

The Eye

5.1 Introduction

A knowledge of the characteristics of the human eye is important to the practice of optical engineering because the majority of optical systems utilize the eye as the final element of the system in one way or another. Thus, it is vital that the designer of an optical system understand what the eye can and cannot accomplish. For example, if a visual optical system is required to recognize a certain size target or to measure to a certain degree of accuracy, the magnification of the image presented to the eye must be sufficient to allow the eye to detect the necessary details. On the other hand, it would be wasteful to design a system with a perfection of image rendition which the eye could not utilize.

The human eye is a living optical system and its characteristics vary widely from individual to individual. For a given individual, the characteristics may vary from day to day, indeed from hour to hour. Therefore, the data presented in this chapter must be considered as central values in a range of values; in fact, some data are useful only as an indication of the order of magnitude of a certain characteristic. The conditions under which the eye is used play a large role in determining the behavior of the eye and must *always* be taken into account.

In physiological optics, the unit of measure for the power of a lens or optical system is the *diopter*; the abbreviation for which is *D*. The diopter power of a lens is simply the reciprocal of its effective focal length, when the focal length is expressed in meters. For example, a lens with a 1-m focal length has a power of 1 diopter; a $\frac{1}{2}$ -m focal length, 2 diopters; and a lens of 1-in focal length has a power of 40

diopeters (or more exactly, 39.37 D). For a single surface, the dioptric power is given by $(n' - n)/R$, with R the radius in meters. A 1-diopter prism produces a deviation of 1 cm in a 1-m distance, i.e., a deviation of 0.01 radians, or about 0.57 degrees.

5.2 The Structure of the Eye

The eyeball is a tough plasticlike shell filled largely with a jellylike substance under sufficient pressure to maintain its shape. It rides in a bony socket of the skull on pads of flesh and fat. It is held in place and rotated by six muscles.

Figure 5.1 is a horizontal section of the right eye; the nose is to the left of the figure. The outer shell (sclera) is white and opaque except for the cornea, which is clear. The *cornea* supplies most (about two-thirds) of the refractive power of the eye. Behind the cornea is the *aqueous humor*, which (as its name implies) is a watery fluid. The *iris*, which gives the eye its color, is capable of expanding or contracting to control the amount of light admitted to the eye. The pupil formed by the iris can range in diameter from 8 mm in very dim light to less than 2 mm under very bright conditions. The *lens* of the eye is a flexible capsule suspended by a multitude of fibers, or ligaments, around its periphery. The eye is focused by changing the shape of the lens. When the sphincter muscles to which the suspensory ligaments are connected are relaxed, the lens has its flattest shape and the normal eye is focused at infinity. When these muscles contract, the lens bulges, so that its radii are shorter and the eye is focused for nearby objects. This process is called *accommodation*.

Behind the lens is the *vitreous humor*, a material with the consistency of thin jelly. All of the optical elements of the eye are largely

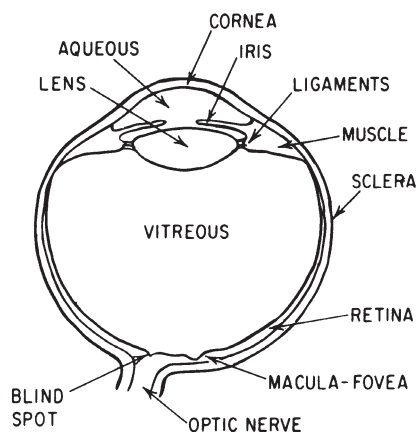


Figure 5.1 Schematic horizontal section of right eyeball (from above).

water; in fact, a reasonable simulation of the optics of the eye can be made by considering the eye as a single refracting surface of water ($n_D = 1.333$, $V = 55$).

The following table lists typical values for the radii, thicknesses, and indices of the optical surfaces of the eye. These, of course, vary from individual to individual.

R_1 (air to cornea) + 7.8 mm	t_1 (cornea) 0.6	n_1 1.376
R_2 (cornea to aqueous) + 6.4 mm	t_2 (aqueous) 3.0	n_2 1.336
R_3 (aqueous to lens) + 10.1 mm	t_3 (lens) 4.0	n_3 1.386–1.406
R_4 (lens to vitreous) –6.1 mm	t_4 (vitreous) 16.9	n_4 1.337

The principal points are located 1.5 and 1.8 mm behind the cornea, and the nodal points are 7.1 and 7.4 mm behind the cornea. The first focal point is 15.6 mm outside the eye; the second is, of course, at the retina. The distance from the second nodal point to the retina is 17.1 mm; thus the retinal size of an image can be found by multiplying the angular subtense of the object in radians (from the first nodal point) by this distance. When the eye accommodates (focuses), the lens becomes nearly equiconvex with radii of about 5.3 mm, and the nodal points move a few millimeters toward the retina. The center of rotation of the eyeball is 13 to 16 mm behind the cornea.

