

Ti:sapphire laser cavity mode and pump-laser mode calculations

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Comprehensive calculations of the cavity mode size throughout a Ti:sapphire laser, made with the *ABCD* Gaussian beam formalism are reported. These calculations show that the beam is not collimated, in general, in what are normally referred to as the collimated arms of the laser cavity. Additionally, the mode size and volume (in the gain medium) of the argon-ion laser, which is used to pump the Ti:sapphire laser optically, are evaluated for different focusing geometries, and graphs that can be used to select suitable mode-matching optics are produced. It is concluded that an appropriate strategy for mode matching the pump beam to the Ti:sapphire laser mode is to use a zoom telescope to tailor the collimated pump-laser beam diameter to an optimum value. Finally, comparisons of the pump-laser mode and the Ti:sapphire laser mode are presented for selected pumping geometries.

Key words: Ti:sapphire laser, laser cavity calculations, Gaussian beam.

Introduction

A number of reports on the analysis of laser cavity stability and cavity waist spot size, for cavities of practical importance, have been published.¹⁻³ Such analyses are most simply carried out by use of the *ABCD* law for laser resonators and matrix methods⁴ to write general computer programs that permit any linear or ring laser cavity to be modeled. Previous studies¹⁻³ have been concentrated on determining the position and the spot size of the waists in multimirror, multifoci, off-axis laser resonators as a function of mirror separations in the cavity. This has often been done with the cavity reduction method, but the tedious algebraic manipulation of this method is not necessary given the ease of writing general computer programs that can be used to model any laser cavity. In the present analysis the ease of graphically presenting large amounts of data from such cavity calculations with a personal computer and appropriate software allows us to extend the calculations to include consideration of the mode size at all positions in the cavity and the mode volume of the cavity mode in the gain medium for the laser. A Ti:sapphire laser cavity is used as an example to illustrate the analysis

and the information on mode size, beam divergence, and mode volume that is obtained. In particular it is noted that the beam is not collimated in any section of such a cavity, although some sections are commonly referred to as collimated arms.

The Ti:sapphire laser, along with many other laser systems, is optically pumped with a longitudinal pumping geometry. For the Ti:sapphire laser a common pump source is an argon-ion laser. Thus the efficiency of the Ti:sapphire laser is dependent on the overlap of the Ti:sapphire laser mode with the laser mode of the focused pump-laser beam throughout the entire length of the gain medium. The Ti:sapphire crystal is typically 10–30 mm in length, and the laser's wavelength (700–1100 nm) differs from the wavelength of the argon-ion laser (515 nm), so exact mode matching is not possible. The efficient transfer of pump power to output power also depends on the gain for the system, and an analysis of the gain of a longitudinally pumped Ti:sapphire laser has been reported.⁵ This analysis predicts that the optimum waist size of the pump mode is 30 μm for a cavity waist size of 50 μm . This requirement that the waist size of the pump mode be somewhat smaller than the cavity waist size for optimum output power has also been predicted and observed experimentally in other laser systems, such as dye lasers. The gain analysis also predicts that for the Ti:sapphire laser the pump beam should be focused slightly to the front of the crystal rather than in the center, but this is a small dependence compared with selection of the optimum spot sizes. Thus it is important to evalu-

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ate the focused pump-beam mode size and volume and to compare them with the mode size and volume of the Ti:sapphire laser mode. The analysis presented in this paper is for an argon-ion laser at 515 nm that pumps a Ti:sapphire laser operating at 850 nm. The results demonstrate that optimum overlap can be achieved with a pump-beam focusing mirror (or lens) of any focal length, provided that the size of the collimated pump beam is tailored appropriately. This tailoring could be achieved in practice with a zoom telescope beam expander. This also allows the pumping geometry to be easily reoptimized for a different wavelength of operation within the broad tuning range of systems such as Ti:sapphire.

Ti:Sapphire Laser Cavity Design

General computer programs based on the $ABCD$ law for cavities, which allow the mode size in both the sagittal and the tangential planes of any linear laser cavity to be analyzed, have been applied to the design of a Ti:sapphire laser cavity, as depicted in Fig. 1(a). The complex beam parameter q at one plane with position z_1 in this cavity is determined with the $ABCD$ law for resonators:

$$q(z_1) = \frac{A_q(z_1) + B}{Cq(z_1) + D}, \quad (1)$$

where A , B , C , and D are the components of the $ABCD$ matrix that describes one complete round-trip in the cavity. Once the complex beam parameter for a single plane is known, the complex beam parameter at any other position can be found with the $ABCD$ law for propagation:

$$q(z_2) = \frac{A'q(z_1) + B'}{C'q(z_1) + D'}, \quad (2)$$

where A' , B' , C' , and D' are the components of the $A'B'C'D'$ matrix that describes propagation through the optical system from the plane of z_1 to the plane of z_2 . The off-axis mirrors and the Brewster-cut Ti:sapphire crystal in Fig. 1(a) have different effective focal lengths and optical path lengths in the tangential and the sagittal planes of the cavity, respectively. In general this leads to an astigmatic cavity mode. However, an anastigmatic output beam is predicted¹ in a linear cavity such as the one in Fig. 1(a) if

$$R \sin \theta \tan \theta = \frac{(n^2 - 1)L}{n^3}, \quad (3)$$

where n is the refractive index of the Ti:sapphire crystal.

