

Multibeam-waist modes in an end-pumped Nd:YVO₄ laser

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We studied both numerically and experimentally the transverse modes near degenerate resonator configurations such as $g_1g_2 = 1/4, 1/2, 3/4$ in a simple plano-concave Nd:YVO₄ laser with a small pump size. We found that there are stationary modes that show an additional beam waist besides the well-known waist on the flat mirror end near $g_1g_2 = 1/4, 3/4$. When the specific modes of the three degeneracies propagate through a lens, they are capable of exhibiting multiple beam waists. The multibeam-waist mode possesses a small waist size without a ring structure in the near field on the flat mirror end; nevertheless, it has a far-field pattern with many concentric rings. The numerical results show good agreement with the experimental observations. Furthermore, by simultaneously considering the wavelike and the raylike character of the multibeam-waist mode, we found that it can be represented as a superposition of N consecutive round-trip electric fields of period- N solution in the degenerate empty cavity, where $N = 2$ for $g_1g_2 = 1/2$ and $N = 3$ for $g_1g_2 = 1/4, 3/4$. © 2003 Optical Society of America
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1. INTRODUCTION

Lasers with various geometrically stable resonators usually support definite transverse modes, such as Hermite-Gaussian and Laguerre-Gaussian modes. However, a specific resonator configuration with a round-trip transfer matrix $M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$ that satisfies a value of M^n equal to an identity matrix, where n is an integer, does not prefer a specific mode but is capable of supporting arbitrary transverse modes. For these resonator configurations the gain effect may dominate the transverse mode distribution. As a matter of fact, it has been demonstrated that the transverse mode can self-adjust its pattern to fit the pumped volume in the self-imaging confocal cavity.¹ Besides, because of gain guiding, some unusual transverse modes were observed in an end-pumped Nd:YVO₄ laser.^{2,3} Recently, using a sufficiently tightly focused pump, we observed unusual modes^{4,5} in an end-pumped Nd:YVO₄ laser with a plano-concave cavity near degeneracy of $g_1g_2 = 1/4$ (i.e., $1/3$ transverse frequency degeneracy) as well as $g_1g_2 = 1/2$ ($1/4$ -degeneracy). This mode for $g_1g_2 = 1/4$ has a near-field pattern with a small waist on the flat mirror end; nevertheless, it has a far-field pattern with many concentric rings.⁵ In particular, this mode exhibited three beam waists when it was propagated through a transform lens.⁴ In this paper we numerically simulate the features of the multibeam-waist mode by using a mode-calculation procedure, and the results not only show good agreement with our previous experiments but also lead to more experimental observations. In addition, by simultaneously considering the wavelike and the raylike properties of the mode, we found that the multibeam-waist mode at $1/3$ degeneracy can be repre-

sented as superposition of three consecutive round-trip fields that form a set of period-3 solutions in an empty cavity. From such a period-3 superposition representation we explain why the modes exhibit such behavior. Similarly, there are period-2 superposition representations for $g_1g_2 = 1/2$ and period-3 superpositions for $g_1g_2 = 3/4$ ($1/6$ degeneracy). In our mode-calculation procedure we used the Collins integral⁶ together with rate equations.

In Section 2 our numerical model is formulated, and the experimental setup is described in Section 3. In Section 4 the numerical data from mode calculation are compared with the experiments; in addition, a method for bringing about multibeam-waist modes is described, followed by a detailed discussion. In Section 5 our conclusions are stated.

2. MODELING

Consider the laser shown in Fig. 1. It consists of a laser crystal with one of its end faces high-reflection coated as a flat mirror and of a curved mirror with radius of curvature R_c , which are separated by distance L . Let the reference plane be the place where the light beam just leaves the laser crystal toward the curved mirror. In cylindrical symmetry, propagation of the light field toward the curved mirror and back to the flat mirror according to the Collins integral⁶ is

$$E_{m+1}^-(r) = \frac{2\pi j}{B\lambda} \int \exp(-jk2L) E_m^+(r') \exp[-(j\pi/B\lambda) \times (Ar'^2 + Dr^2)] J_0(2\pi r r' / B\lambda) r' dr', \quad (1)$$