An often overlooked fact is that the commonly accepted eye data tabulated above do not give an adequate picture of the quality of the visual system. First, the surfaces of the eye are not spherical. Some surfaces, especially those of the lens, depart significantly from true spheres. In general, the surface curvature tends to be weaker toward the margin of the surface. Second, the index of the lens is not uniform, but is higher in the central part of the lens. This sort of index gradient produces convergent refracting power in and of itself; it also reduces the surface refracting power at the margin of the lens. Note that both the gradient index and the surface asphericities introduce overcorrected spherical aberration, which offsets the undercorrected spherical of the outer surface of the cornea.

The *retina* contains blood vessels, nerve fibers, the light-sensitive rod and cone cells, and a pigment layer, in that order in the direction that the light travels. The optic nerve and the associated blind spot are located where the nerve fibers leave the eyeball and proceed to the brain. Slightly (about 5°) to the temporal (outer) side of the optical axis of the eye is the macula; the center of the macula is the fovea. At the fovea, the structure of the retina thins out and, in the central 0.3-mm diameter, only cones are present. The fovea is the center of sharp vision. Outside this area rods begin to appear; further away only rods are present.

There are about 7 million cones in the retina, about 125 million rods, and only about 1 million nerve fibers. The cones of the fovea are 1 to 1.5 μm in diameter and are about 2 to 2.5 μm apart. The rods are about 2 μm in diameter. In the outer portions of the retina, the sensitive cells are more widely spaced and are multiply connected to nerve fibers (several hundred to a fiber), accounting for the less distinct vision in this area of the retina. In the fovea, however, there is one cone cell per fiber.

The field of vision of an eye approximates an ellipse about 150° high by about 210° wide. The binocular field of vision, seen by both eyes simultaneously, is approximately circular and about 130° in diameter.

5.3 Characteristics of the Eye

Visual acuity

The characteristic of the eye which is probably of greatest interest to the optical engineer is its ability to recognize small, fine details. Visual acuity (VA) is defined and measured in terms of the angular size of the smallest character that can be recognized. The characters most frequently used to test VA are uppercase letters or a heavy ring with a break in the outline. Many uppercase letters can be considered as made up of five elements; e.g., the letter E has three bars and two spaces. Visual acuity is the reciprocal of the angular size (in minutes of arc) of one of the elements of the letter. "Normal" VA is considered to be 1.0, i.e., when the smallest recognizable letter subtends an angular height of 5 minutes from the eye and each element of the letter subtends 1 minute. Acuity is frequently expressed as the ratio between the distance to the target (usually 20 ft) and the distance at which the target element would subtend 1 minute. Thus, a VA of one-half, or 20/40, indicates that the minimum recognizable letter subtends 10 minutes and its elements 2 minutes. In the Landolt broken ring test, the width of the ring and the width of the break correspond to the letter element size, and recognition consists of determining the orientation of the break. Visual acuity may reach 2 (or 3 in unusual individuals) under ideal conditions.

As indicated above, the normal visual acuity is 1 minute, and this is also the value for the angular resolution of the eye which is conventionally assumed in connection with the design of optical instruments. Note that a resolution of one line pair (or one cycle) per minute of arc actually corresponds to a VA of 2, or 20/10. However, this is the value of VA under what might be termed "normal conditions," and it is the value *only* for that part of the field of view which corresponds to

the fovea of the retina. Outside the fovea, the acuity drops rapidly, as indicated in Fig. 5.2, which is a logarithmic plot of visual acuity (relative to that at the fovea, which is arbitrarily set at unity) versus the angular position of the test target in the field of view. Also note that the vertical VA is 5 to 10 percent higher than horizontal and that the horizontal and vertical VA are about 30 percent higher than oblique (45°) VA.

As the brightness of a scene is diminished, the iris opens wider and the rods take over from the cones. At low illuminations, the eye is color blind and the fovea becomes a blind spot, since the cones lack the necessary sensitivity to respond to low levels of illumination. One result of this process is that the visual acuity drops as the illumination drops. This relationship is plotted in Fig. 5.3, which also indicates the normal pupil size. Note that the brightness of the area surrounding the test target affects the acuity. A uniform illumination seems to maximize the acuity. Figure 5.4 shows that, as might be expected, reducing the contrast of the target will also reduce the acuity.

Because the eye has about 0.75 *D* of chromatic aberration (C-light to F-light), VA is affected by the wavelength of light illuminating the target. Normally, VA is given for white light. In monochromatic light, the acuity is very slightly higher for the yellow and yellow-green wavelengths and slightly lower for red wavelengths. In blue (or far red) light, VA may be 10 to 20 percent lower, and in violet light the reduction in VA is 20 to 30 percent. The chromatic of the eye can be corrected or doubled

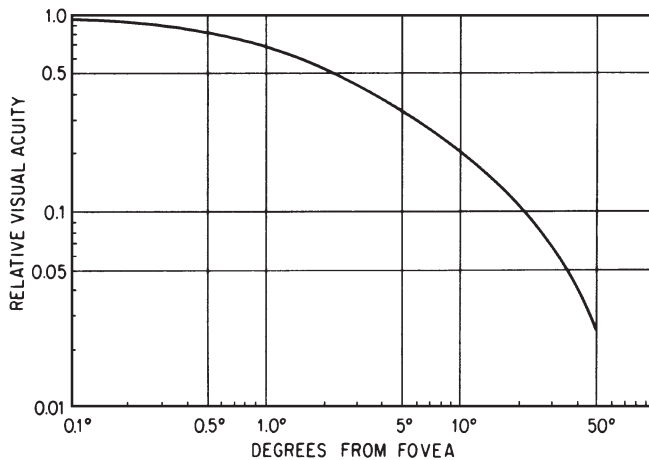


Figure 5.2 The variation of visual acuity (relative to the fovea) with the retinal position of the image. Note that because of the logarithmic scales of the figure, the falloff in visual acuity is far more rapid than the shape of the curve might indicate.

