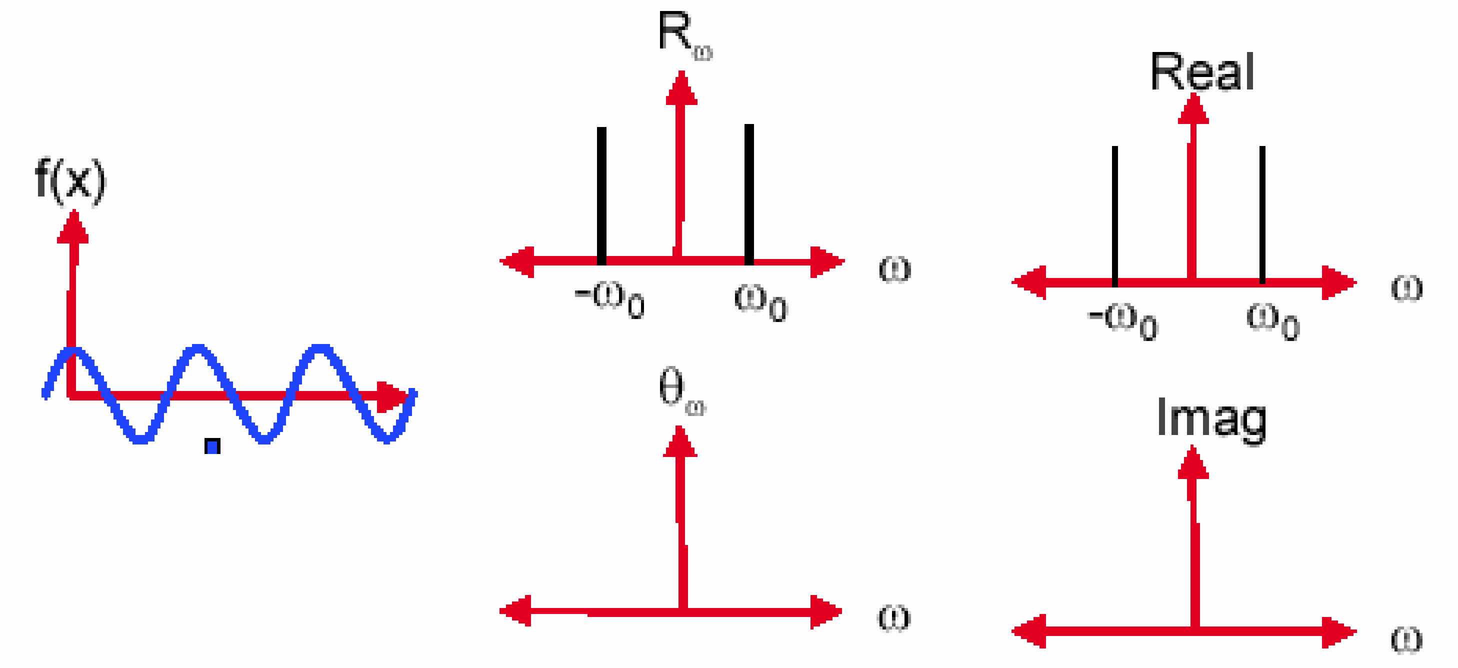


一些函数的富里叶变换

4、正弦函数

$$\text{Let} \quad f(x)=\cos(2\pi\omega_0 x)$$

$$F(\omega)=\int_{-\infty}^{\infty}\frac{1}{2}\left(e^{i2\pi\omega_0 x}+e^{-i2\pi\omega_0 x}\right)e^{-i2\pi\omega x}\,dx=\frac{1}{2}\left[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right]$$



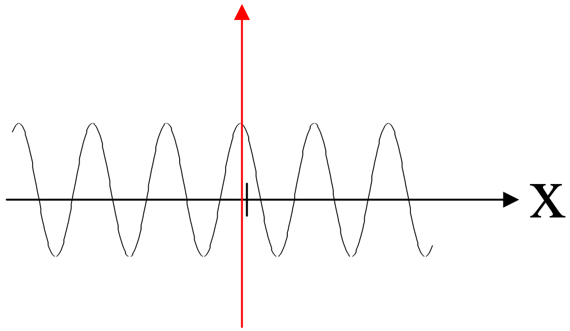
一些函数的富里叶变换

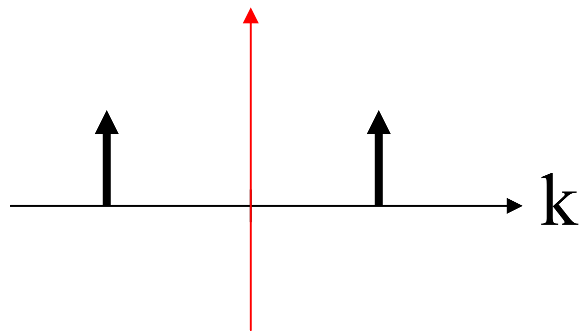
4、正弦函数：进一步

$$\mathrm{F}\{\cos(2\pi f_0 t)\}=\frac{1}{2}[\delta(s-f_0)+\delta(s+f_0)]$$

$$\mathrm{F}\{\sin(2\pi f_0 t)\}=\frac{j}{2}[\delta(s+f_0)-\delta(s-f_0)]$$

Periodic Signals at f_0 in spatial Domain are Impulses in Frequency Domain

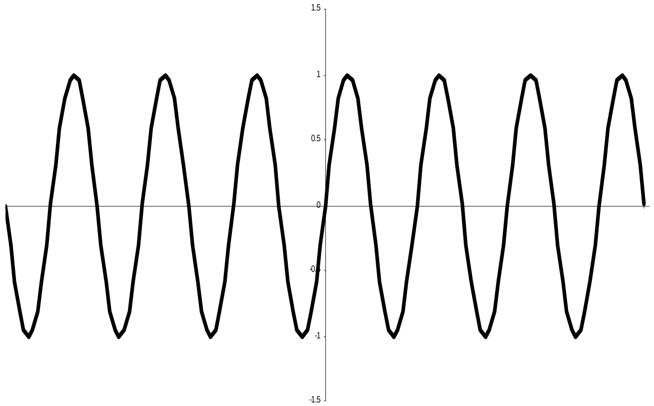


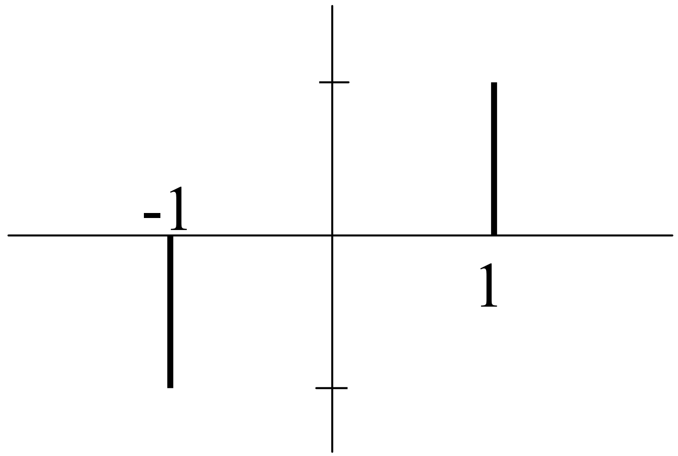


Pls tell me the difference between cos and sin FTs

一些函数的富里叶变换

4、正弦函数：进二步





If *f(x)* is odd, so is *F(ω)*

$$G(f)=\left[\int_{-\infty}^{\infty} g_r(x)\cos 2\pi fx dx+\int_{-\infty}^{\infty} g_i(x)\sin 2\pi fx dx\right]$$

$$+\,j\left[\int_{-\infty}^{\infty} g_i(x)\cos 2\pi fx dx-\int_{-\infty}^{\infty} g_r(x)\sin 2\pi fx dx\right]$$

