

## **I. Introduction to the Aberrations of Optical Systems**

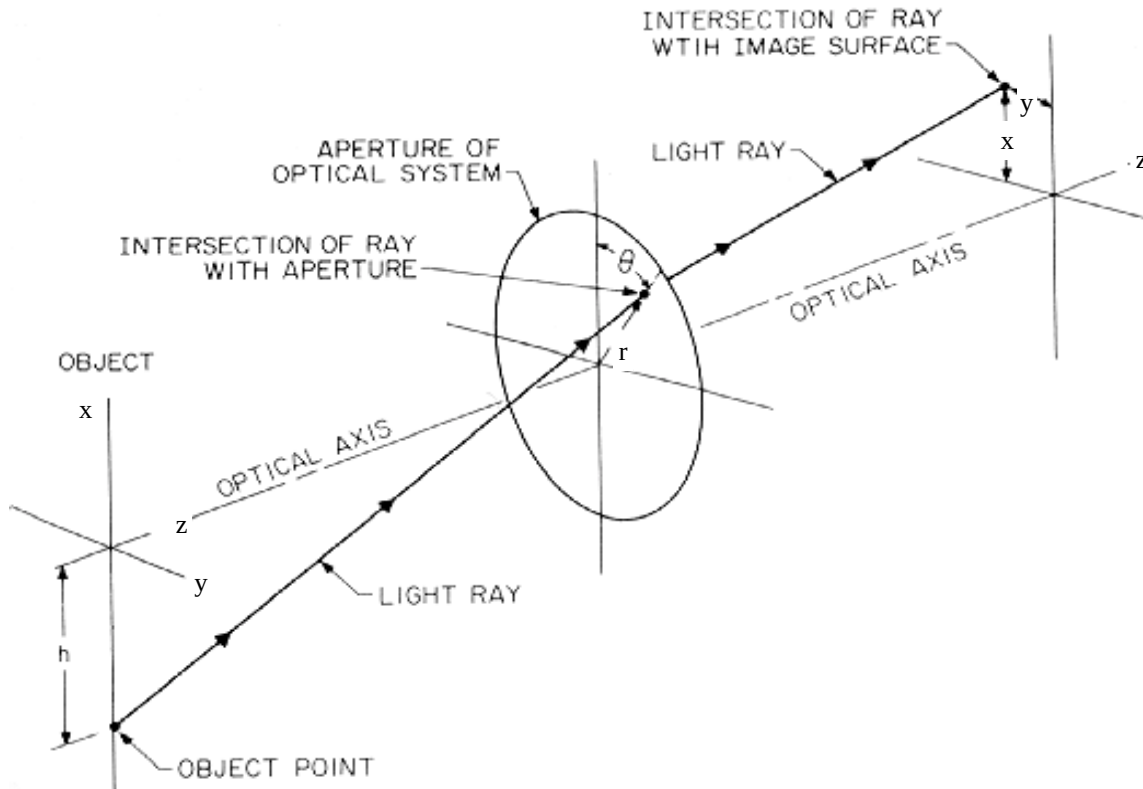
Perfect image formation in an optical system occurs when all of the rays originating from a single object point cross at a single image point, or equivalently, when the geometrical wavefront in image space has a spherical shape centered on the image point. Lord Rayleigh provided a mathematical proof that in order for an optical system to produce a perfect image of an object, all of the optical path lengths (OPL's) for each possible ray connecting an object point to its corresponding image point (conjugate points) must be equal. The only surfaces to rigorously satisfy this condition are the Cartesian oval and the conic sections at one pair of conjugates. This situation is never satisfied for a real optical system with spherical optics, or for the oval and conic sections away from their unique pair of conjugate points (the only optical element that exhibits stigmatic imaging for all pairs of conjugate points is the plane mirror). This leads us to expect deviations from ideal (stigmatic) imaging which we refer to as aberrations. Aberrations are highly dependent upon surface curvatures, conjugate ratios, aperture stop location and diameter, and field angles.

- 1). Monochromatic aberrations are aberrations that arise due to geometrical deviations from paraxial (Gaussian) theory. First-order theory corresponds to the approximation  $\sin\theta \approx \theta$ . If we extend the approximation to the next term, we can predict deviations from paraxial theory when  $\sin\theta \approx \theta - \theta^3/6$ . This third-order theory describes the five primary or Seidel aberrations: spherical aberration, coma, astigmatism, field curvature, and distortion.
- 2). Chromatic aberrations result from the dependence of index of refraction on wavelength ( $n \rightarrow n(\lambda)$ ). This causes the properties of an optical system, such as magnification, focal length, and location of all the principal points, to be different for each color of light passing through the system.
- 3). Diffractive aberrations are caused by deviations from geometrical optics due to the wave nature of light. Diffraction effects place the ultimate limit on a system's best possible imaging performance.
- 4). Aberrations may also result from the physical limitations of an optical system (surface quality, surface accuracy, limited aperture size, etc.).

## **II. Third-Order Aberration Theory**

- 1). The first step in analyzing deviations from paraxial theory is to model the behavior of an optical system as a function that is an infinite series of terms of which the first term reproduces paraxial theory and the remaining terms correspond to successively smaller corrections. These expansions may be obtained many different ways: optical Hamiltonian, wavefront distortion, optical surfaces,  $\sin\theta$ , ... Generally, the five Seidel terms are the only aberration terms that maintain the same functional form regardless of method of derivation.

- 2). Consider the situation depicted below: a ray originates from an object point at  $-h$  on the  $x$  axis, passes through the system's aperture stop at position  $(r, \theta)$ , and intercepts the chosen image plane at position  $(x, y)$ .



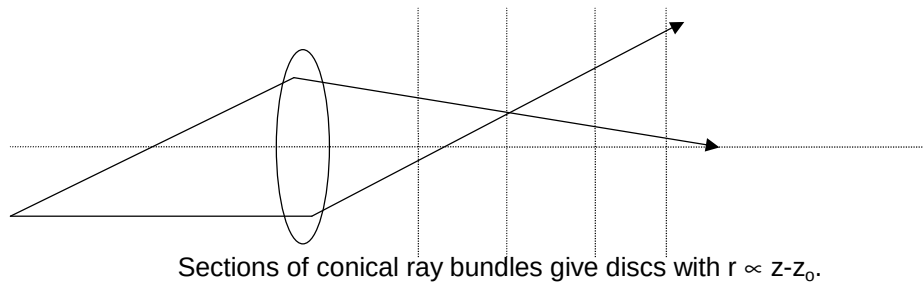
- 3). Using a Taylor expansion of the optical Hamiltonian (the optical Hamiltonian is derived from Fermat's principle of minimizing the optical path length integral), we can find a series solution for the final position  $(x, y)$ :

$$x = A_1 r \cos \theta + A_2 h + B_1 r^3 \cos \theta + B_2 r^2 h (2 + \cos 2\theta) + (3B_3 + B_4) r h^2 \cos \theta + B_5 h^3 + \dots$$

$$y = A_1 r \sin \theta + B_1 r^3 \sin \theta + B_2 r^2 h \sin 2\theta + (B_3 + B_4) r h^2 \sin \theta + \dots$$