To evaluate the $ABCD$ matrix for one round-trip of the cavity, the cavity is linearized as shown in Fig. 1(b). The off-axis mirrors are replaced with lenses with focal length $f_s = R/(2 \cos \theta)$ in the sagittal plane and focal length $f_t = (R \cos \theta)/2$ in the tangential plane. The effective length of the Brewster-cut Ti:sapphire crystal is L/n in the sagittal plane and L/n^3 in the tangential plane. The complex beam parameter in the plane of M_1 for the sagittal plane can be determined from the $ABCD_s$ matrix:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_s = D_1 F_{2s} D_2 B_L D_2 F_{3s} D_3 D_3 F_{3s} D_2 B_L D_2 F_{2s} D_1. \quad (4)$$

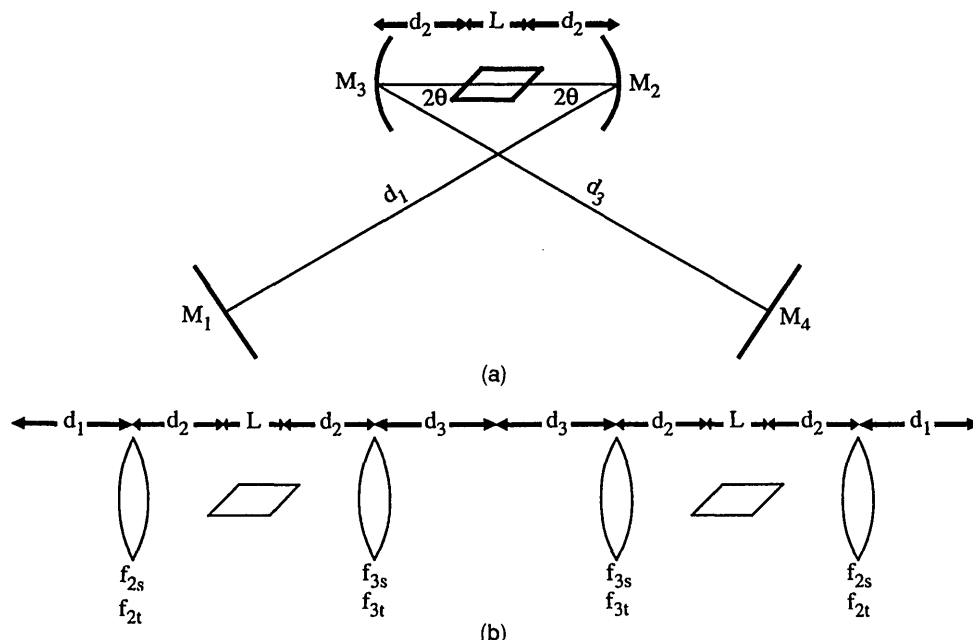


Fig. 1. (a) Design of the Ti:sapphire laser cavity. Mirrors M_1 and M_4 are planes. Mirrors M_2 and M_3 have a radius of curvature $R = 10$ cm. The gain medium is a Brewster-cut Ti:sapphire crystal of length $L = 20$ mm and refractive index $n = 1.76$. (b) Equivalent lens system for one complete round-trip for the Ti:sapphire laser cavity, starting and finishing at the plane of M_1 .

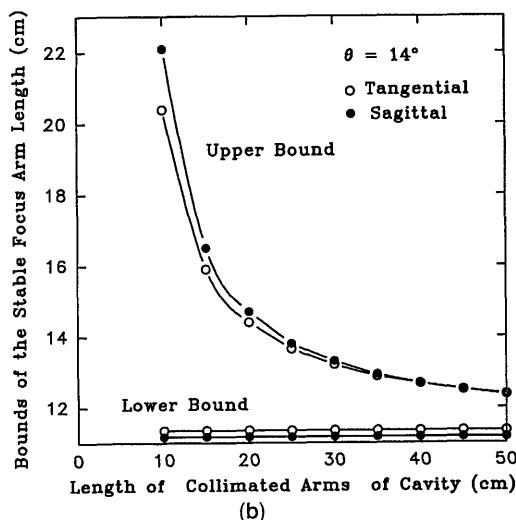
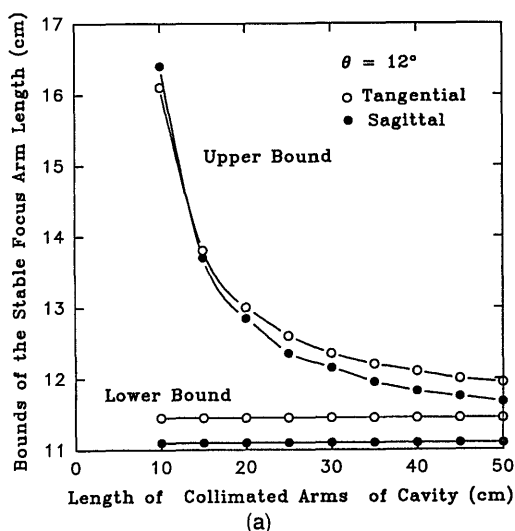