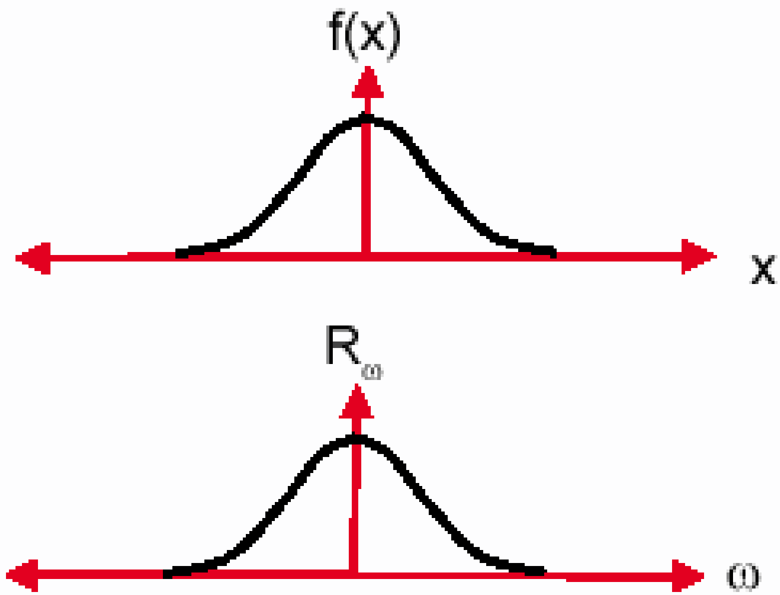
$$=R(f)+jI(f)$$

$$G(f)=-2j\int_{0}^{\infty} g(x)\sin 2\pi fx dx$$

一些函数的富里叶变换

5、高斯函数

$$f(x)=e^{-\pi x^2} \qquad F(\omega)=e^{-\pi\omega^2}$$



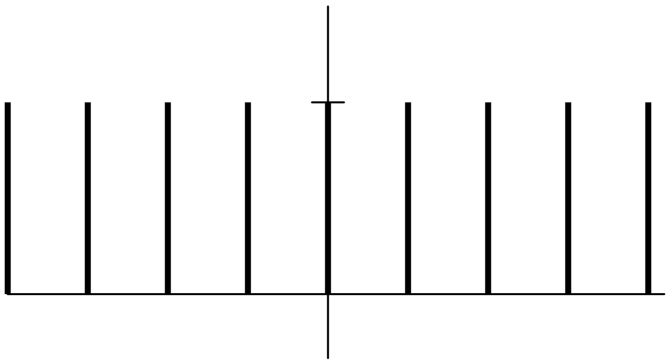
一些函数的富里叶变换

6、Comb 函数

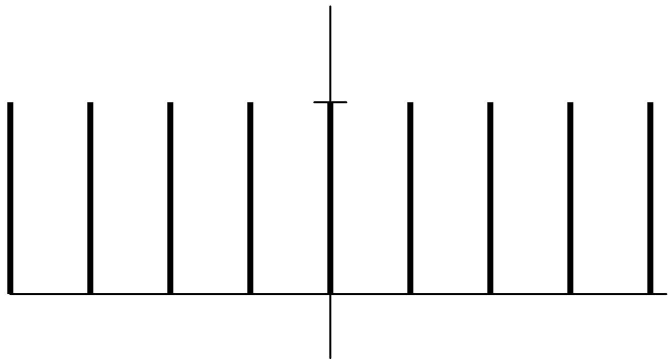
$$g(x)=\frac{1}{\tau}comb(\frac{x}{\tau})=\sum_{n=-\infty}^{\infty}\delta(x-n\tau)=\frac{1}{\tau}\sum_{n=-\infty}^{\infty}\exp(j2\pi nf_0x)-(f_0=\frac{1}{\tau})$$

可利用(1)、展开为富里叶级数，(2)直接利用Delta函数的富里叶变换

$$\Im\left\{\frac{1}{\tau}comb(\frac{x}{\tau})\right\}=\frac{1}{\tau}\sum_{n=-\infty}^{\infty}\delta(f-\frac{n}{\tau})=comb(\tau f)$$



period =τ

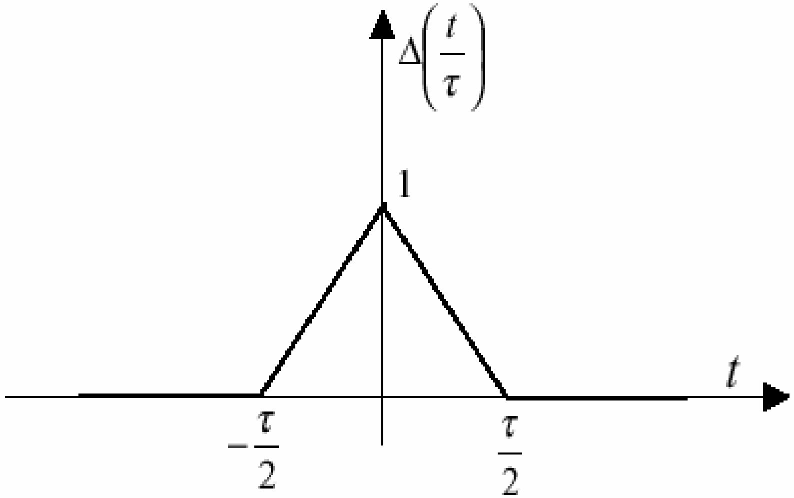


period is 1/ τ

一些函数的富里叶变换

7、三角形函数

Recall that $\mathrm{rect}(t/\tau) * \mathrm{rect}(t/\tau) = \tau\Delta(t/2\tau)$. Using this result and convolution property of the Fourier transform find the Fourier transform of



$$\wedge(x)\wedge(y) \longleftrightarrow \mathrm{sinc}^2(u)\,\mathrm{sinc}^2(v)$$

一些函数的富里叶变换

8、园域函数

Equivalent to a two-dimensional Fourier transform with a radially symmetric integral kernel, and also called the Fourier-Bessel transform.

$$\begin{aligned} g(u,v) &= \mathcal{F}_r[f(r)](u,v) \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(r)e^{-2\pi i(ux+vy)}\,dx\,dy. \end{aligned}$$