### III. The First-Order Terms

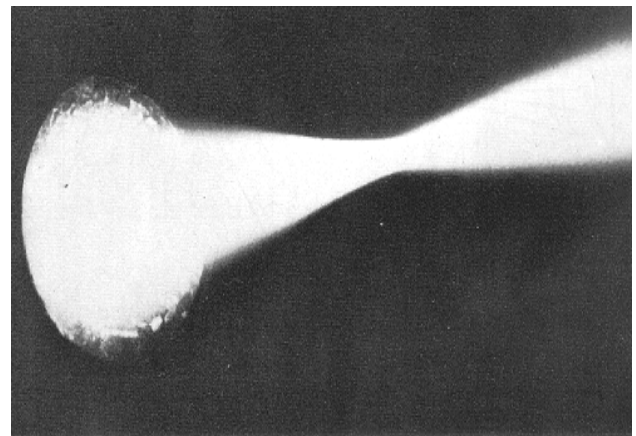
- 1). We expect that the first-order terms will correspond to paraxial optics.
- 2).  $x = r \cos \theta$  and  $y = r \sin \theta$  are the parametric equations for a disk if  $r$  and  $\theta$  are allowed to vary; if  $A_1$  is allowed to vary, a cone of light rays will be traced. From this we can see that  $A_1$  determines the location of our  $x$ - $y$  plane ( $A_1 \rightarrow A_1(z)$ ), where  $A_1 = 0$  is the Gaussian image plane.



- 3). The  $A_2$  term is proportional to the convergence angle of the principal ray. Notice that in the Gaussian image plane,  $A_2$  is just the simple magnification.

#### IV. Spherical Aberration

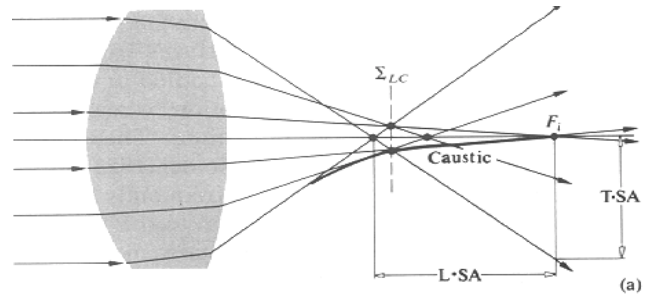
- 1). The  $B_1$  term results in spherical aberration. A simple explanation of SA is that it is the aberration resulting from the deviation of spherical surfaces from the ideal shape as aperture radius increases.



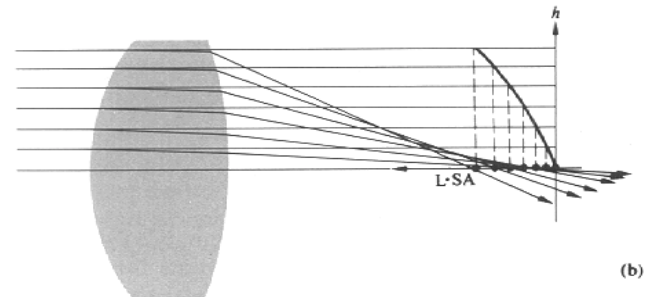
The spherical aberration of a lens (courtesy of K. K. Gupta and A. Sharma).

- 2). It is the only third-order term that doesn't depend upon  $h$ ; thus, spherical aberration is the only aberration that occurs for on-axis imaging.
- 3). The  $B_1$  term represents a disk of radius  $B_1 r^3$ , so there is a "blur" spot instead of a point image. If  $r$  is held constant, there is one value of  $A_1$  that regains focus by canceling the effect of the SA terms. This implies that the position of the image plane varies with  $r$  (wavefronts are no longer spherical).
- 4).  $B_1 > 0$  (positive SA) corresponds to the marginal rays focusing more sharply, while  $B_1 < 0$  (negative SA) corresponds to the paraxial rays focusing more sharply.
- 5). The longitudinal spherical aberration (LSA) is the distance between the marginal and paraxial focal planes. The transverse spherical aberration (TSA) is the radius of the blur disc at the Gaussian image plane. LSA and TSA are not independent of each other since they are related by the marginal convergence angle.

- 6). The “circle of least confusion” is the smallest blur spot that can be obtained by “de-focusing” from the paraxial image plane. The CLC plane is located  $\frac{3}{4}$  LSA from the paraxial image plane, and  $r_{CLC} = \frac{TSA}{4}$ .

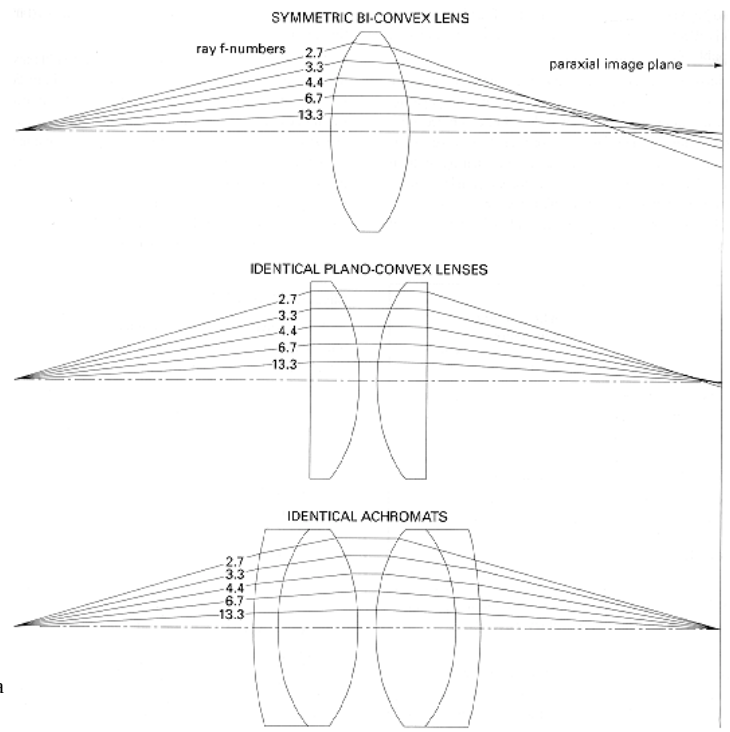


- 7). All aberrations (especially SA) may be minimized by choosing a lens shape (“bending”) that minimizes all of the refraction angles, thus making  $\sin\theta \approx \theta$  a better approximation. The “best form” is the singlet lens shape that minimizes spherical aberration ( $n=1.69$  gives plano-convex for an infinite conjugate ratio with the collimated light on the curved side).

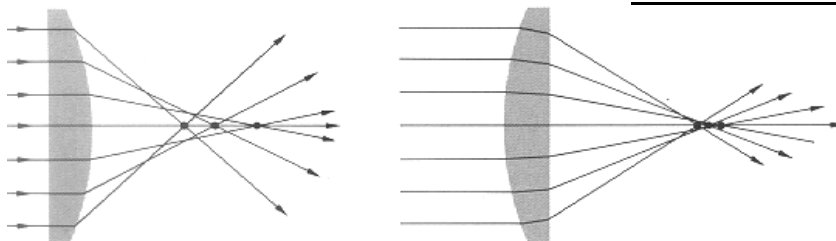


Spherical aberration for a lens. The envelope of the refracted rays is called a caustic. The intersection of the marginal rays and the caustic locates  $\Sigma_{LC}$ .