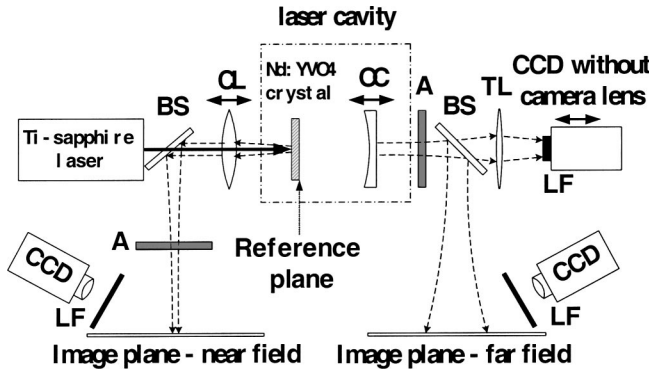


Fig. 1. Schematic diagram of the experimental setup: BS, beam splitter; A, attenuator; other abbreviations defined in text.

with round-trip transmission matrix $M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$. Here $E_m^+(r')$ and $E_{m+1}^-(r)$ are the electric fields of the m th and the $(m+1)$ st round trips, respectively, at the planes immediately after and before the gain medium (denoted by the superscripts $+$ and $-$); r' and r are the corresponding radial coordinates, λ is the wavelength of the laser, and J_0 is a Bessel function of zero order. In a thin-slab approximation, we can relate the electric fields E_{m+1}^+ to E_{m+1}^- (after and before the gain medium) in the same round trip as

$$E_{m+1}^+(r) = \rho E_{m+1}^-(r) \exp(\sigma \Delta N d) \quad (r/a), \quad (2)$$

where $1 - \rho^2$ is the round-trip energy loss, σ is the stimulated-emission cross section, ΔN is the population inversion per unit volume, d is the length of the active medium, and $\Pi(r/a)$ is an aperture function that equals 1 for r less than aperture radius a and equals 0 otherwise. Furthermore, assuming that the evolution of the population inversion follows the rate equation of a four-level system, we can write the rate equation as

$$\Delta N_{m+1} = \Delta N_m + R_{pm} \Delta t - \gamma \Delta N_m \Delta t - \frac{|E_m|^2}{E_s^2} \Delta N_m \Delta t, \quad (3)$$

where R_{pm} is the pump rate (per unit time per unit volume), Δt is the round-trip time, E_s is the saturation parameter of the active medium, and γ is the spontaneous decay rate. It was found that a standing-wave resonator can be approximated by a ring resonator if a thin gain medium is placed close to one of the end mirrors.⁷ The method is similar to the Fox-Li approach⁸ and has been used to analyze the decay rates of standing-wave laser cavities.⁹ For a continuous Gaussian pump profile $R_{pm} = R_{p0} \exp(-2r^2/w_p^2)$ with constant pump beam radius w_p throughout the active medium (thin slab), the total pump rate over the entire active medium is

$$\int R_{pm} dV = P_p / h \nu_p, \quad (4)$$

where P_p is the effective pump power and $h \nu_p$ is the photon energy of the pumping laser. We maintain the values of R_c and ρ fixed through this paper; therefore we have three control parameters, L , w_p and P_p , that play important roles in our laser systems.

Because we are concerned chiefly with transverse mode distribution, we do not consider the dispersion of the ac-

tive medium or frequency detuning between the atomic transition and the cavity mode; thus the gain is assumed to be real. Thermal effects are also neglected. Furthermore, in an ordinary axially pumped solid-state laser the round-trip propagation time is many orders of magnitude shorter than the spontaneous decay time, especially in a short cavity; as a result, it would take a large number of iterations to arrive at the convergent steady state. To reduce computation time, we used the scaling method⁹ to magnify the value γ of the Nd:YVO₄ laser by 100 times to obtain the continuous-wave solution because the transverse mode distribution is independent of γ as long as $\Delta t \ll 1/\gamma$. Given initial ΔN and E , after the power output undergoes a procedure similar to relaxation oscillation the field distribution E converges to a cw steady-state solution. Such a numerically convergent solution is equivalent to a theoretical one-round-trip self-consistent solution. In our simulation, the aperture was chosen to be 1-mm, which is much larger than the empty cavity beam radius of 108 μm for $g_1 g_2 = 1/4$. To implement the Collins integral by the Romberg method, we divided the 1-mm aperture radius into 2048 segments.