令

$$\begin{aligned} x+iy &= re^{i\theta} \\ u+iv &= qe^{i\phi} \end{aligned}$$

于是我们有

$$\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \\ r &= \sqrt{x^2+y^2} \end{aligned} \qquad \begin{aligned} u &= q\cos\phi \\ v &= q\sin\phi \\ q &= \sqrt{u^2+v^2}. \end{aligned}$$

一些函数的富里叶变换

8、园域函数

$$g(q) = \int_0^{\infty}\int_0^{2\pi} f(r)e^{-2\pi irq(\cos\phi\cos\theta+\sin\phi\sin\theta)}r\,dr\,d\theta$$

$$= \int_0^{\infty}\int_0^{2\pi} f(r)e^{-2\pi irq\cos(\theta-\phi)}r\,dr\,d\theta$$

$$= \int_0^{\infty}\int_{-\phi}^{2\pi-\phi} f(r)e^{-2\pi irq\cos\theta}r\,dr\,d\theta$$

$$= \int_0^{\infty}\int_0^{2\pi} f(r)e^{-2\pi irq\cos\theta}r\,dr\,d\theta$$

$$= \int_0^{\infty} f(r)\left[\int_0^{2\pi} e^{-2\pi irq\cos\theta}\,d\theta\right]r\,dr$$

$$= 2\pi\int_0^{\infty} f(r)J_0(2\pi qr)r\,dr,$$

一些函数的富里叶变换

8、园域函数

这样我们可以得到HANKEL 变换对为

$$g(q) = 2\pi \int_0^\infty f(r) J_0(2\pi q r) r\, dr$$

$$f(r) = 2\pi \int_0^\infty g(q) J_0(2\pi q r) q\, dq.$$

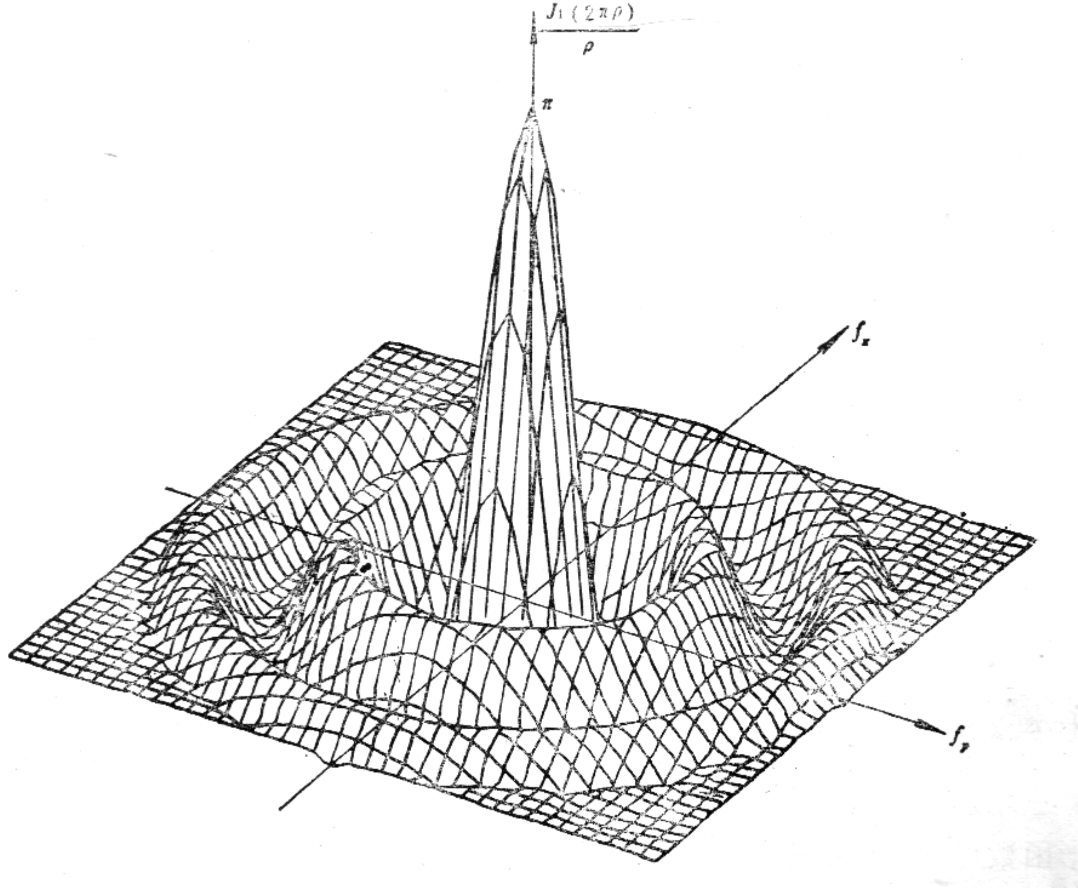
对园域函数我们有：

$$F(cir(r)) = 2\pi \int_{-\infty}^{\infty} cir(r) J_0(2\pi \rho r) dr = \boxed{\frac{J_1(2\pi\rho)}{\rho}}$$

J_1 为一阶第一类贝塞尔函数

一些函数的富里叶变换

8、园域函数



傅里叶变换定理—列表

– 常见函数的傅里叶变换式(熟记)

$g(x,y)$	$G(u,v)$
$g(-x,-y)$	$G(-u,-v)$
$g^*(x,y)$	$G^*(-u,-v)$
$G(x,y)$	$g(-u,-v)$
$G^*(\pm x,\pm y)$	$g^*(\pm u,\pm v)$
$g(ax,by)$	$\frac{1}{\lvert ab\rvert}G(\frac{u}{a},\frac{v}{b})$
$g(x\pm x_0,y\pm y_0)$	$\exp[\pm i2\pi(ux_0+vy_0)]G(u,v)$
$\delta(x\pm x_0,y\pm y_0)$	$\exp[\pm i2\pi(ux_0+vy_0)]$
$\exp[\pm i2\pi(ux_0+vy_0)]$	$\delta(u\mp u_0,v\mp v_0)$
$\exp[\pm i2\pi(ux_0+vy_0)]g(x,y)$	$G(u\mp u_0,v\mp v_0)$
$g(x)h(y)$	$G(u)H(v)$
$g(x)$	$G(u)\delta(v)$
$\delta^{(k,l)}(x,y)$	$(i2\pi u)^k(i2\pi v)^l$
$g^{k,l}(x,y)$	$(i2\pi u)^k(i2\pi v)^lG(u,v)$
x^ky^l	$(-i2\pi)^{-k-l}\delta^{(k,l)}(u,v)$
$\nabla^2g(x,y)$	$-4\pi^2(u^2+v^2)G(u,v)$
$\int_{-\infty}^{x}\int_{-\infty}^{\infty}g(\zeta,\eta)d\zeta d\eta$	$\frac{1}{2}[G(0,0)\delta(u)+\frac{G(u,0)}{i\pi u}]\delta(v)$
$g(x,y)*h(x,y)$	$G(u,v)H(u,v)$
$g(x,y)h(x,y)$	$G(u,v)*H(u,v)$
$g(x,y)\otimes h(x,y)$	$G^*(u,v)H(u,v)$
$g(x,y)^*h(x,y)$	$G(u,v)\otimes H(u,v)$
$\exp[\pm i\pi(x^2+y^2)]$	$\pm i\exp[\mp i\pi(u^2+v^2)]$
$\exp[-\pi(x^2+y^2)]$	$\exp[-\pi(u^2+v^2)]$
$rect(x)rect(y)$	$\sin c(u)\sin c(v)$
$\sin c(x)\sin c(y)$	$rect(u)rect(v)$
$\wedge(x)\wedge(y)$	$\sin c^2(u)\sin c^2(v)$