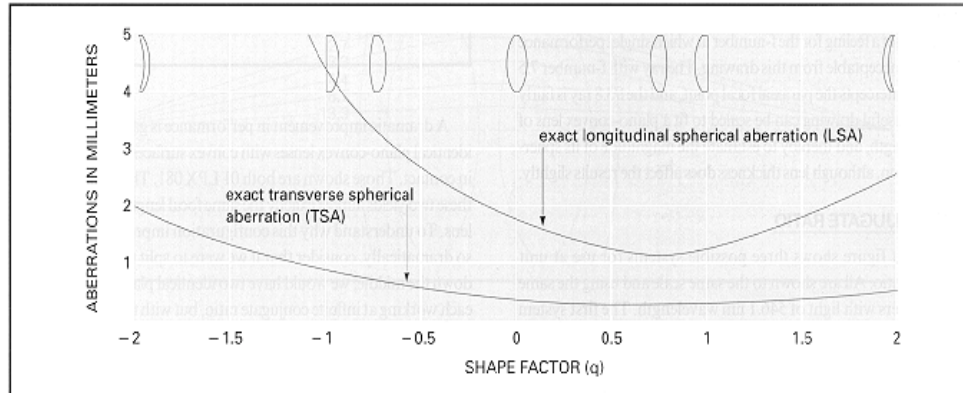
- 8). The CLC spot size for the infinite conjugate best form may be approximated as  $d \approx \frac{0.067 f}{(f/\#)^3}$
- 9). Pairs of infinite conjugate best forms may be used at finite conjugates to reduce spherical aberration if the conjugate ratio is equal to the focal ratio (the conjugate points are the focal points of the two lenses).



In the first diagram, all the bending occurs at the second surface, so  $\sin\theta \approx \theta$  is a worse approximation than in the second diagram; thus, the SA is much worse.



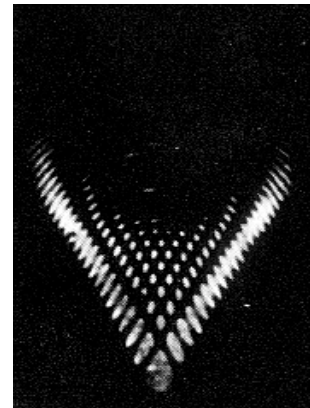
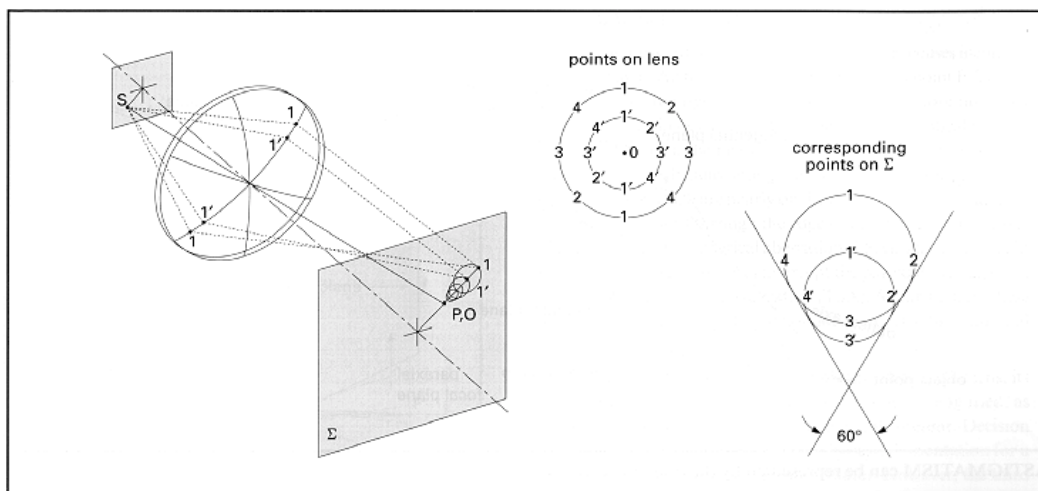
Scaled diagram of SA for several optical systems.



ABERRATIONS OF POSITIVE SINGLETs at infinite conjugate ratio as a function of shape.

## V. Comatic Aberration

- 1). The  $B_2$  terms represent coma. Coma was often called the worst of all aberrations due to its asymmetrical shape (a problem for precise distance measurements). Conceptually, coma has been viewed as a result of the curvature of a system's principal "planes".
- 2). Notice that the  $B_2$  term has two parts in  $y$ . The  $x=B_2r^2h\cos 2\theta$  and  $y=B_2r^2h\sin 2\theta$  terms represent two overlapped circles (one for each half of the lens) of radius  $B_2r^2h$ . The remaining term,  $x=2B_2r^2h$ , acts to vertically shift the center of each circle with given  $r$  value. The combined result produces a cone shaped blur spot with the apex at the Gaussian image point and an opening angle of  $\pi/3$ .

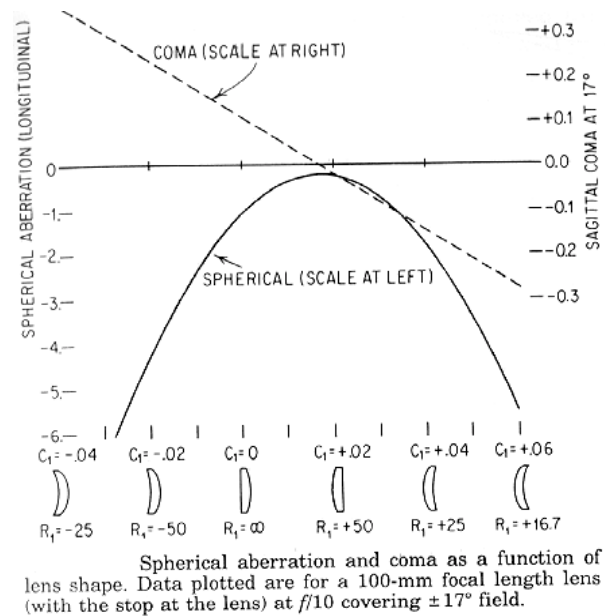


IMAGING OF AN OFF-AXIS POINT SOURCE by a lens with positive transverse coma. Each cone of rays passing through a circle of the lens surface is imaged not to a point, but as a comatic circle. The overall image is shaped like a comet with a conical flare. Rays passing through the central region of the lens form a point image at the vertex of the cone.

- 3).  $B_2 > 0$  is positive coma: the cone opens away from the optical axis.  $B_2 < 0$  is negative coma: the cone opens toward the optical axis.
- 4). The linear dependence on  $h$  implies that coma will be a significant effect even for small field angles.
- 5). Defocusing coma can obtain a more symmetrical shape, but the area of the blur spot will not be reduced. There is no CLC in the sense of maximizing power density.



Figure 3.5 The coma patch. The image of a point source is spread out into a comet-shaped flare.