Fig. 2. Lower and upper bounds of separations of M_2 and M_3 for which the cavity is stable for the sagittal and the tangential planes of the cavity, as a function of $d_1 = d_3$: (a) $\theta = 12^\circ$ and (b) $\theta = 14^\circ$. The lasing wavelength is 850 nm. All other cavity parameters are as in Fig. 1.

The complex beam parameter in the plane of M_1 for the tangential plane can be determined from the $ABCD_t$ matrix:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_t = D_1 F_{2t} D_2 B_{Lt} D_2 F_{3t} D_3 D_3 F_{3t} D_2 B_{Lt} D_2 F_{2t} D_1, \quad (5)$$

where the matrices D_1 , D_2 , and D_3 refer to propagation over distances d_1 , d_2 , and d_3 , respectively; $F_{2t} = F_{3t}$ and $F_{2s} = F_{3s}$ describe propagation through the equivalent lenses of focal lengths $(R \cos \theta)/2$ and $R/(2 \cos \theta)$, respectively; and B_{Ls} and B_{Lt} are the matrices for propagation through the Brewster-cut Ti:sapphire crystal. The forms of these matrices are given in Appendix A. The cavity will be stable in a plane if

$$(2 - |A + D|) > 0. \quad (6)$$

For overall stability the cavity must be stable in both planes.

Calculations of the range of separations of the two focusing mirrors ($2d_2 + L$) for which the cavity is stable, in both of the sagittal and the tangential planes, as a function of $d_1 = d_3$ are shown in Fig. 2(a) for $\theta = 12^\circ$ and in Fig. 2(b) for $\theta = 14^\circ$. The cavity sections of lengths d_1 and d_3 are often referred to as collimated arms of the cavity. The lasing wavelength is set at 850 nm. It can be seen in Fig. 2 that a complete overlap of the sagittal and the tangential stable ranges of $(2d_2 + L)$ is never achieved because the sagittal stability range is slightly larger than the tangential stability range. Also, the optimum angle varies depending on the value of d_1 . Good overlap of the upper bound of the stable range for d_1 in the range 15–25 cm is achieved for $\theta = 12^\circ$, and $\theta = 14^\circ$ gives us a good overlap of the upper bound for d_1 in the range 40–65 cm. Overlap of the lower bound of the stable range is achieved for $\theta = 16^\circ$. This is the angle predicted from Eq. (3), indicating that Eq. (3) only predicts astigmatism compensation at the lower bound of the stable range of separations of M_2 and M_3 . Mirror separations in the middle of the stable range yield optimum operation of the laser.

The $1/e$ radius for the electric-field amplitude (which corresponds to the $1/e^2$ radius in intensity) at the waist that occurs in the middle of the Ti:sapphire crystal has been calculated for the center of the stable range. The $1/e$ radius, varies little with angle and is shown for $\theta = 14^\circ$ in Fig. 3, in which the tangential $1/e$ radius is divided by n for direct comparison with the sagittal $1/e$ radius. A smaller $1/e$ radius is achieved for larger d_1 , but the stable range of operation is then much reduced.

In the practical laser a suitable value of d_1 is defined by pump-laser mode matching and focusing of

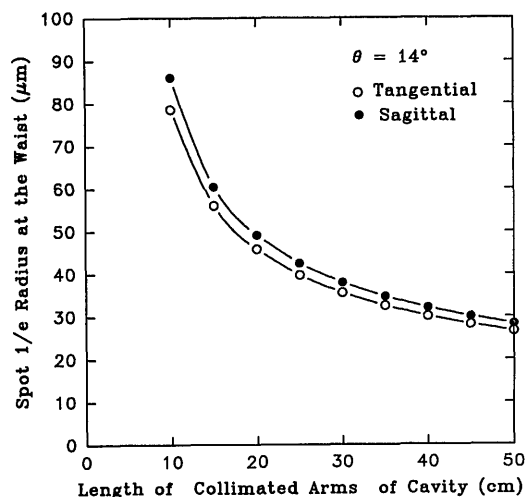


Fig. 3. $1/e$ radius at the waist, in the center of the Ti:sapphire crystal, calculated for the separation of M_2 and M_3 that corresponds to the middle of the stable range of separations shown in Fig. 2(b), as a function of $d_1 = d_3$. The tangential $1/e$ radius has been divided by $n = 1.76$ for direct comparison with the sagittal $1/e$ radius $\theta = 14^\circ$. All other cavity parameters are as in Fig. 1.