傅里叶变换：正变换与反变换

对-变换，首先我们有：

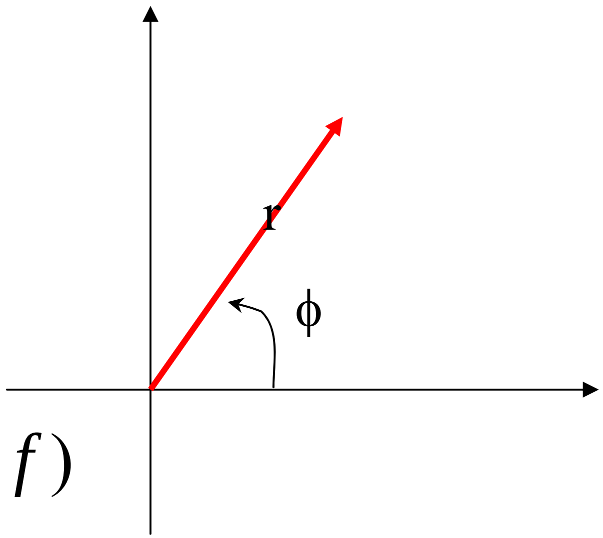
$$G(f)=\int_{-\infty}^{\infty} g(x)e^{-j2\pi fx}dx$$

另一方面， 对+变换， 我们有

$$G(f)=\int_{-\infty}^{\infty} g(x)e^{j2\pi fx}dx$$

令$f=-f'$,则我们有

$$G(-f')=\int_{-\infty}^{\infty} g(x)e^{-j2\pi f'x}dx=G(f)$$



复矢量可看做复平面上的相矢量来解释：对正变换， 我们选择的是随幅角增加而反时针方面旋转的相矢量或者是反而道之， 都成立。

即顺时针与反变换， 逆时针与正变换相联系

傅里叶变换：正变换与反变换的应用

在所有g连续的点上

$$FF\{g(x,y)\} = F^{-1}F^{-1}\{(g(x,y)\} = g(-x,-y)$$

$$Suppose\ F\{g(x,y)\} = G(f_x, f_y),\ then,\ we\ have$$

$$FF\{g(x,y)\} = F\{G(f_x, f_y)\} = \int\int_{\infty}^{\infty} G(f_x, f_y)\exp[-j2\pi(xf_x + yf_y)]df_x df_y$$

$$= \int\int_{\infty}^{\infty} G(f_x, f_y)\exp[j2\pi((-x)f_x + (-y)f_y)]df_x df_y$$

$$= g(-x,-y)$$

$$F^{-1}\{g(x,y)\} = \int\int_{\infty}^{\infty} g(x,y)\exp[-j2\pi(x(-f_x) + y(-f_y))]dxdy = G(-f_x,-f_y)$$

傅里叶变换：正变换与反变换的应用

$$F^{-1}F^{-1}\{g(x,y)\} = F^{-1}\{G(-f_x,-f_y)\}$$

$$= \iint_{-\infty}^{\infty} G(-f_x,-f_y)\exp[j2\pi(f_x x + f_y y)]df_x df_y$$

$$= \iint_{-\infty}^{\infty} G(-f_x,-f_y)\exp[j2\pi((-f_x)(-x)+(-f_y)(-y))]d(-f_x)d(-f_y)$$

$$= g(-x,-y)$$

于是我们证明了在所有g连续的点上

$$FF\{g(x,y)\} = F^{-1}F^{-1}\{(g(x,y)\} = g(-x,-y)$$

相位与振幅，谁更重要

Fig a 原图

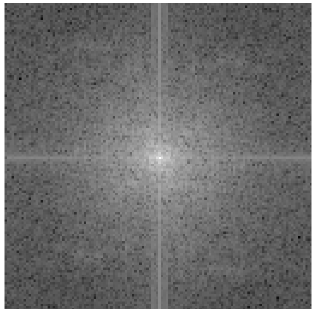
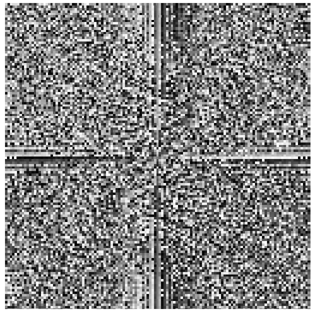