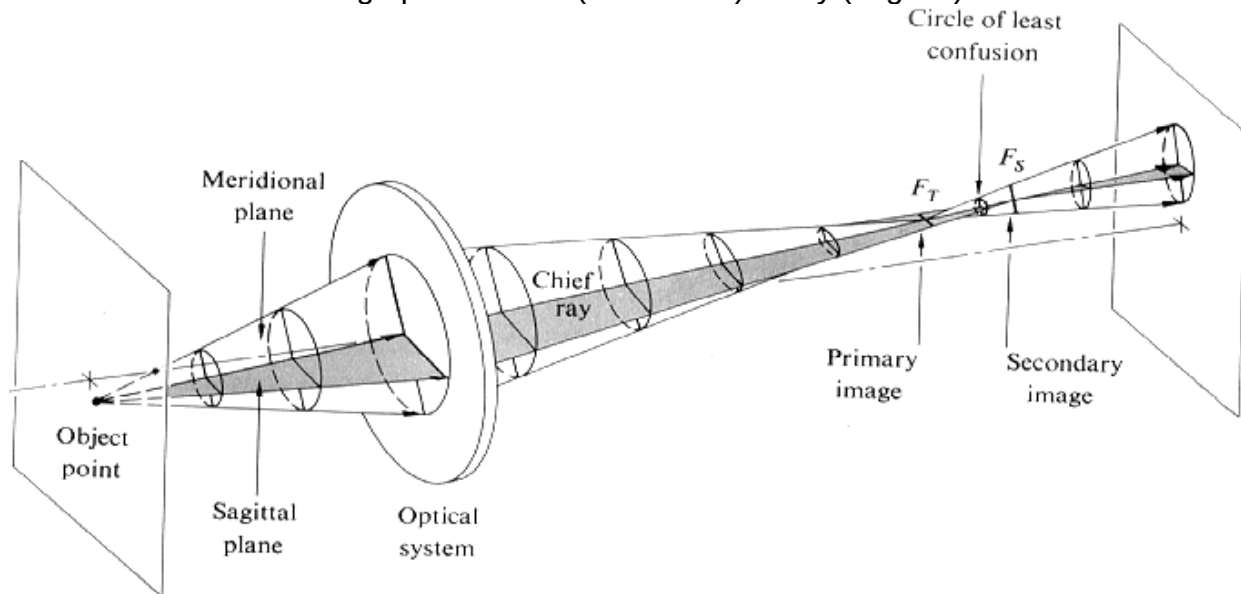


- 6). An important aplanatic (no spherical or comatic aberration) condition is the Abbe sine condition. If a system satisfies the relation  $\frac{\sin U}{u} = \frac{\sin U'}{u'}$  ( $U$  is the marginal convergence angle,  $u$  is paraxial approximation of  $U$ , unprimed/primed represent object/image space), there will be no contributions from any of the spherical or comatic aberration terms. The sine condition effectively requires that the “error” made in using paraxial theory is the same for both entering and exiting rays. Most achromatic doublets are designed to satisfy the sine condition.

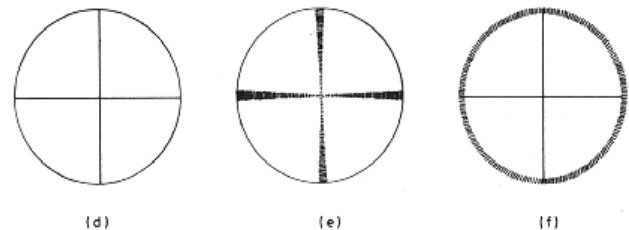
## VI. Astigmatism

- 1). The  $B_3$  term corresponds to astigmatism. This term depends upon  $h^2$  so it becomes more important as the object point moves farther away from the axis. Astigmatism is due to the asymmetry between the rays in the meridional (the plane containing the optical axis and the object point) and sagittal (the plane perpendicular to the meridional plane which contains the chief ray) planes. The meridional rays intercept the lens at a larger angle than the sagittal rays, so they are bent more (and thus focus more sharply for a positive lens).

- 2). The  $x$  and  $y$  terms represent an ellipse in the Gaussian image plane that has a major axis three times longer than the minor axis (if  $B_4=0$ ).
- 3). As the image plane is translated away from the Gaussian image plane, we can only eliminate either the  $x$  or the  $y$  term (with the linear  $A_1$  terms); there are two different image planes for  $x$  (meridional) and  $y$  (sagittal).



- 4). The linear  $r$  dependence implies that perfect images may be obtained for either plane—the image degenerates into a line in each plane.
- 5). The CLC is located halfway between the sagittal and meridional focal planes. At this plane, the CLC is a disk with a diameter that is half the length of the astigmatic focal lines.



If the object is of the form shown in (a) [or (d)], then the image formed in the tangential plane will be of the form (b) [or (e)] and the image in the sagittal plane will be of the form (c) [or (f)].

- 6). Anamorphic lenses (lenses without cylindrical symmetry about the optical axis) may be used to correct astigmatism (cylindrical lenses for example).

## VII. Field Curvature

- 1). The  $B_4$  terms result in field curvature. This term is grouped with astigmatism due to their similar functional forms.

- 2). This aberration produces a blur disc, but unlike SA, an image plane may be found that focuses a ring for each value of  $h$ . The focus of each  $h$  is located at  $A_1 = -B_4 h^2$ . This is equivalent to having a parabolic image “plane”: the image is formed on a parabolic surface (the image surface is approximately spherical in the paraxial region).

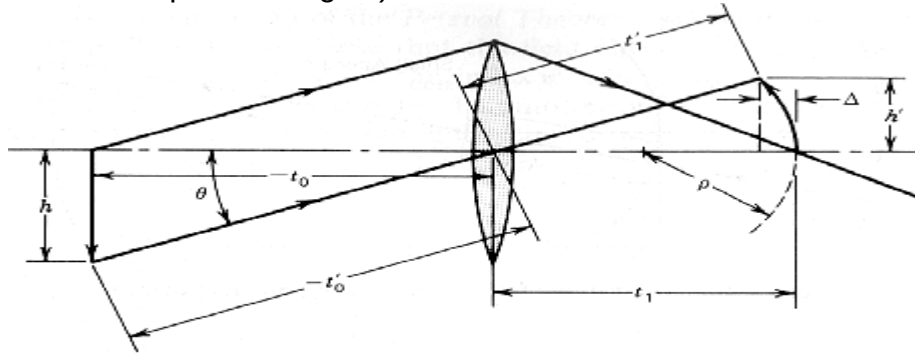
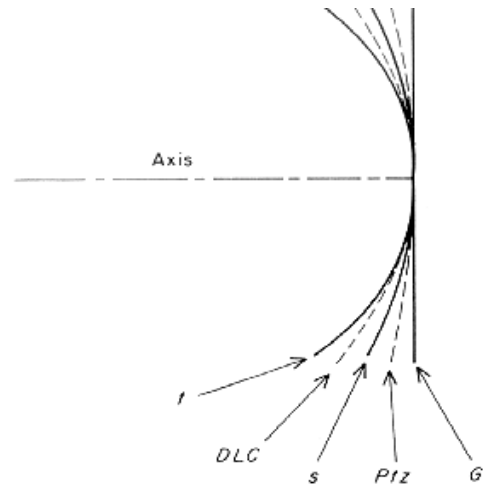


Illustration of field curvature with a thin lens.