3. EXPERIMENTAL SETUP

The experimental setup is shown in Fig. 1. This laser contains a 1-mm-thick Nd:YVO₄ crystal and an output coupler with $R_c = 8$ cm that has 10% transmission at a lasing wavelength of 1.064 μm . The face of the crystal that faced the pump beam had a dichroic coating with greater than 99.8% reflection at 1.064 μm and greater than 99.5% transmission at the pump wavelength of 808 nm; the other surface comprised an antireflection layer at 1.064 μm to avoid the effects of intracavity etalons. The output coupler (OC) was mounted upon a translation stage so we could tune the cavity length about the degenerate configurations. The pump beam source was a Ti:sapphire laser operated in cw output with a nearly TEM₀₀ mode. A collimating lens (CL) was added in front of the crystal such that we could tune the pump size, and the lens was simultaneously used to transfer the near-field mode pattern on the crystal to the screen. The laser output was split into two beams, one of which was used to project the far-field pattern onto the screen or was sent to a rf spectrum analyzer via a photodetector; the other was propagated through a transform lens and then was detected with a charge-coupled device (CCD). To directly image the mode pattern behind the transform lens and reduce the noise, we replaced the camera lens of the CCD by a laser line filter (LF). The near-Gaussian pump size is determined by the standard knife methods. The degeneration point is determined by the position where the lowest lasing threshold occurs.^{4,5}

4. RESULTS

A. Comparison of Numerical and Experimental Results
We focused the simulations primarily on the configuration near 1/3-degeneracy to compare it with the experimental data. The parameters that we used were $\rho = 0.95$, $d = 1$ mm, $R_c = 8$ cm, $\lambda = 1.064$ μm , $1/\gamma = 50$ μs , $\sigma = 25 \times 10^{-19}$ cm^2 , $N_0 = 1.7 \times 10^{20}$ cm^{-3} , and

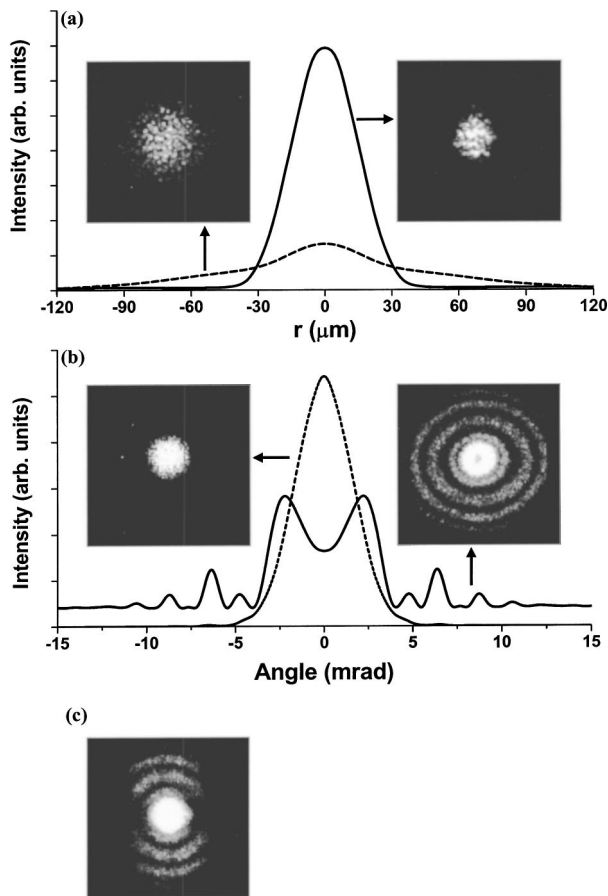


Fig. 2. (a) Numerical intensity profiles on the flat mirror end for $L = 6$ cm (solid curve) and $L = 6.15$ cm (dashed curve) together with the corresponding experimental photographs of beam-waist patterns in the near field. The graininess in the photographs is due to reflection from the screen. (b) Far-field intensity profiles that corresponding to the profiles in (a). Solid curve, degeneracy. (c) Experimental far-field mode pattern as a knife set in the cavity.

$E_s^2 = 1.12 \times 10^{10} \text{ J/F m}^{-2}$. We chose $w_p = 30 \mu\text{m}$ and $P_p = 50 \text{ mW}$; in Fig. 2(a) we show the numerical intensity distributions of the output fields on the flat mirror end, the near field for $L = 6$ cm (at degeneracy) and $L = 6.15$ cm (far from degeneracy), and the corresponding experimental mode patterns. Their far-field intensity profiles are shown in Fig. 2(b). To illustrate clearly the far-field ring structure for the case of degeneracy, we elevated its zero-intensity baseline in Fig. 2(b). We can see that the numerical results are in agreement with the experiments. The numerical near-field profiles in Fig. 2(a) show a small spot size of $27 \mu\text{m}$ at degeneracy, whereas a spot size of $87 \mu\text{m}$ far from degeneracy for a $1/e^2$ spot size is defined. The spot size shrinks to approximately the pump size when the cavity is tuned toward degeneracy; this means that the gain-guiding effect dominates the transverse mode pattern near degeneracy. From Figs. 2(a) and 2(b) the transverse mode near degeneracy exhibits several rings with low intensity on the axis in the far field but does not have a ring in the near field with peak intensity on the axis. This reveals that the mode differs significantly from a usual mode and that its pattern depends on the propagation distance.