Fig b 振幅

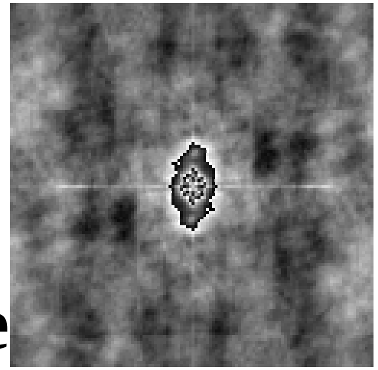


相位分布

original

$$F^{-1}\left(\left|F(x)\right|\right)$$

amplitude



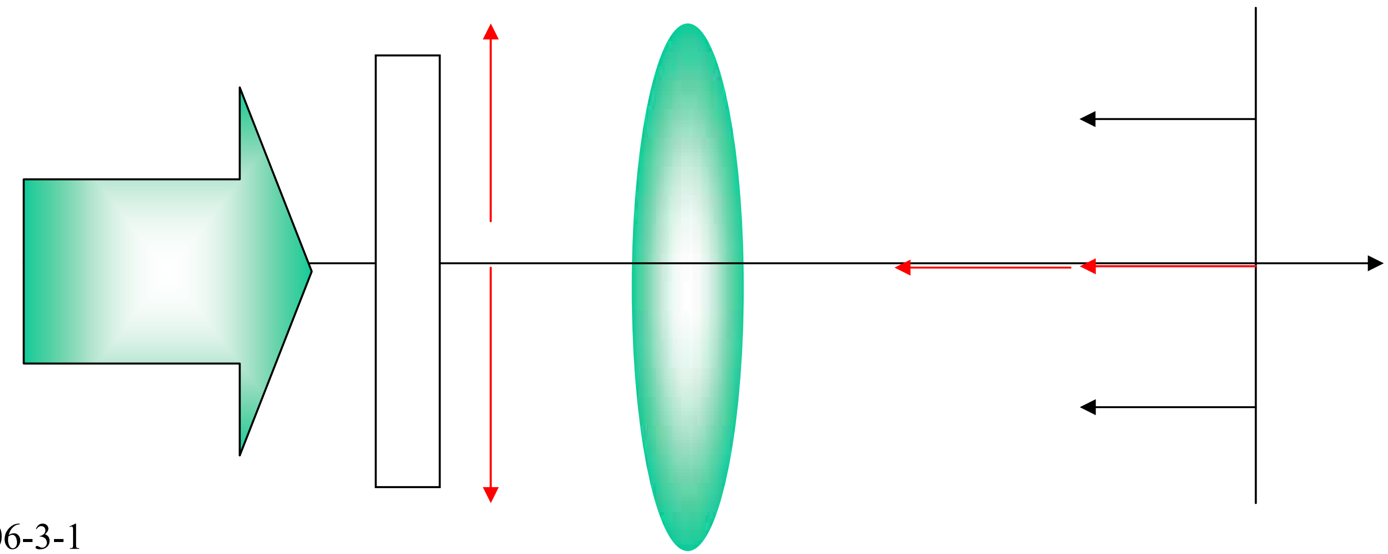
phase

$$F^{-1}\left(F(x)/\left|F(x)\right|\right)$$

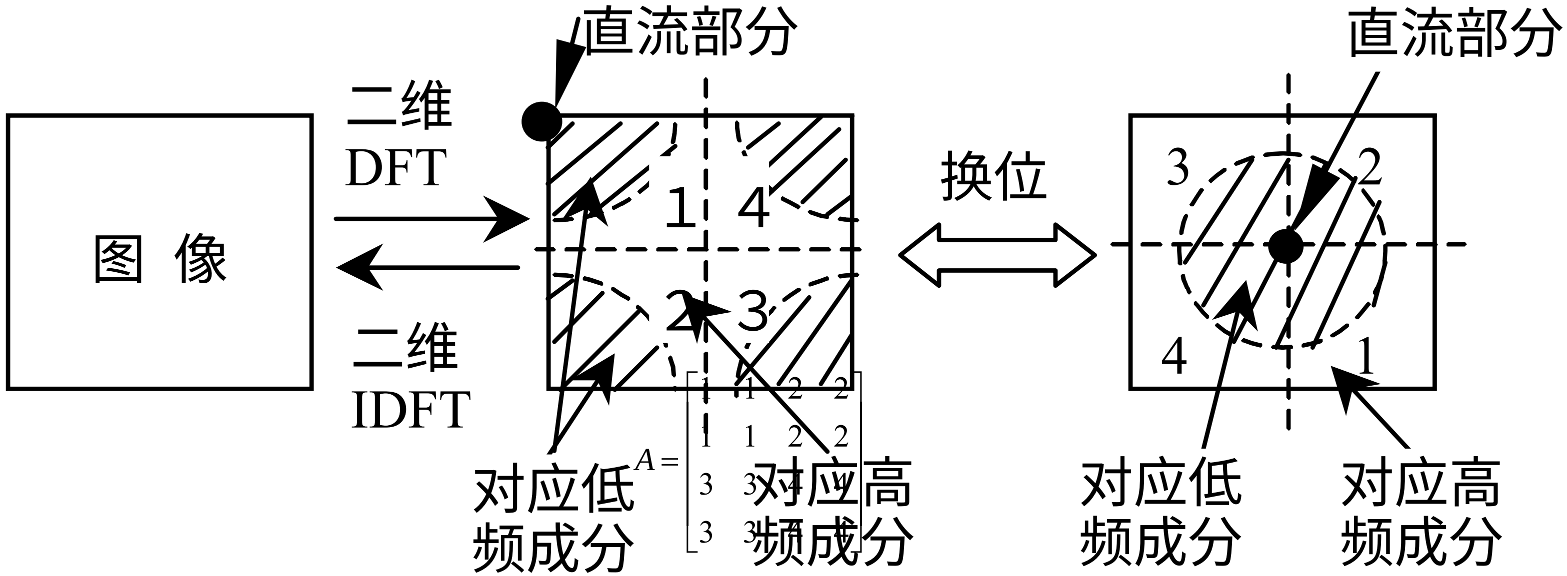
渐变以及与光学系统的关系

当正弦函数的频率f_0渐渐接近0时, 我们发现，它对应的两个富里叶变换谱 $\delta(f-f_0)$和$\delta(f+f_0)$ 也向中心靠拢，当等于0时， 两个谱也合二为一， 其振幅由两个二分之一相当，也等于1

其结果相当于常函数1的富里叶变换结果。



FT频谱分布

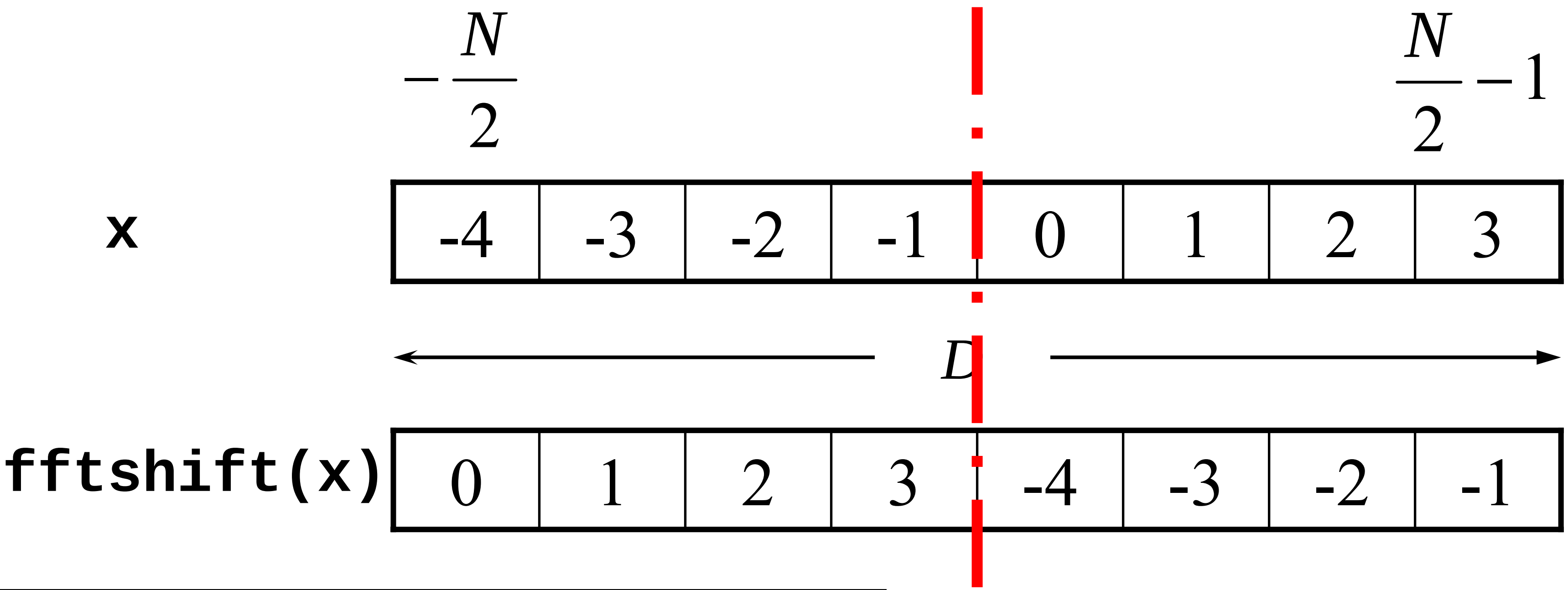


计算使然

$$A=\begin{bmatrix}1&1&4&4\\1&1&4&4\\2&2&3&3\\2&2&3&3\end{bmatrix}$$

$$fftshift(A)=\begin{bmatrix}3&3&2&2\\3&3&2&2\\4&4&1&1\\4&4&1&1\end{bmatrix}$$

FT 的建立




```
>> N=8;
>> x = [-N/2:N/2-1]

x =

    -4     -3     -2     -1      0      1      2      3

>> fftshift(x)

ans =

     0      1      2      3     -4     -3     -2     -1
```