- 3). This implies that the meridional and sagittal focal surfaces are also parabolic (with different curvatures).
- 4). Since positive lenses have  $B_4 > 0$ , and negative lenses have  $B_4 < 0$ , combinations can be found to cancel the field curvature. The condition that must be satisfied is known as the Petzval condition which states that  $n_1 f_1 + n_2 f_2 = 0$  for a “flattened field”.

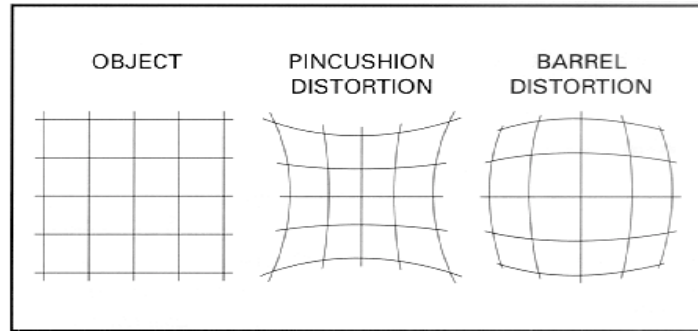


The image surfaces; G—Gaussian, t—tangential, s—sagittal, Prz—Petzval, DLC—disc of least confusion

- 5). A “field flattener” is a lens that is placed very near the image plane to remove the effect of field curvature. By choosing the correct shape, the Petzval condition may be satisfied without introducing any additional aberration since the flattener is so near to the image plane.

## VIII. Distortion

- 1). The effect of the  $B_5$  term is called distortion. Notice that this term is independent of  $r$  and  $\theta$ ; each object point is imaged perfectly as a point in the Gaussian image plane.
- 2). The  $B_5 h^3$  term may be grouped with the magnification term. This results in a magnification that depends upon the cube of the distance of the object from the axis. This will cause the shape of an extended object to become distorted.

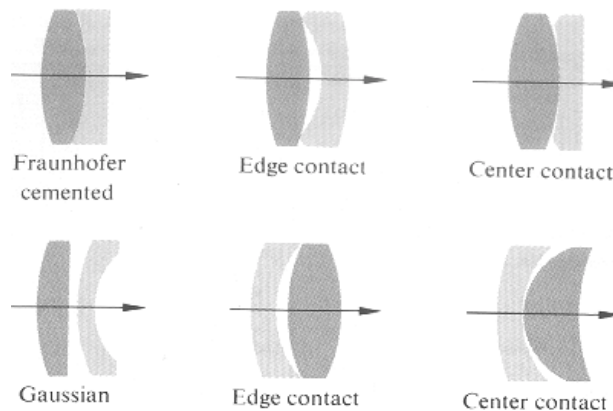


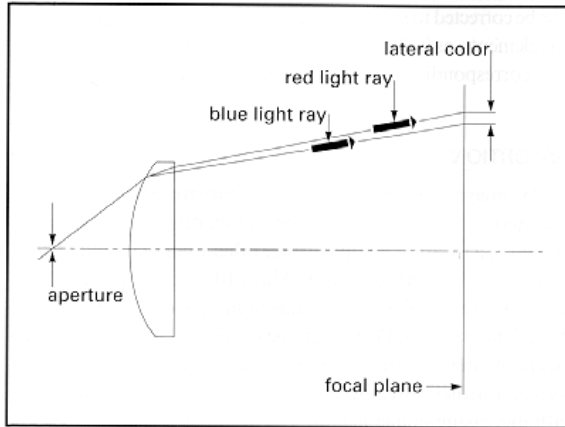
**DISTORTION** is illustrated by these two image types.

- 3). For  $B_5 > 0$ , the outer edges of the object will be magnified more (pincushion distortion). For  $B_5 < 0$ , the outer edges of the object will be magnified less (barrel distortion).
- 4). If the lens is turned around, the sign of  $B_5$  changes. This implies that if two identical lenses are used symmetrically “back to back”, the system will be free of distortion (the aperture stop must be at the center of the system).
- 5). Thin lenses have no distortion when the aperture stop is located at the lens.
- 6). Distortion is the only aberration in a system with a pinhole exit pupil.

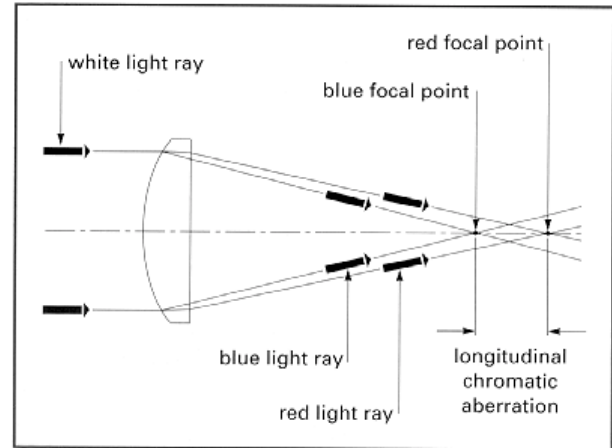
## IX. Chromatic Aberrations

- 1). Since the index of refraction depends upon the wavelength of light ( $n$  grows larger for higher frequency), an optical system will focus each color differently, causing an aberration.
- 2). Since  $n_{\text{blue}} > n_{\text{red}}$ , positive lenses bend blue light more sharply toward the axis while negative lenses bend blue light more sharply away from the axis. This suggests that a combination of lenses (known as an achromatic doublet) could reduce chromatic aberrations.





LATERAL COLOR is the difference in image height between red and blue rays.



LONGITUDINAL CHROMATIC ABERRATION

- 3). Longitudinal chromatic aberration is the distance between the red focal plane and the blue focal plane. Lateral chromatic aberration (or color) is the difference in magnification between red and blue images.

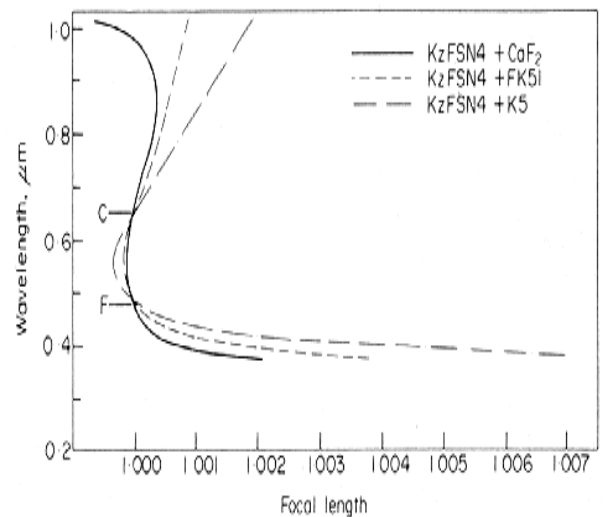
- 4). A measure of dispersion for a lens is its Abbe number  $V$ . The Abbe number

is  $V = \frac{n_{\text{yellow}} - 1}{n_{\text{blue}} - n_{\text{red}}}$  where the usual wavelengths used are the sodium D line

and the hydrogen C and F lines (the larger  $V$ , the less dispersion exhibited).