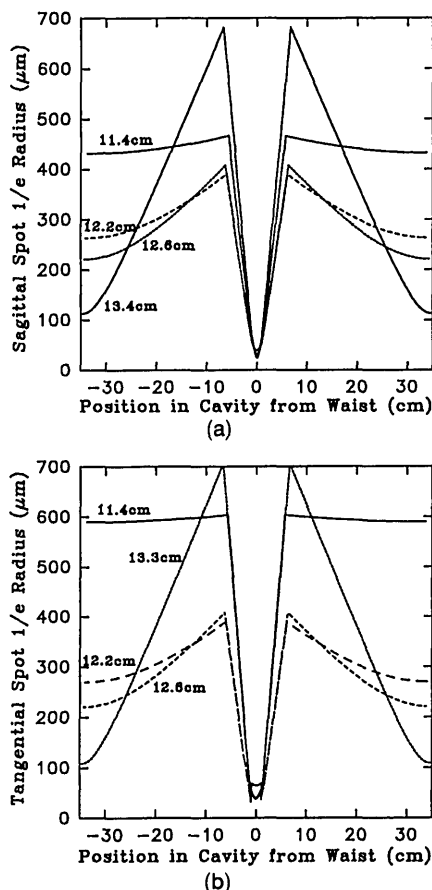


Fig. 4. $1/e$ radius for (a) the sagittal plane and (b) the tangential plane of the Ti:sapphire cavity for several separations of M_2 and M_3 [Fig. 1(a) (separation $2d_2 + L$)]. $d_1 = d_3 = 28$ cm, $\theta = 14^\circ$, $\lambda = 850$ nm.

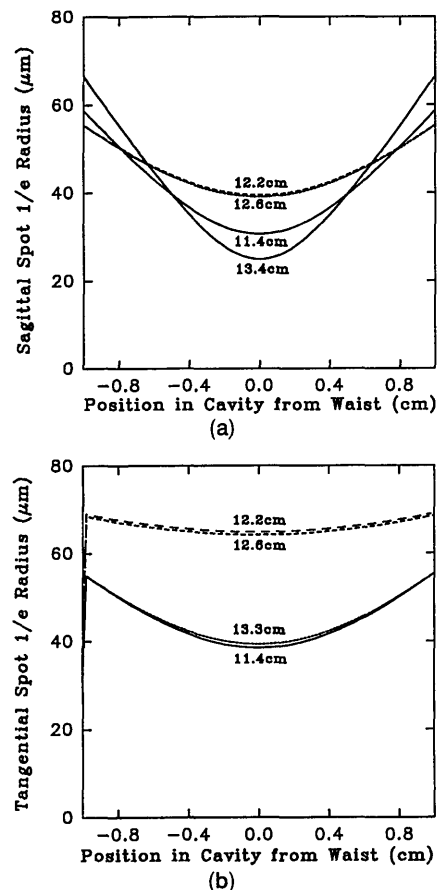


Fig. 5. $1/e$ radius for (a) the sagittal plane and (b) the tangential plane in the Ti:sapphire crystal for several separations of M_2 and M_3 . Cavity parameters are as in Fig. 4.

the near-infrared fluorescence from the Ti:sapphire crystal in the cavity. Values of $d_1 = d_3 = 28$ cm and $\theta = 14^\circ$, which provide good astigmatism compensation in the center of the stable range of separations of M_2 and M_3 for this d_1 value, are used in the remaining analysis. The $1/e$ radii for the sagittal and the tangential planes of the cavity for several values of the separation of mirrors M_2 and M_3 are shown in Figs. 4(a) and 4(b), respectively. Expanded graphs of the spot size in the Ti:sapphire crystal are shown in Figs. 5(a) and 5(b). The Gaussian beam in the (M_1 - M_2) and (M_3 - M_4) arms of the cavity is collimated only for the minimum stable separation of M_2 and M_3 [$d_2 = 4.7$ cm; the separation of 11.4 cm is approaching this minimum stable separation in Figs. 4(a) and 4(b)]. The far field of the laser beam leaving the cavity, calculated from the waist size at the output mirror, will vary between approximately 0.03° for the minimum stable separation and approximately 0.15° for the maximum stable separation. The maximum output power is expected to be obtained from the cavity configuration with the maximum cavity mode volume in the gain medium. The elliptical mode volume in the crystal as a function of d_2 is shown in Fig. 6 and the maximum value of the elliptical mode

volume corresponds to the center of the stable range of $d_2 = 5.2$ cm. A far-field divergence of approximately 0.08° is calculated for this d_2 [the M_2 - M_3 mirror separation is 12.4 cm (10.4 cm in air plus a 2-cm length of Brewster-cut Ti:sapphire)]. It is worth