变量周期为 $D=8$

基频为：

$$\Delta\xi_0=\frac{1}{D}$$

离散 FT

$-\frac{N}{2}$ … $\frac{N}{2}-1$

x

-4	-3	-2	-1	0	1	2	3

```
>> N=8;
>> x = [-N/2:N/2-1]
```

fftshift(x)

0	1	2	3	-4	-3	-2	-1

阵列 **x** 周期性:

物空间：$\Delta X = 1, N = 8$

FT空间满足 $\Delta\xi_0 = \frac{1}{N\Delta X}$

0	1	2	3	-4	-3	-2	-1	0	1	2	3	-4	-3	-2	-1	0	1	2	3	-4	-3	-2	-1

$\longleftarrow D \longrightarrow$

$\pm\frac{N}{2}\Delta\xi_0$

周期$D = 8$

基频：$\Delta\xi_0 = \frac{1}{D}$

folding frequency

定义 f(x)

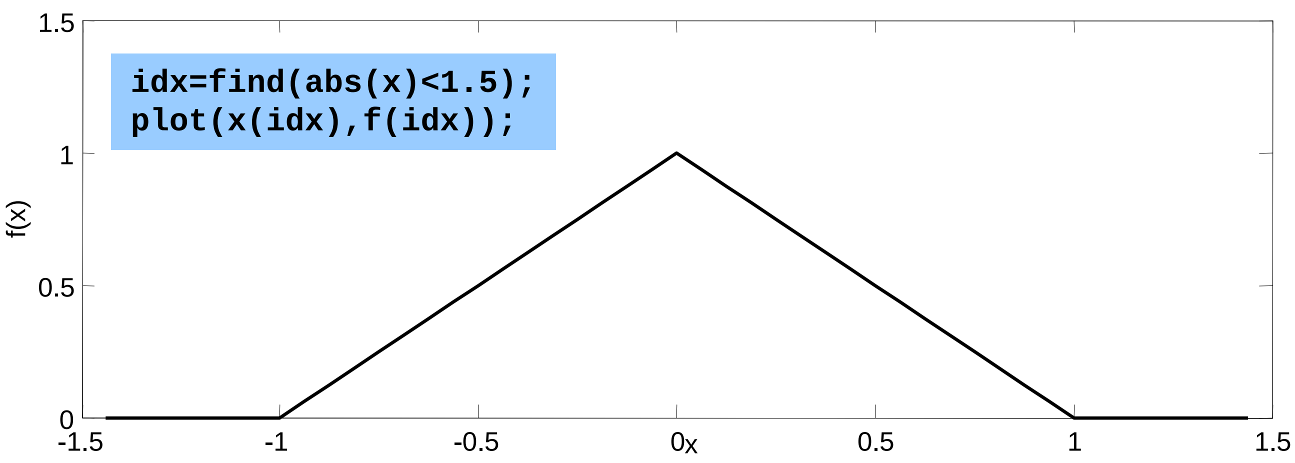
k	-128		127
x	-8		7.9375

```
N=256;
D=16;

k=-N/2:N/2-1;
x = (D/N)*k;
dx = x(2)-x(1);
f = tri(x);
```

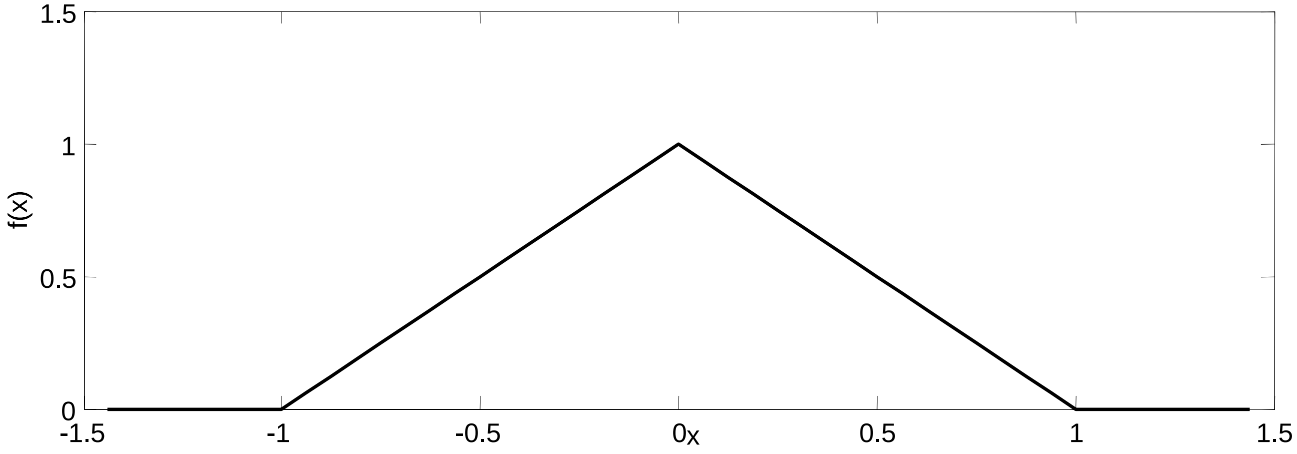
dx = (1/16) =0.0625

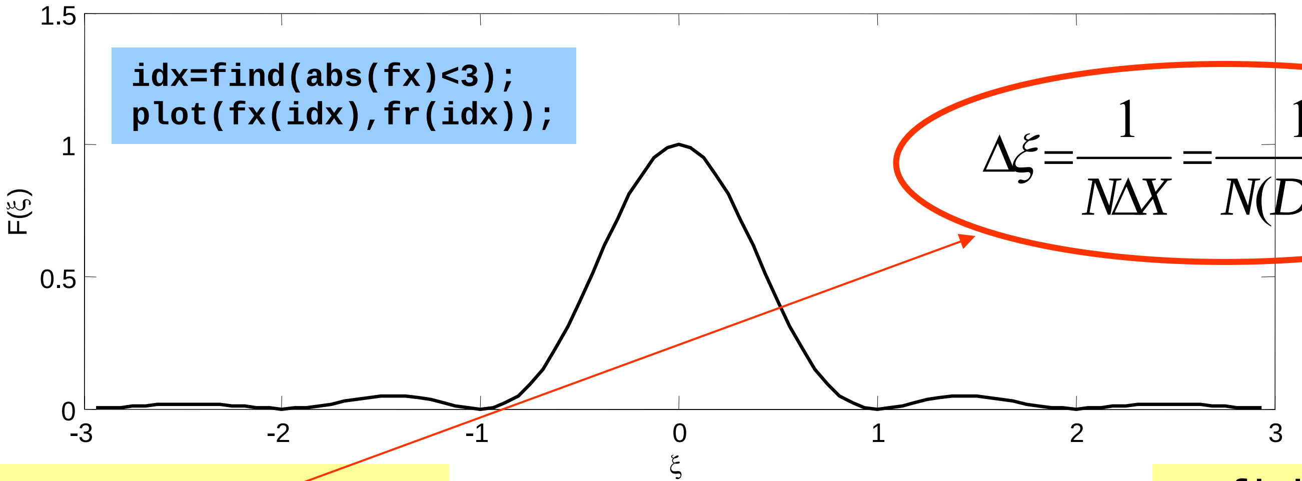
$\Delta X = 1/16, N = 256$

```
idx=find(abs(x)<1.5);
plot(x(idx),f(idx));
```