We now discuss the beam propagation shown in Fig. 3. The numerical data also show that the number of rings in the far field increases with decreasing pump size. However, far from degeneracy the near-field spot size in Fig. 2(a) approaches the cold-cavity beam waist size, and its far-field profile in Fig. 2(b) has a near-Gaussian distribution. This indicates that the transverse mode is a normal one and that the diffraction effect is more important than the gain effect in pattern formation far from degeneracy. Indeed, we suggest using small-size pumping to obtain fundamental-mode operation with a low lasing threshold in end-pumped four-level solid-state lasers.¹⁰ Moreover, if we vertically block the cavity beam with a knife in the near-degenerate cavity, parts of the far-field rings still survive, rather than completely disappear, as shown in Fig. 2(c). It must be noted that, because of a lack of transverse mode beating, the transverse profiles described above are immune from detection in different parts of the profile; the use of an InGaAs fast detector with a 200-ps response time and a 60-dB dynamic range shows that these profiles are time independent.

Propagating the field solution at degeneracy in Fig. 2(a) toward the output coupler ($z = 6$ cm) in the $+z$ direction, we showed a beam-profile variation with propagation distance z [Fig. 3(a)]. Note that the numerical reference plane is on the flat mirror ($z = 0$ cm), where the well-

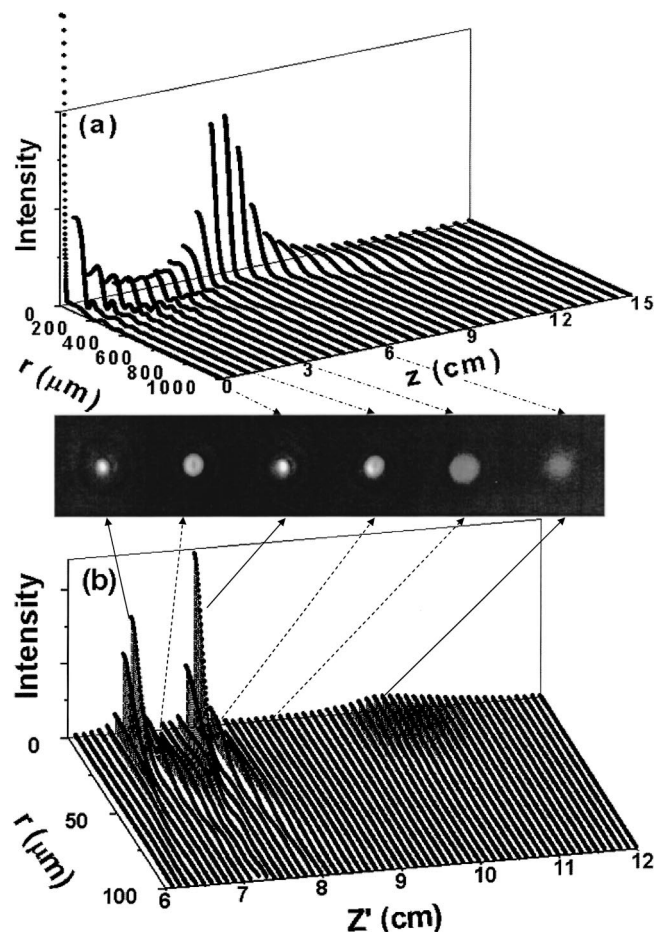


Fig. 3. Propagating behavior for $L = 6$ cm in Fig. 2(a): (a) through intracavity space and (b) through a transform lens. Note the correspondence between photographs in (a) and (b).

known beam waist is located. We see a peculiar feature: the beam does not diverge but self-converges to form a waist close to the output coupler. In addition, the profile variation along z is unlike that of a beam whose profile maintains the same pattern, except that it increases its spot size with propagation distance. We can add a transform lens (Fig. 1, TL) after the output coupler to monitor the beam-profile variation that reveals the z dependence of the stationary mode and concurrently uncovers the main character of the three beam waists as follows.