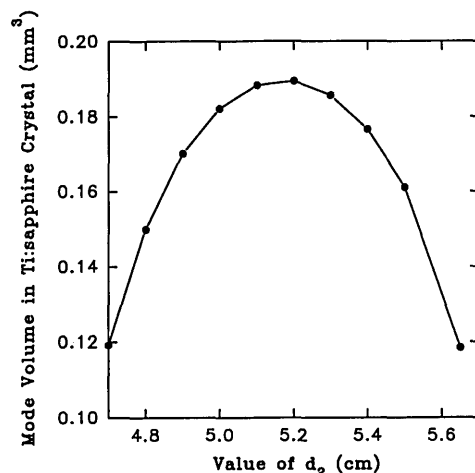


Fig. 6. Cavity mode volume in the Ti:sapphire crystal, calculated with the elliptical beam cross section, as a function of d_2 [Fig. 1(a)].

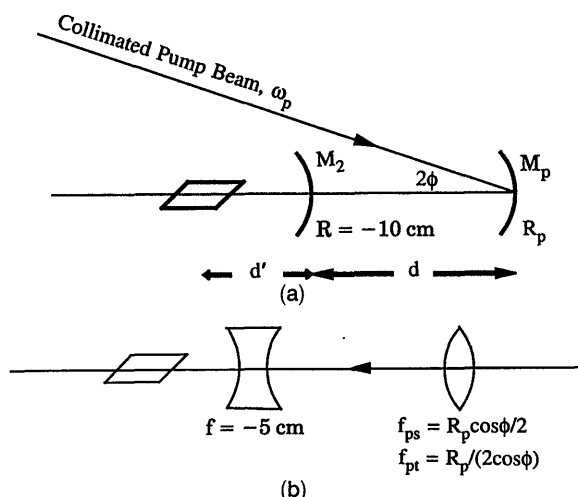


Fig. 7. (a) Argon-ion longitudinal pumping geometry and (b) equivalent lens system.

emphasizing that the divergence is not a minimum when the mode volume is a maximum.

Argon-Ion Pump-Laser Beam Focusing

The following analysis describes a collimated argon-ion laser beam with variable beam diameter ω_p , but constant power, that is focused by a spherical mirror M_p with radius of curvature R_p , followed by the Ti:sapphire laser focusing mirror M_2 , with a radius of curvature of -10 cm for the pump-laser beam [Fig. 7(a)]. The equivalent lens system is shown in Fig. 7(b). We calculate the distance d by requiring that the waist of the focused pump beam occur in the center of the Ti:sapphire crystal. In the first instance, ϕ is taken to be 0° but is subsequently chosen to give an anastigmatic pump focus. The collimated pump-laser beam that is incident upon the pump-focusing mirror has a complex beam parameter

$$\frac{1}{q_1} = \frac{i\lambda_p}{\pi\omega_p^2}. \quad (7)$$

The complex beam parameter at the plane of the cavity waist is given by

$$\frac{1}{q_2} = \frac{i\lambda_p}{\pi\omega_2^2} = \frac{Cq_1 + D}{Aq_1 + B}, \quad (8)$$

where A , B , C , and D are the elements of the $ABCD$ matrix that describes the beam transformation from the input plane to the plane of the cavity waist:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & d' \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2/R_2 & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2/R_p & 1 \end{bmatrix} = \begin{bmatrix} 1 - \frac{2d}{R_p} - \frac{2d'}{R_2} + \frac{4dd'}{R_p R_2} - \frac{2d'}{R_p} & d + d' - \frac{2dd'}{R_2} \\ \frac{4d}{R_p R_2} - \frac{2}{R_p} - \frac{2}{R_2} & 1 - \frac{2d}{R_2} \end{bmatrix}. \quad (9)$$

The waist of the focused pump beam occurs where q_2 is pure imaginary. For this we require that

$$AC \frac{\pi^2 \omega_p^4}{\lambda_p^2} + BD = 0. \quad (10)$$

This gives us

$$\left[\left(\frac{4d}{R_p R_2} - \frac{2}{R_p} - \frac{2}{R_2} \right) d' + \left(1 - \frac{2d}{R_p} \right) \right] \times \left(\frac{4d}{R_p R_2} - \frac{2}{R_p} - \frac{2}{R_2} \right) \left(\frac{\pi^2 \omega_p^4}{\lambda_p^2} \right) + d \left(1 - \frac{2d}{R_2} \right) + d' \left(1 - \frac{2d'}{R_2} \right)^2 = 0. \quad (11)$$

Equation (11) is used to calculate d with the required value of d' , with $R_2 = -10$ cm, and by taking ω_p and R_p as independent variables. Once d is calculated, it is used along with the other parameters in Eq. (8) to find the radius of the focused pump beam at the cavity waist position.