计算 F(ξ)





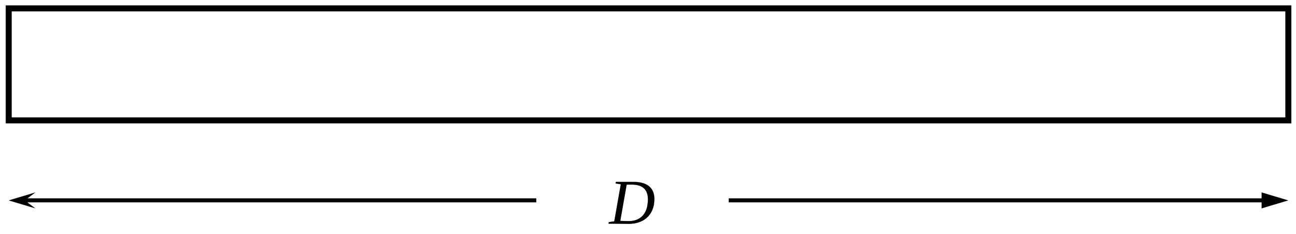
$$\Delta\xi = \frac{1}{N\Delta X} = \frac{1}{N(D/N)} = \frac{1}{D}$$

```
ft = fftshift(fft(fftshift(f)));
fx = (1/D)*k;
fr = real(ft)*dx;
```

经检查发现虚部为0.

```
>> fi=imag(ft);
>> max(abs(fi))

ans = 7.7716e-016
```

空域-频域　等价性



基频: $$\Delta\xi_0 = \frac{1}{D}$$

频宽(共有N倍基频宽度): $N\Delta\xi_0$ 分布范围从 $\frac{N\Delta\xi_0}{2}$ to $\frac{N\Delta\xi_0}{2}$

令空间带宽D=频带宽.

$$N\Delta\xi_0 = D \quad \text{or} \quad \boxed{D = \sqrt{N}}$$

```
N=256;
D=16;
```

1

Space-Bandwidth Product $N\Delta\xi_0 \cdot D = N$

傅里叶变换：MATLAB实现

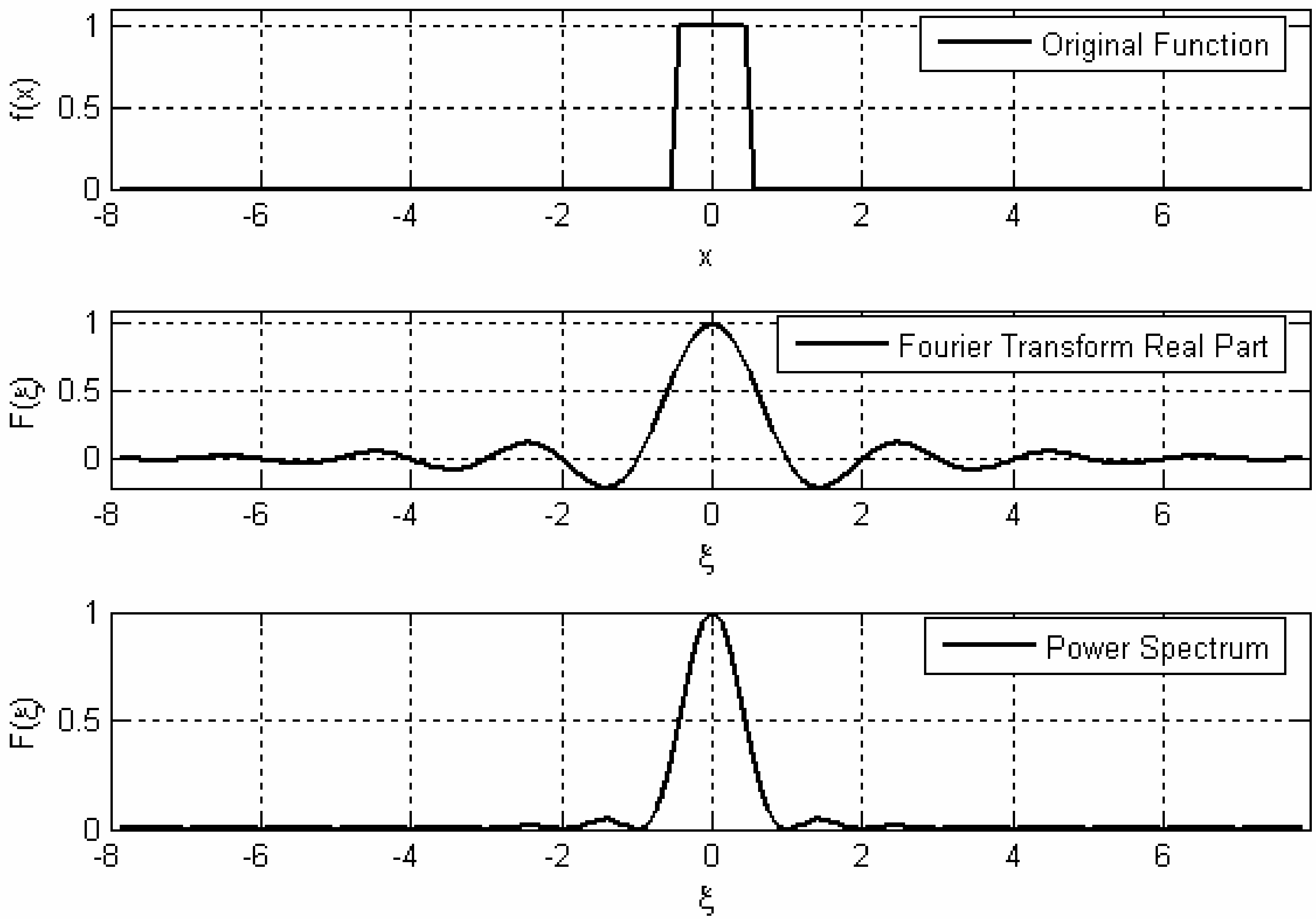
```
function FT(fun,D)
% function to be transformed
% N the transform point
% D the transform range
%check the number of parameters; if not 3 error

error(nargchk(2,2,nargin));
fun=fcnchk(fun);
N=D^2;

k=-N/2:N/2-1; %the number of discrete points
x = (D/N)*k ;%the range of x position
dx =D/N; %the spacing in x direction
```

```
% Original function operation
f=feval(fun,x); % the input function
xrange =max(x);
idx=find(abs(x)<xrange);
figure(1);
subplot(3,1,1);% Draw original function
plot(x(idx),f(idx),'k','LineWidth',2);
axis([min(x) max(x) min(f) max(f)+0.1]);
xlabel('x');
ylabel('f(x)');
legend('Original Function');
grid on;
```

```
% FFT
ft = fftshift(fft(fftshift(f)));
fx = (1/D)*k ;% the spacing in spectrum
fr = real(ft)*dx;

fi = imag(ft);
max(abs(fi));
frange= max(x);
idx = find(abs(fx)<frange);%Determin the
    range to draw
subplot(3,1,2);
plot(fx(idx),fr(idx),'k','LineWidth',2);
axis([min(fx) max(fx) min(fr) max(fr)+0.1]);
xlabel('\xi');
ylabel('F(\xi)');
legend('Fourier Transform Real Part')
grid on;
```

```
subplot(3,1,3);
idx = find(abs(fx)<frange);
%Determin the range to draw
A = sum(f)*dx;
ftn=ft*(dx/A);
fps=ftn.*conj(ftn);

plot(fx(idx),fps(idx),'k','LineWidth',2);
frange=max(x);
axis([min(fx) max(fx) 0 1]);
xlabel('\xi');
ylabel('F(\xi)');
legend('Power Spectrum')
grid on;
```