B. Three Beam Waists and Beam Geometry

If we place a transform lens with a focal length of 5.2 cm a distance of 10.5 cm from the output coupler as in the experiment,⁴ this is equivalent to propagating the field solution a distance of 16.5 cm and then through the transform lens. Here we neglect the refraction from the finite thickness of the output coupler. The variation of beam profile with propagation distance Z' is shown in Fig. 3(b), where $Z' = 0$ is the position of the transform lens. We found there are three beam waists, at positions $Z' = 6.8, 7.6, 10.0$ cm, that match quite well the experimental observation,⁴ as is also shown in Fig. 3(b). Moreover, the intracavity beam profiles in Fig. 3(a) are transmitted to the respective positions from $Z' = 7.6$ to $Z' = 10.3$ cm that follow the Gaussian lens law. For instance, the intracavity profiles at $z = 1.0$ and $z = 3.0$ cm have the same patterns as those of at $Z' = 7.82$ and $Z = 8.46$ cm, respectively, where the observed profiles have low intensity on axis, as shown in Fig. 3(b). The additional waist close to the output coupler in Fig. 3(a) is transformed on the plane $Z' = 10.0$ cm. Thus by ray tracing we show the three beam-waist locations, at Z'_A, Z'_D , and Z'_{D_I} , that are the images of the three objects A, D, and D_I , respectively, as illustrated in Fig. 4(a), where a paraxial ray from point A via B to F and back to A forms a closed ray path after three round trips in the cavity. So it is reasonable to suppose that the peculiar beam may be a superposition of three sources, which are located at $z = -L, 0, +L$ on the optical axis.

We found from a stability analysis of a conservative map involving only Gaussian-beam propagation in a plano-concave empty cavity that the resonator configuration with $g_1 g_2 = 1/4$ has a period-3 orbit of a q parameter.¹¹ We plot in Fig. 4(b) the q parameter's evolution in $(w, 1/R)$ space with reference plane z_0 at the flat mirror ($z = 0$ cm) for the 1/3-degenerate empty cavity, where w is the spot size and $1/R$ is the curvature of the wave front. We can see that the three states 1–3 of consecutive round trips form a set of period-3 solutions and that $1/R$ is the mismatch of the mirror curvature in states 2 and 3 if we assume that the wave front of state 1 matches the flat mirror. We use $E_{gi}(w_i, R_i; z_0)$, $i = 1, 2, 3$ to label a set of period-3 solutions (three successive round-trip electric fields of Gaussian distribution at the flat mirror) in Fig. 4(b) and sometimes abbreviate it as $E_{gi}(z_0)$ if w and R need not be emphasized. Any positive w_1 will follow a periodic evolution, which means there are infinite sets of period-3 solutions for an empty cavity of 1/3 degeneracy.

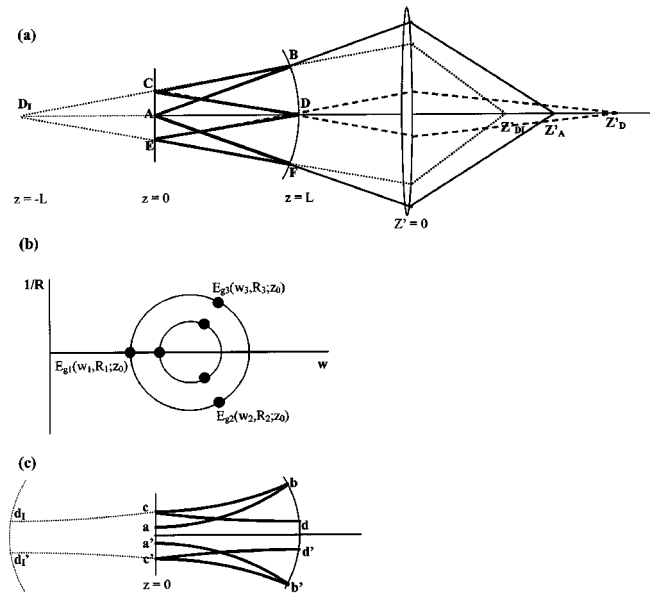


Fig. 4. (a) Ray diagram for illuminating three sources of three beam waists for $g_1 g_2 = 1/4$. Each round-trip ray emanating from the flat mirror toward the output coupler goes through a focus behind the convergent lens. (b) Periodic orbits of the q parameter for the empty cavity. The two concentric circles mean that there are infinite sets of period- N solutions. (c) Gaussian-beam evolution in the empty cavity.