If  $f_{1y}V_1 + f_{2y}V_2 = 0$  for a pair of lenses in a doublet, the system will be achromatized for red and blue (they have the same focal length). Notice that this does not eliminate longitudinal chromatic aberration since the positions of the principal planes depend on wavelength.

- 5). Although doublets are achromatized for two wavelengths, there will be a residual dispersive effect for other wavelengths. This remaining chromatic aberration is called "secondary spectrum". Notice that while monochromatic aberrations may theoretically be reduced to zero by adding many optical elements, chromatic aberrations are generally worsened by using more than a few (2-3) optical elements (this is due to the difference in the slopes of the dispersion curves for different glasses).



The secondary spectrum of achromats; the KzFSN4 is the "flint", i.e. the material of higher dispersion. The graphs are calculated for a doublet of unit power at C and F

| Aberration                                     | vs. Aperture | vs. Field Size or Angle |
|------------------------------------------------|--------------|-------------------------|
| Spherical (longitudinal)                       | $y^2$        | —                       |
| Spherical (transverse)                         | $y^3$        | —                       |
| Coma                                           | $y^2$        | $h$                     |
| Petzval curvature (longitudinal)               | —            | $h^2$                   |
| Petzval curvature (transverse)                 | $y$          | $h^2$                   |
| Astigmatism and field curvature (longitudinal) | —            | $h^2$                   |
| Astigmatism and field curvature (transverse)   | $y$          | $h^2$                   |
| Distortion (linear)                            | —            | $h^3$                   |
| Distortion (in percent)                        | —            | $h^2$                   |
| Axial chromatic (longitudinal)                 | —            | —                       |
| Axial chromatic (transverse)                   | $y$          | —                       |
| Lateral chromatic                              | —            | $h$                     |
| Lateral chromatic (CDM)                        | —            | —                       |

The variation of the primary aberrations with aperture and field.

## X. The Seidel Sums

- 1). Seidel derived expressions that give the aberration coefficients in terms of summations over all the optical surfaces in a system. This allows the magnitude of the primary and chromatic aberrations to be calculated from knowledge of the geometrical properties of the system. In practical applications, all information needed for the summation is found by tracing two rays from an axial object point.
- 2). Spherical aberration  $\rightarrow S_I$   
Comatic aberration  $\rightarrow S_{II}$   
Astigmatism  $\rightarrow S_{III}$   
Field Curvature  $\rightarrow S_{IV}$   
Distortion  $\rightarrow S_V$   
Longitudinal Chromatic  $\rightarrow C_I$   
Lateral Chromatic  $\rightarrow C_{II}$
- 3). The Seidel sums are related to the aberration coefficients through inverse multiples of the marginal convergence angle and index of refraction.
- 4). The aperture stop equations are some of the most important equations in aberration theory. They relate a small change in aperture stop position ( $\delta E$ ) to a resulting change in the Seidel sums ( $\delta S$ ). They are written in terms of the "stop shift parameter"  $\delta E$ , where  $\delta E = \frac{\Delta z}{n_s r_s (r_s + \Delta z u_s)}$  and the Lagrange invariant  $H$  for the marginal rays, where  $H = nuh$  (this is a more general version of the optical sine theorem).

$$\delta C_I = 0$$

$$\delta C_{II} = H(\delta E)C_I$$

$$\delta S_I = 0$$

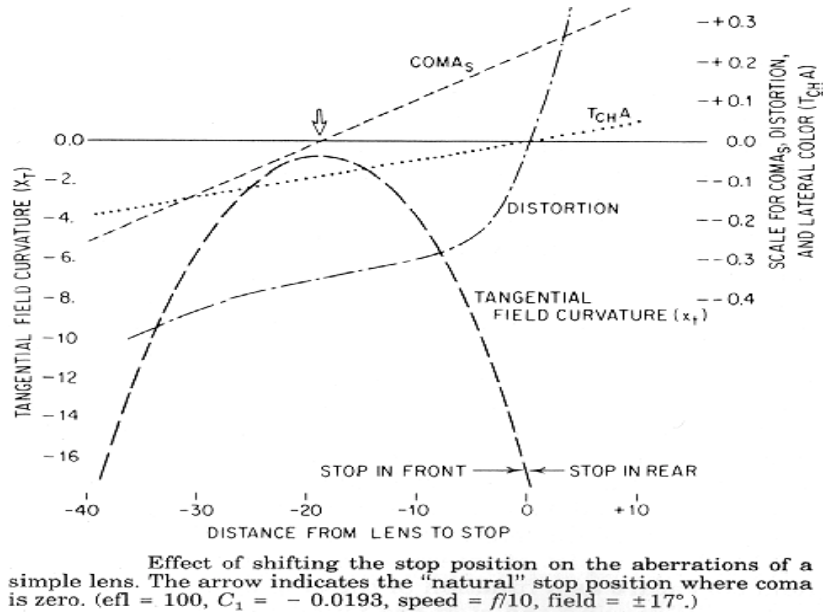
$$\delta S_{III} = 2H(\delta E)S_{II} + H^2(\delta E)^2 S_I$$

$$\delta S_{II} = H(\delta E)S_I$$

$$\delta S_{IV} = 0$$

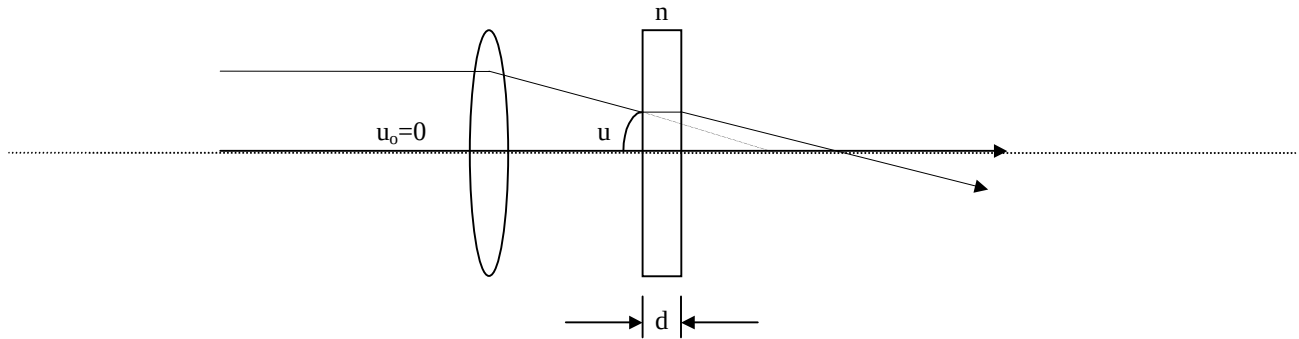
$$\delta S_V = H(\delta E)(3S_{III} + S_{IV}) + 3H^2(\delta E)^2 S_{II} + H^3(\delta E)^3 S_I$$

- 5). Notice that the largest effects from aperture shift occur for small stops (although the absolute magnitude of the aberration is reduced for small stops).