For each radius of curvature R_p there is a minimum radius ω_{pm} that is required for us to achieve a focus at the distance d' . Radii below this value form a focus at shorter distances from M_2 , and there is a second limit ω_{pmo} , below which there is no focus of the pump beam inside the Ti:sapphire laser cavity. This radius ω_{pmo} is the same as ω_{pm} at the level of precision quoted below. The minimum collimated pump beam $1/e$ radius ω_{pm} for $d' = 6.96$ cm is a linear function of the pump-mirror radius of curvature R_p and is given by

$$\omega_{pm}(\text{mm}) = 1.2 \times 10^{-3} [R_p(\text{mm}) - 14]. \quad (12)$$

This minimum collimated pump beam $1/e$ radius also corresponds to a maximum waist $1/e$ radius and, therefore, a larger collimated pump beam $1/e$ radius will be used in practice to achieve a small pump waist $1/e$ radius at the position of the cavity waist. The pump-waist radius as a function of R_p is given for different values of ω_p in Fig. 8. The same data are plotted as a function of ω_p for different values of R_p in Fig. 9. The corresponding value of d is almost constant for each value of R_p . Figures 8 and 9 can be used to select a suitable radius of curvature for the pump-focusing mirror if the radius of the pump beam is known and fixed. Alternatively, having selected a certain radius of curvature for the pump-focusing mirror, we can select a suitable pump-beam diameter (using, for example, a beam-expanding telescope) to

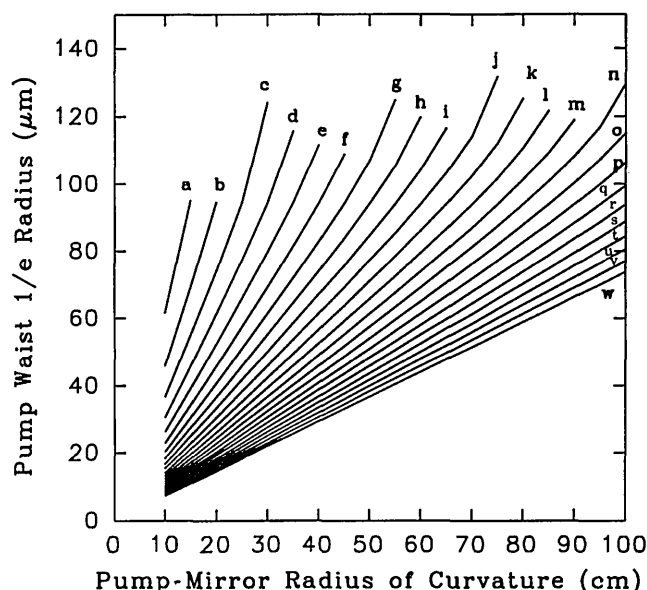


Fig. 8. Pump beam 1/e radius at the cavity waist (also the pump-beam focus) as a function of the radius of curvature for the pump mirror for different values of the radius of the collimated pump beam, ω_p . Values of ω_p (in millimeters) are a, 0.6; b, 0.8; c, 1.0; d, 1.2; e, 1.4; f, 1.6; g, 1.8; h, 2.0; i, 2.2; j, 2.4; k, 2.6; l, 2.8; m, 3.0; n, 3.2; o, 3.4; p, 3.6; q, 3.8; r, 4.0; s, 4.2; t, 4.4; u, 4.6; v, 4.8; w, 5.0.

yield the required pump-beam spot size at the cavity waist. In general, optically pumped lasers yield optimum output for a pump-laser mode somewhat smaller than the cavity mode of the laser being pumped.⁵ The waist for the astigmatically compensated Ti:sapphire laser cavity mode, for the center of the

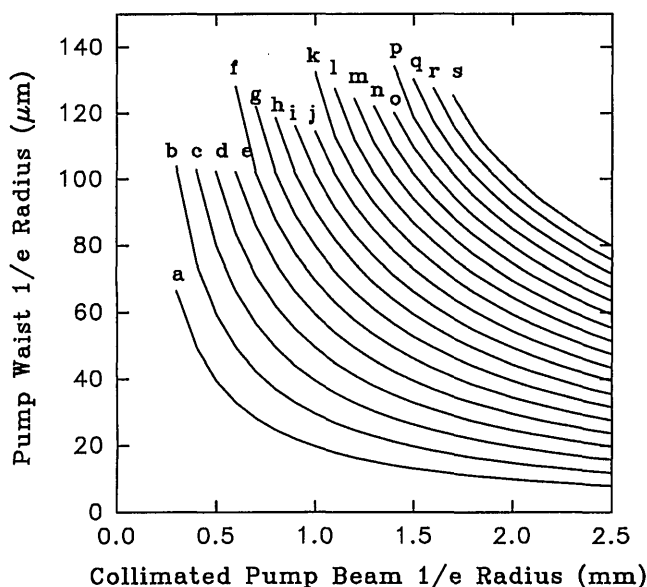


Fig. 9. Pump beam 1/e radius at the cavity waist position as a function of collimated pump beam 1/e radius ω_p for different pump mirror radii of curvature. Values of R_p (in centimeters) are a, 5; b, 10; c, 15; d, 20; e, 25; f, 30; g, 35; h, 40; i, 45; j, 50; k, 55; l, 60; m, 65; n, 70; o, 75; p, 80; q, 85; r, 90; s, 95.