In accordance with Fig. 4(b), in Fig. 4(c) we depict the Gaussian-beam evolution in which the first round-trip wave begins with $E_{g1}(w_1 = aa'/2, R = \infty; z_0)$ and reproduces itself after three round trips in the cold cavity. Note that a positive R represents a divergent wave riding in the propagation direction. The second round trip begins when $E_{g2}(w_2 = cc'/2, R_2; z_0)$ converges at dd' owing to negative R_2 ; which means that the light wave emanates from $E_{g2}(z_0)$ just as from dd' . The third round trip, with $E_{g3}(w_3 = w_2, R_3 = -R_2; z_0)$, is divergent from cc' ; however, it seems to emanate from $d_I d_I'$. Thus the beam sources with waist sizes at aa', dd' , and $d_I d_I'$ correspond to $E_{gi}(z_0)$, $i = 1, 2, 3$, respectively. Moreover, in comparison with that in Figs. 4(a) and 4(c), the Gaussian-beam evolution from aa' to bb' is similar to rays AB and AF; then bb' and cc' act as the rays BC and FE, and so on. Point sources A, D, and D_I are therefore equivalent to the three beam sources at aa', dd' , and $d_I d_I'$, respectively. So far we have learned that three waists come from three point sources that act as three beam sources that correspond to $E_{gi}(z_0)$, $i = 1, 2, 3$.

C. Period-3 Superposition Representation

We know from both experimental observation and mode calculation that the stationary cw laser output of 1/3 degeneracy is a one-round-trip self-consistent mode. It exhibits the raylike character of three beam waists. The stationary cw laser field on any plane within the cavity is the accumulated electric fields of the successive round trips. So we suspect that the electric field of the multibeam-waist mode of 1/3 degeneracy can be represented as the superposition of three consecutive fields of period-3 solution for the 1/3-degenerate empty cavity.

Because the gain medium lies against the flat mirror, we choose $w_1 = 27 \mu\text{m}$, the waist size of the intensity profile in Fig. 2(a), and propagate $E_{g1}(w_1 = 27 \mu\text{m}, R = \infty; z_0)$ in the empty cavity of $L = 6 \text{ cm}$ by using integral formula (1), which is equivalent to choosing a set of period-3 fields according to the gain effect. Then we obtain $E_{g2}(z_0)$ and $E_{g3}(z_0)$ after one and two round trips, respectively. Because the Guoy phase shift was not included in Eq. (1), we added a phase shift of π when the beams passed through the waists of aa' and dd' in Fig. 4(c). The period-3 superposed intensity profile $|E_{g1}(z_0) + E_{g2}(z_0) - E_{g3}(z_0)|^2$ is shown in Fig. 5 by open circles on a logarithmic vertical scale; also shown, by filled circles, is the one-round-trip self-consistent mode normalized with respect to the superposed peak intensity. We see that the two profiles match well, except for small differences near the edge of the 1-mm aperture and near the intensity minimum at $r = 200 \mu\text{m}$. These deviations are discussed in Subsection 4.D below. From such a representation the observed behaviors can easily be understood. The small waist in the near-field pattern is due mainly to $|E_{g1}(z_0)|^2$ because it is much larger than $|E_{g2}(z_0)|^2$ and $|E_{g3}(z_0)|^2$. The self-convergent phenomenon in Fig. 3(a) results from the negative curvature of $E_{g2}(z_0)$ that converges there. The additional transformed waists at $Z' = 6.8 \text{ cm}$ and $Z' = 10.0 \text{ cm}$ in Fig. 3(b) are due mainly to the individual contributions of $E_{g3}(z_0)$ and $E_{g2}(z_0)$, respectively. Because the propagation behaviors of $E_{g1}(z_0) + E_{g2}(z_0) - E_{g3}(z_0)$ are similar to those shown in Figs. 3(a) and 3(b), the far-field ring pattern or any profiles on the transverse planes of z are the result of interference of the three Gaussian fields at z that emanates from $E_{gi}(w_i, R_i; z_0)$, $i = 1, 2, 3$, respectively. Therefore we conclude that the multibeam-waist mode distribution at z can be written as a period-3 superposition, namely, $\sum_{i=1}^3 E_{gi}[w_i(z), R_i(z); z]$.