## XI. Seidel Sums for a Plane-Parallel Plate

Consider a plane-parallel window of thickness  $d$  and index  $n$ :



The Seidel sums take a particularly simple form for this situation. The values can calculate to be the following (where  $u$  is the marginal convergence angle, and  $u_0$  is the angle of the chief ray as it passes through the aperture stop).

$$S_I = -\frac{n^2 - 1}{n^3} u^4 d$$

$$S_{III} = \left(\frac{u_0}{u}\right)^2 S_I$$

$$C_I = -\frac{\delta n}{n^2} u^2 d$$

$$S_{II} = \frac{u_0}{u} S_I$$

$$S_{IV} = 0$$

$$C_{II} = -\frac{\delta n}{n^2} u u_0 d$$

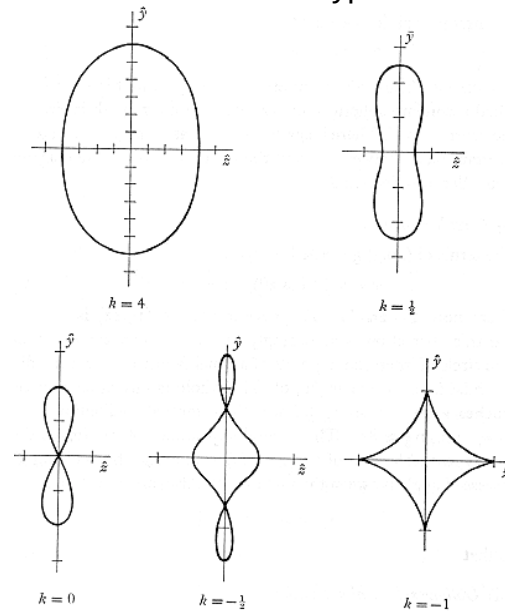
$$S_V = \left(\frac{u_0}{u}\right)^3 S_I$$

And, since transverse spherical aberration is given by  $TSA = \frac{S_I}{2u}$ , we have

that  $TSA = -\frac{n^2 - 1}{2n^3} u^3 d$ . Thus, for a plate 2mm thick, with an index of 1.5, and a convergence angle of  $\pi/15$  (resulting from a lens with  $f/\# = 4.8$  for example), we have  $TSA \cong 4\mu\text{m}$  which gives the radius of the CLC to be  $\sim 1\mu\text{m}$ . Notice that if we assume that the focusing lens is an infinite conjugate best form with a typical focal length of 100 mm, the CLC radius due to SA is  $30\mu\text{m}$  for uniform irradiance of the aperture—the aberration due to the plate would only be important if a system corrected for SA were used (such as a doublet).

## XII. Fifth-Order Aberrations

- 1). Generally, third-order theory is used as a simple test of optics, as well as a conceptual basis for design considerations. The theory is rarely pursued to higher order due to the complexity and ambiguity of results.
- 2). The most common treatment of fifth-order theory results in fifth-order versions of each of the primary aberrations as well as two new types.
- 3). One of the new aberrations is elliptical coma. This is similar to primary (or “circular”) coma except that displaced ellipses are formed within a conical flare.
- 4). The other new aberration is oblique spherical aberration, which has no analogue in Seidel theory (the name is due to similar  $r$  dependence). Oblique SA has a more complicated functional form than any of the other aberrations, resulting in a variety of aberrated spot shapes for different values of the expansion coefficient.

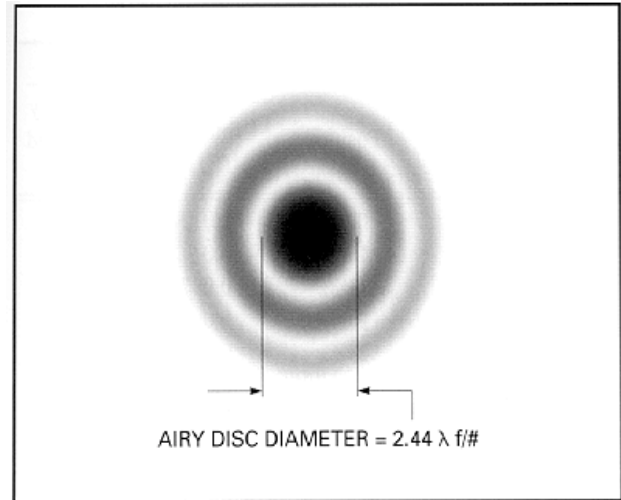


Some of the possible images of a point source suffering from oblique SA.

## XIII. Diffraction Effects

- 1). Since any real optical system only collects a portion of the object's emitted wavefront, we expect to observe diffraction in the transmitted wavefront. This diffraction is what ultimately limits a system's performance, although few systems are actually diffraction limited.

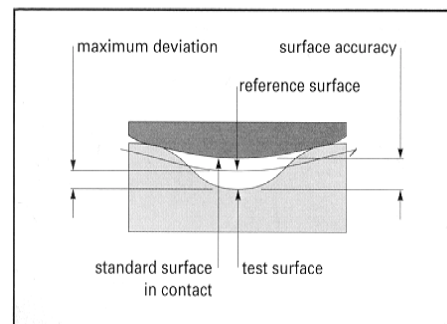
- 2). For axially symmetric optics, a uniformly irradiated circular aperture will produce an Airy function diffraction pattern at the Gaussian image point. If any aberration is introduced, the intensity of the central "Airy disk" will be reduced. The Strehl condition for classifying a system as diffraction limited requires that the central intensity of the Airy disk be at least 80% of the ideal intensity.
- 3). The size of the Airy disk is  $d=2.44\lambda(f/\#)$ . Notice that diffraction becomes greater as  $f/\#$  increases, while geometrical aberration increases with decreasing  $f/\#$ .
- 5). For a Gaussian beam of waist  $\omega_0$ , a pupil radius of  $2r = \omega_0$  approximates uniform irradiance, and thus the focused spot will be an Airy pattern (which will be narrower than the spot size predicted by beam calculations). For a pupil radius of  $r=2\omega_0$ , the intensity approximates a Gaussian profile, and beam calculations for the Gaussian spot size must be used. A popular compromise between transmitted power and spot size is to choose optics such that  $r=\omega_0$ .



CENTER OF A TYPICAL DIFFRACTION PATTERN for a circular aperture.

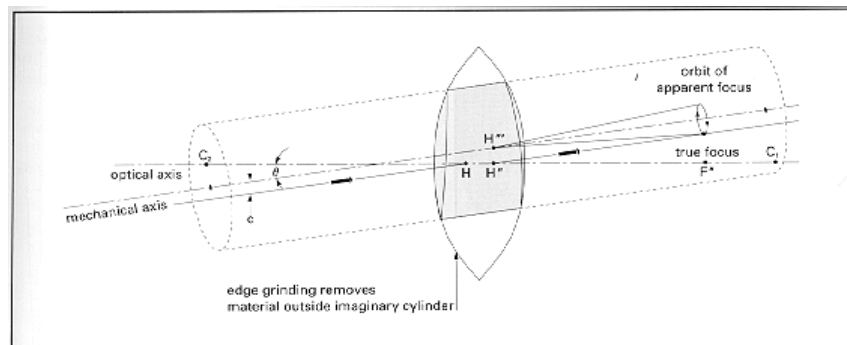
#### XIV. Physical Limitations

- 1). Surface Accuracy—Due to inevitable variations in the grinding process, optical surfaces are never identical with the mathematical functions that we use to model them. This variation from the theoretical surface shape can introduce additional aberrations into an optical system. Typically, surface accuracy is measured by comparing a highly accurate surface ( $\lambda/20$  or better) to a test surface and measuring the difference in shape with an interferometer. The results of such tests are given in terms of fractions of the wavelength of the light used in the interferometer (usually a helium-neon laser at 632.8 nm or the mercury green line at 546.1 nm). Surface accuracy is defined as twice the maximum deviation of the test surface from its ideal mathematical form: a  $\lambda/10$  surface accuracy would mean a maximum deviation of about 30 nm.