stability range ($2d_2 + L = 12.4$ cm), is approximately $40 \mu\text{m}$ in the sagittal plane and $65 \mu\text{m}$ in the tangential plane. To achieve a focused pump-beam spot size smaller than this, at the same position in the cavity for $\omega_p = 0.8$ mm (the $1/e$ beam radius of a Spectra-Physics 2030 argon-ion laser), a pump-focusing mirror with a radius of curvature of 15 cm or less would be required. This radius of curvature is shorter than could be used easily in the focusing scheme shown in Fig. 7(a). Selection of $R_p = 25$ cm, a more convenient value from a practical point of view, would require that the pump beam be expanded to $\omega_p = 1.5$ mm to achieve good mode matching of the pump and the cavity modes. The calculated values of the pump-beam mode volume in the gain medium as a function of R_p and ω_p are shown in Fig. 10. The mode volume for the cavity mode ($2d_2 + L = 12.4$

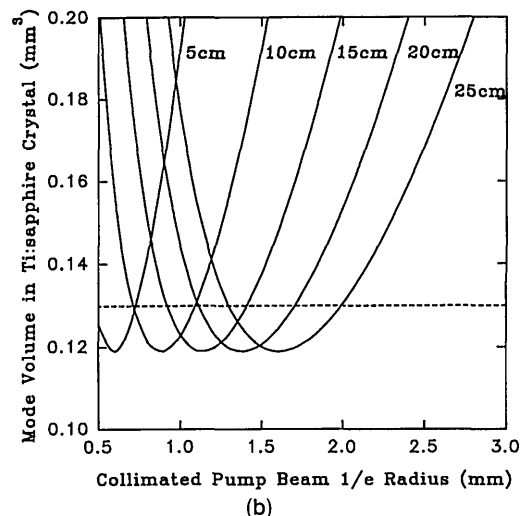
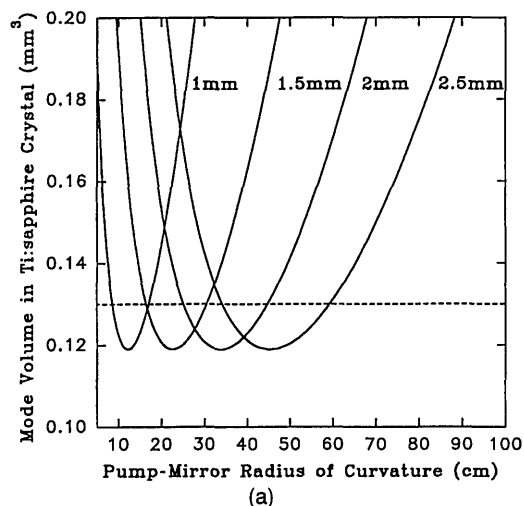


Fig. 10. Pump-mode volume in the Ti:sapphire crystal as a function of (a) the radius of curvature of the pump mirror for different values of collimated pump-beam radius and (b) the collimated pump-beam radius for different values of the pump-mirror radius of curvature. The Ti:sapphire cavity mode volume (with the sagittal plane $1/e$ radius) is represented by the dotted line.