For $g_1g_2 = 1/2$ the results of period-2 superposed field $E_{g1}(z_0) + E_{g2}(z_0)$ match those of mode calculation and of experiments that show two waists behind a proper transform lens. Similarly, $E_{g1}(z_0) + E_{g2}(z_0) + E_{g3}(z_0)$ also simulates $g_1g_2 = 3/4$ well. In particular, one can detect an additional waist outside the cavity without adding a transform lens, as shown in Fig. 6.

D. Discussion and Dependence of Pump Power, Pump Size, and Cavity Length

The multibeam-waist modes discussed so far exhibit the duality of wave and ray optics, similarly geometric modes,² in which the gain guide induces a preferred W-shaped closed ray path such that the light spots appear at the ray trace in an off-axis end-pumped Nd:YVO₄ laser near $g_1g_2 = 1/2$. Ramsay and Degnan¹² have shown that the vertices of closed ray paths can be identified with the points of maximum field intensity in a CO₂ laser, so a closed ray path might be regarded as a mode. However, our experimental observations indicate points B, C, E, and F of Fig. 4(a) are not local intensity maxima. Therefore, the relationship of Fig. 4(a) to 4(c) is different from those shown in Refs. 2 and 12. Furthermore, the ratio of the pump size to the fundamental mode waist size is not an important factor for obtaining the geometric modes described in Ref. 2, for which a size ratio larger than 1 was

used. Also, because of the large pump region near degeneracy, the CO₂ laser described in Ref. 12 was operated in multitransverse modes, so there are mode patterns that correspond well to closed ray paths. However, small pump size is necessary and is an important parameter for achieving multibeam-waist modes.

The deviation between the two profiles in Fig. 5 near the aperture edge comes from the high-order Laguerre–Gaussian mode because we can add a small Laguerre–Gaussian component of high order to E_{g1} to eliminate the small difference. Note that now E_{g1} is no longer a lowest-order Gaussian; however, the mode still preserves the character of periodic superposition to exhibit three beam waists. That the high-order component is due to high pump power is known from the fact that E_{g1} consists only of the lowest-order Gaussian as P_p decreases to 12 mW. Indeed, we observed three beam waists for pump powers from a little above the lasing threshold (7 mW) to 400 mW in the experiments. Even for high pump power, the higher-order Laguerre–Gaussian component plays only a minor role and never changes the basic character of period-3 superposition. It must be mentioned that the multibeam-waist mode is independent of longitudinal beating because it was detected for a large pump power range. Furthermore, increasing the pump power will in-

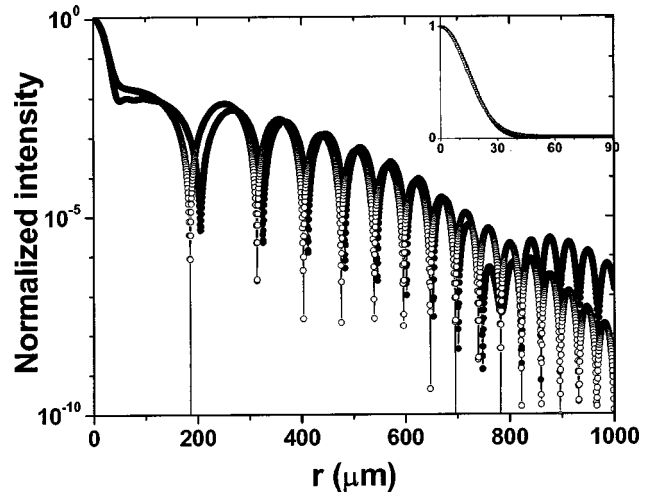


Fig. 5. Normalized intensity profiles of the period-3 superposition mode (open circles) and the self-consistent mode from the mode calculation (filled circles). Inset, their central parts with a linear scale on the vertical axis.

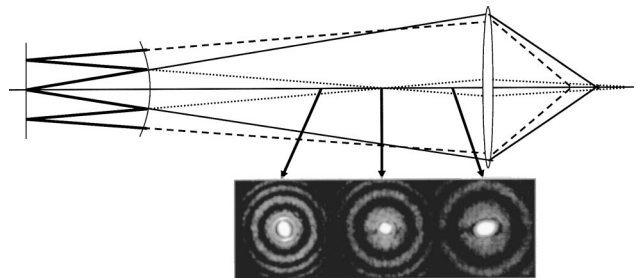


Fig. 6. Ray picture similar to Fig. 4(a) for $g_1g_2 = 3/4$ together with three photographs taken before, at, and after the additional beam waist before the beam has gone through the transform lens. The elliptic central parts are due to astigmatism of the attenuator.