FT('rect(x)',16)



1
0.5
0
f(x)
-8
-6
-4
-2
0
2
4
6
x
Original Function
F(ξ)
Fourier Transform Real Part
ξ
F(ξ)
Power Spectrum
ξ

傅里叶变换：数值实现

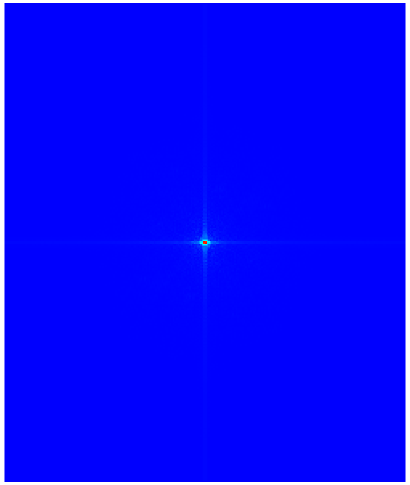
2、FFT自适应算法特性及其实现

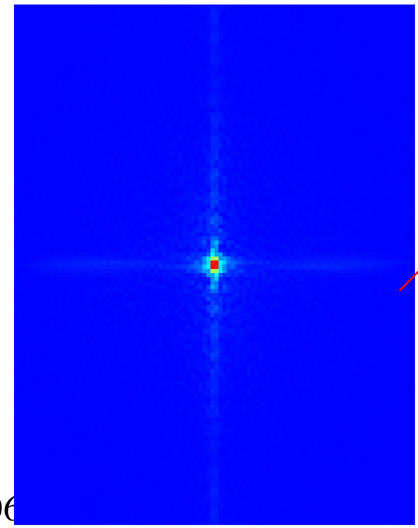
1）一维复数变换（FFTW）；2）多维复数变换（FFTWND）；3）一维实数变换（RFFTW）；4）多维实数变换（RFFTWND）。每一种模式都有它自己的计算方案和执行器。

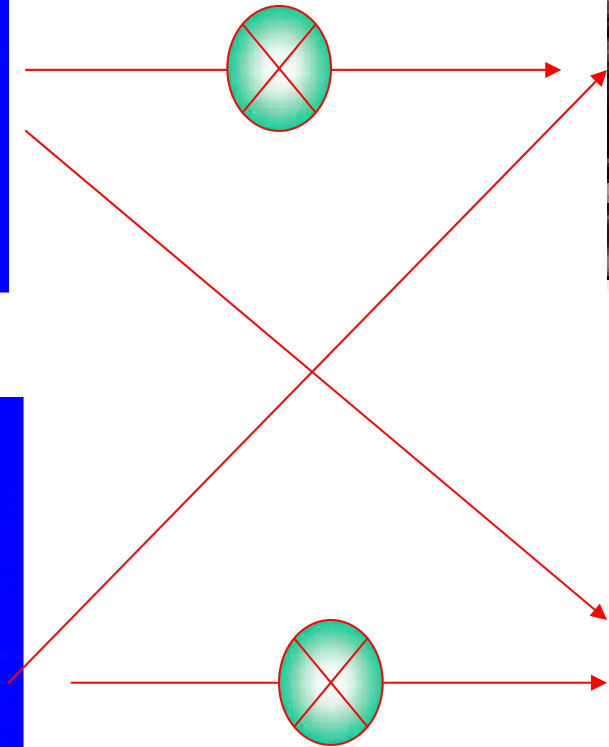
```
#include <fftwnd.h>
…
{
    fftw_complex  in[M][N] , out[M][N] ;
    fftwnd_plan p ;//FFTWND 计算方案
    p=fftw2d_create_plan(M, N , FFTW_FORWARD, FFTW_ESTIMATE) ;
    //根据机器情况自动创建一个计算方案
    fftwnd_one(p, in, out) ;//将方案与输入一起， 进行FFT， 并输出
    fftwnd_destroy_plan(p);//删除方案
}
```

3、其他实现方法。。。。。。

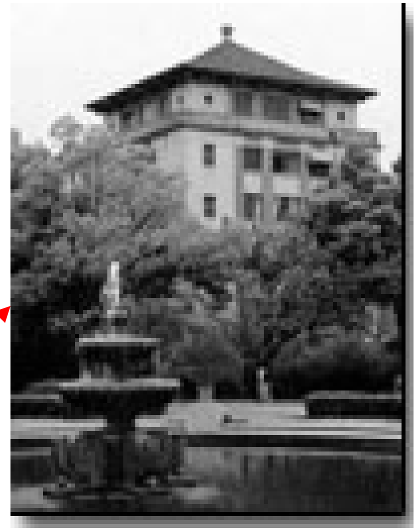
你能找出对应关系吗？？







Block 3 Opt. Eng. Dept.



Karen Mok Man-Wai

作业

试着编写一个针对普通函数的傅里叶变换算法，实现函数的傅里叶二维变换，并将函数、傅里叶变换分别以灰度图像的方式或立体图像形式出来，注意要求有通用性

1、最好的形式应该是函数形式，如FT2D(’rect(x).*rect(y)’,d1,d2)；
2、显示的应该是二维灰度图或三维立体图，
3、针对一些特殊函数，试着用你的函数进行变换，看看是不是同理论一致，若不一致，试分析原因(以上要求也要附在文档里）；
4、要求做文字说明，并附上你的.m程序同DOC文档；不要忘记你的大名同学号。
5、程序注释要清楚、文档排版要整洁（用标题方式来排版）
6、yufeihong@gmail.com