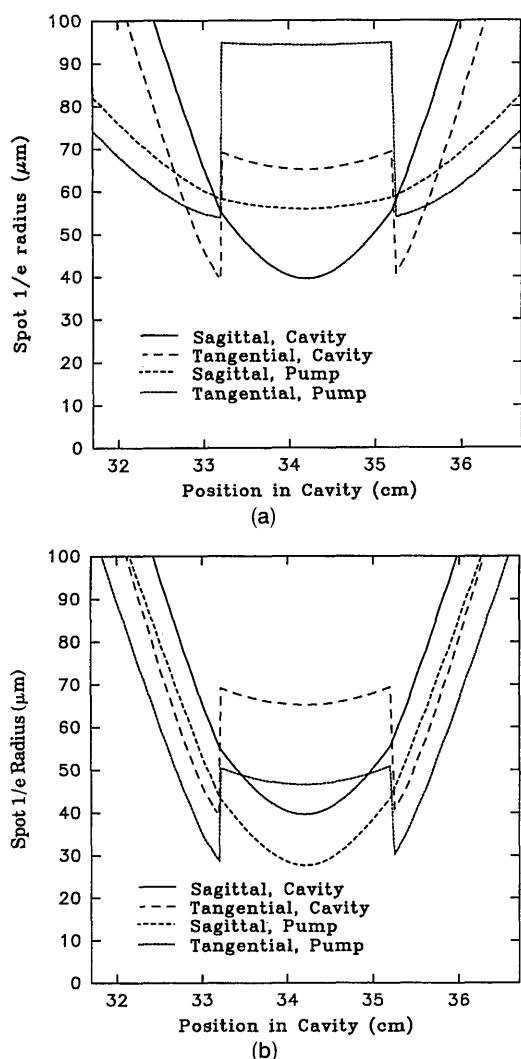


Fig. 11. Argon-ion pump beam and the Ti:sapphire beam $1/e$ radii for the region about the center of the Ti:sapphire crystal [which occurs 34.2 cm from either M_1 or M_4 in Fig. 1(a)] for both the sagittal and the tangential planes of the cavity. $d_1 = d_3 = 28$ cm, $d_2 = 5.2$ cm, $L = 2$ cm, $\theta = 14^\circ$ [Fig. 1(a)]. In (a) $R_p = 25$ cm, $\omega_p = 0.8$ mm, $\phi = 4.8^\circ$. In (b) $R_p = 25$ cm, $\omega_p = 1.5$ mm, $\phi = 4.8^\circ$.

cm), with the sagittal mode size as the beam radius, is also shown in Fig. 10. It can be seen from Fig. 10 that there is no differential optimization of the pump-mode volume that can be achieved by use of pump-focusing mirrors with different radii of curvature. Exactly the same range of pump-mode volume (also, with similar waist size and mode size throughout the length of the gain medium) can be achieved with any radius of curvature of the pump-focusing mirror. All that is required is that a larger-diameter collimated pump beam be used with longer-focal-length focusing mirrors. Thus it is concluded that a useful strategy for optimizing the mode matching between the pump mode and the cavity mode, once a particular pump-focusing mirror has been selected, involves the use of a zoom telescope to achieve a continuous variation in the collimated pump-beam diameter and

a consequent variation in the pump-mode volume. We are undertaking the design of such a zoom beam expander to allow us to test this conclusion experimentally.

Finally, a comparison of the pump-beam mode with the Ti:sapphire cavity mode in the Ti:sapphire crystal is made, as shown in Fig. 11, for both the sagittal and the tangential planes of the cavity, for which the angle ϕ [Fig. 7(a)] is chosen to achieve astigmatism compensation of the two foci of the pump beam. Figure 11(a) shows that the pump-beam focus achieved with $R_p = 25$ cm, $\omega_p = 0.8$ mm, and $\phi = 4.8^\circ$ gives a focused pump spot much larger than the cavity mode. Increasing the collimated pump-beam radius to 1.5 mm [Fig. 11(b)] gives a focused pump spot that is 25% smaller than the cavity spot. This is expected to be much closer to optimum mode matching. In practice we could achieve the optimum mode match by continuously varying ω_p about this value.

The type of data obtained for the argon-ion pumped Ti:sapphire laser presented here could be easily generated for any laser cavity or pump laser with general computer programs for modeling of such cavities.

Conclusion

Using a Ti:sapphire laser that was longitudinally pumped by an argon-ion laser as an example, we have calculated the cavity mode throughout the Ti:sapphire cavity and the focused pump-beam parameters in the Ti:sapphire crystal for a single geometry with variable mirror spacings and focusing optics. It is shown that a collimated beam can only be obtained in the Ti:sapphire cavity at the minimum stable separation of the two focusing mirrors. In practice the laser will lase with poor efficiency compared with its optimum efficiency, if at all, for this condition. In general, a collimated beam is not obtained in the cavity or as the output beam. From the analysis of the pump-beam focusing it is concluded that a useful strategy for optimizing the mode match between the pump and the cavity mode is to use a zoom beam expander to produce a collimated pump beam with variable diameter.

Appendix A: Forms of the Propagation Matrices

The matrix that describes propagation *in vacuo* through a distance d is given by

$$D = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}. \quad (\text{A1})$$

Matrices that describe propagation through a thin lens, which model an off-axis mirror with radius of curvature R and angle of incidence θ in the sagittal and the tangential planes at the cavity, respectively, are given by

$$F_s = \begin{bmatrix} 1 & 0 \\ -\frac{2 \cos \theta}{R} & 1 \end{bmatrix}, \quad F_t = \begin{bmatrix} 1 & 0 \\ -\frac{2}{R \cos \theta} & 1 \end{bmatrix}. \quad (\text{A2})$$

Matrices that describe propagation through a Brewster-cut rhomb of length L with refractive index n in the sagittal and the tangential planes of the cavity, respectively, are

$$B_{Ls} = \begin{bmatrix} 1 & \frac{L}{n} \\ 0 & 1 \end{bmatrix}, \quad B_{Lt} = \begin{bmatrix} 1 & \frac{L}{n^3} \\ 0 & 1 \end{bmatrix}. \quad (\text{A3})$$

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