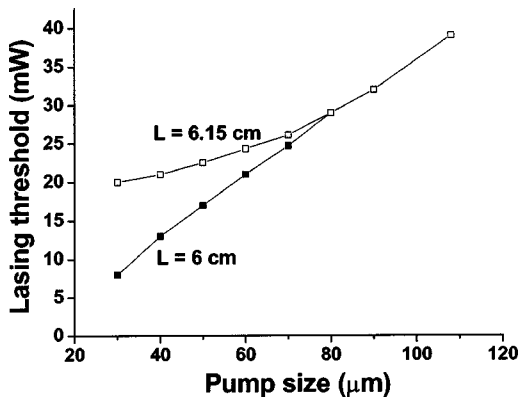


Fig. 7. Lasing threshold versus w_p for $L = 6, 6.15$ cm.

crease w_1 of $E_{g1}(z_0)$ as a result of gain saturation, from ~ 20 to ~ 32 μm , corresponding to P_p from 12 to 200 mW for $w_p = 30$ μm . So the chosen w_1 discussed in Subsection 4.C depends on the pump power. Another difference, near $r = 200$ μm comes from a gain guide that changes configuration parameter g_1g_2 . We can eliminate this deviation by superposing E_{g1} , E_{g2} , and E_{g3} in the cavity of $L = 6 + \delta$ cm by replacing $L = 6$ cm, where δ is a small quantity. This confirms the influence of gain guiding on pattern formation.

Finally, we discuss the pump size and the cavity-length dependence on the multibeam-waist modes. Figure 7 shows that the numerical lasing threshold varies with the pump size for $L = 6$ cm and $L = 6.15$ cm. The degenerate case has a lower lasing threshold, and in fact we use it to determine the degeneration point in the experiments. For instance, the lasing threshold at degeneracy as $w_p = 30$ μm is ~ 8 mW, which matches the experiment well. It was also found that the threshold difference increases with decreasing w_p , so it is easier to excite the multibeam-waist mode with smaller w_p near degeneracy. As $w_p > 80$ μm the gain guide is too weak to excite the multibeam-waist mode. When the cavity was tuned away from degeneracy with w_p fixed, the waist position at Z'_D [see Fig. 4(a)] shifted, and this waist gradually became indistinct. Finally, the three-beam-waist configuration is broken up far from degeneracy. The tuning range for achieving a multibeam-waist mode depends on w_p . Our numerical data show a full tuning range of ~ 700 μm for $w_p = 30$ μm . However, it is not easy to identify the exact tuning range as well as the pump size limit in experiments by observation of additional waists. The far-field ring structure together with the average power output therefore may become a good identifier.

5. CONCLUSIONS

We have demonstrated that a laser mode is capable of exhibiting N beam waists when it propagates through a transform lens, where $N = 2$ corresponds to $g_1g_2 = 1/2$ and $N = 3$ to $g_1g_2 = 1/4, 3/4$ with plano-concave cavity. For $g_1g_2 = 1/4$ the multibeam-waist mode has an additional waist close to the curved mirror end; similarly, for $g_1g_2 = 3/4$ one can detect an additional waist outside the cavity without using a transform lens. It is easier to ex-

cite the multibeam-waist mode with a smaller pump size because of the stronger gain-guiding effect near specific degenerations within a certain range of cavity tuning. The multibeam-waist mode has a small beam-waist size that is close to the pump size in the near field at the flat mirror end; however, its far-field pattern has a concentric-ring structure. The gain-guiding effect dominates the formation of a transverse mode pattern near degeneracy; nevertheless the diffraction effect is more important away from degeneracy.

Moreover, by simultaneously considering the wavelike and the raylike characteristics of the multibeam-waist mode we expand the mode as a superposition of N consecutive electric fields of period- N solution for the empty cavity but not as a superposition of the orthogonal bases. Because these N fields act as independent in-phase sources located in different positions with different waist sizes such that the mode can converge to form N beam waists at different positions after the transform lens, the beam profile variation with propagation distance, in particular for those profiles with low intensity on the axis, is well simulated. The additional waist in the absence of a transform lens is due to the convergence of the field with negative curvature of the period- N solution. The ring pattern in the far field or in any position is the result of interference of the N consecutive round-trip electric fields. Because axially pumped solid-state lasers are widely used, the multibeam-waist modes may be important because there are many degenerate positions within the geometrically stable region.

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