DEFINITION OF SURFACE ACCURACY.

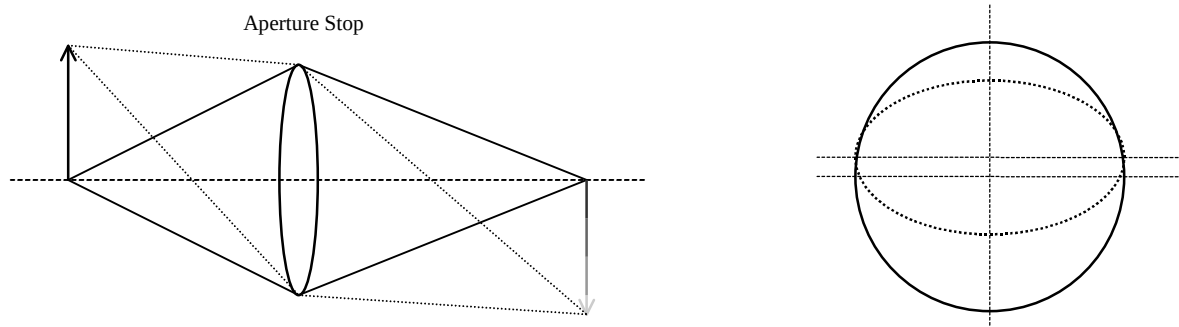
- 2). Cosmetic Surface Quality—Another result of the physical limitations of the grinding process is that optical surfaces are “rough”. Scratches and pits on an optical surface will reduce the quality of images formed by the surface. In the United States, cosmetic surface quality is usually measured in terms of the U.S. Military Surface Quality Specification (MIL) for optical surfaces. In this system, surface quality is given as two numbers separated by a dash. The first number (scratch) represents the maximum width of any scratches on an optical surface in tenths of a micron (the combined length of all maximum width scratches must not exceed a quarter of the diameter of the surface): a 80 scratch would be a surface with no scratches wider than 8 microns. The second number (dig) is the maximum diameter of any pits on the surface of the optic measured in hundredths of a millimeter: a 50 dig would be a surface having no pits with diameter greater than 500 microns. 80-50 is a commonly acceptable standard, 60-40 is typical for scientific applications, and 10-5 is used for precise laser applications.
- 3). Centration—In most of lens theory, it is assumed that the optical axis of the system passes through the centers of curvature for all the optical surfaces. Due to the finite precision of the manufacturing process, the center of curvature for each optical surface of a real system is usually slightly displaced from the optical axis. This misalignment of the surfaces can have a deleterious effect upon image formation. Centration is a measure of this misalignment defined as the maximum angular deviation of a ray entering an optical system along its optical axis (or the mechanical axis of grinding).



DEFINITION OF CENTRATION: A centration specification of 3 arc minutes means that the angular radius of the orbit of apparent focus cannot exceed 3 minutes.

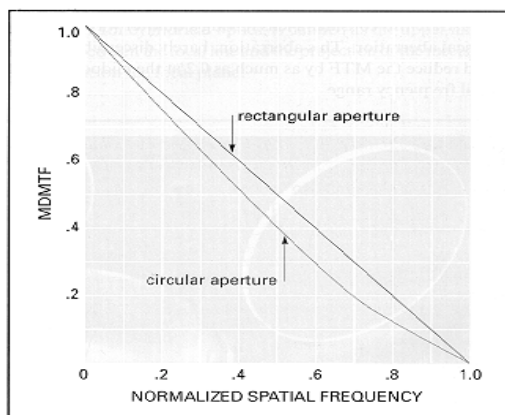
- 4). Vignetting—The amount of light that is collected by an optical system is determined by the size of the aperture stop (or equivalently, the entrance pupil); thus, the intensity of an image will be determined by the system's  $f/\#$  (the intensity of the image is proportional to the square of the  $f/\#$ ). Vignetting is the term used for the reduction in intensity of off-axis image points due to the finite size of the aperture—Vignetting is an aberration in the system's reproduction of the object's intensity profile. Vignetting may be understood by realizing that the angular size of the AS is reduced for off-axis object points; therefore, less light is transferred to the image. If a system has a circular AS, the AS will appear elliptical to off-axis image points, and the

apparent area (or angular size) will be reduced. If  $\theta$  is the angle of the object point's chief ray as it passes through the AS, we find that the intensity of an image point will be given by  $I_i = I_o \cos \theta$ , where  $I_o$  is the image intensity at the optical axis.

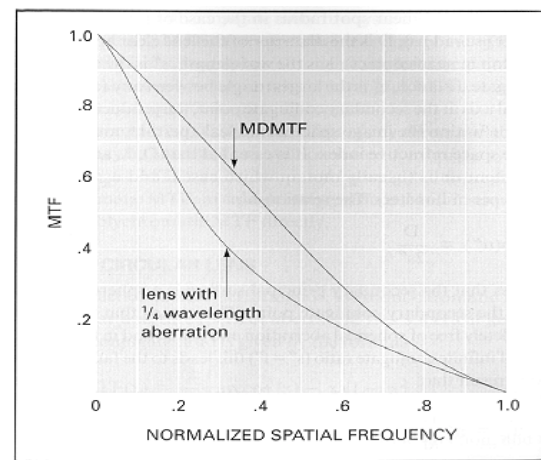


Vignetting is displayed in the figures above. The figure on the left depicts the reduction in intensity for off-axis image points. The figure on the right displays the apparent entrance pupil for the on-axis (solid) and off-axis (dotted) object points. The shift of the center of the pupil is due to the divergence of the intercepted light rays and the unequal distances from the image point to the top and bottom of the AS.

- 5). Modulation Transfer Function (MTF)—In order to specify the imaging performance of an optical system, a quantitative measure of image quality is required. The modulation transfer function is one of the most useful measures of an optical system's ability to transfer object contrast (sharpness) to the image. The MTF( $\omega$ ) is determined by forming the image of a uniformly illuminated object with a spatially sinusoidal transmittance of frequency  $\omega$ . The MTF is a measure of the maximum variation in intensity of the sinusoidal image as compared to the intensity variation of the object (measured as a function of the spatial frequency of the modulation  $\omega$ ): an MTF equal to one would be a perfect image, while an MTF of zero would be a uniform blur. The MTF is a very sensitive measure of aberration; a lens with only a quarter-wavelength spherical aberration (barely observable with the human eye) would have its MTF reduced by as much as thirty percent!



MDMTF(x) vs x. These curves (and the formulas they represent) precisely quantify an important and fundamental property common to all electromagnetic, de Broglie, and acoustic waves. All objects become more difficult, and ultimately impossible, to image as their dimensions diminish toward the radiation wavelength.



MTF AS A FUNCTION of normalized spatial frequency.