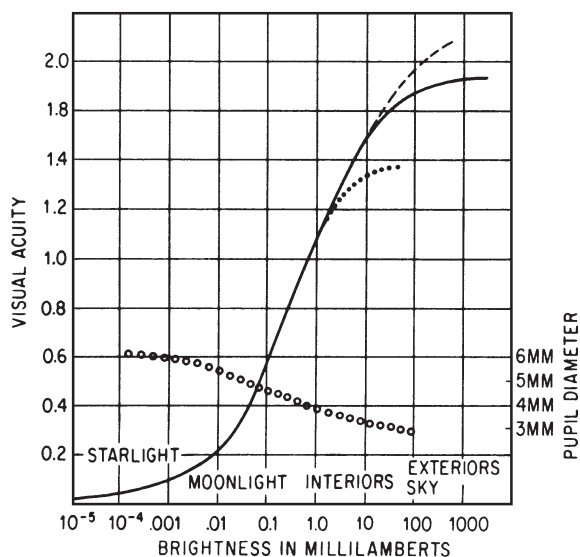


Figure 5.3 Visual acuity as a function of object brightness. Visual acuity in reciprocal minutes. The dashed and dotted lines show the effect of increased and decreased (respectively) surround brightness (1 millilambert is approximately the brightness of a perfect diffuser illuminated by 1 footcandle). The open circle curve indicates the diameter of the pupil; pupil diameters are larger in the young and smaller in the old, especially at lower brightnesses.

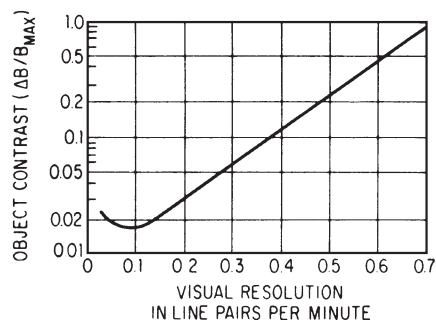


Figure 5.4 The object contrast ($\Delta B/B_{\max}$) necessary for the eye to resolve a pattern of alternating bright and dark bars of equal width. Note that this curve shifts upward in reduced light levels and drops as the light level is increased. For this plot the bright bars had a brightness of $B_{\max} = 23$ foot-lamberts.

(by external lenses) without detection; a quadrupling is noticeable. The effect of the chromatic aberration on the acuity of the eye is less than one might expect because the slightly yellow lens blocks out the ultra-violet, and the macula lutea (which is Latin for yellow spot) filters out the blue and violet light; the spectral response function of the eye is as shown in Fig. 5.8.

Other types of acuity

Vernier acuity is the ability of the eye to align two objects, such as two straight lines, a line and a cross hair, or a line between two parallel lines. In making settings of this type, the eye is extremely capable. In instrument design, it can be safely assumed that the average person can repeat vernier settings to better than 5 seconds of arc and that he or she will be accurate to about 10 seconds of arc. Exceptional individuals may do as well as 1 or 2 seconds. Thus, the vernier acuity is 5 or 10 times the visual acuity. Vernier acuity is best when setting one line between two, next best setting a line on cross hairs or aligning two butting lines, and less effective in superimposing two lines.

The narrowest black line on a bright field that the eye can detect subtends an angle of from $\frac{1}{2}$ to 1 second of arc. In conditions of reversed contrast, i.e., a bright line or bright spot, the size of the line is not as important as its brightness. The governing factor is the amount of energy which reaches and triggers the retinal cell into responding. The minimum level seems to be 50 to 100 quanta incident on the cornea (only a few percent of the energy incident on the cornea actually reaches the cell).

The eye is capable of detecting angular motion to the order of 10 seconds of arc. The slowest motion that the eye will detect is 1 or 2 minutes of arc per second of time. At the other extreme, a point moving faster than 200° per second will blur into a streak.

The eyes judge distance from a number of clues. Accommodation, convergence (the turning in of the eyes to view a near object), haze, perspective, experience, etc., each play a part. Three-dimensional, or stereo, vision results from the separation of the two eyes, which causes each eye to see a slightly different picture of an object. The amount of stereo parallax which can be detected is as small as 2 to 4 seconds. In a clueless surround, a test subject can adjust two rods to be equidistant to within about 1 in when the rods are 20 ft away. The detectable ΔD in millimeters is approximately the square of the distance in meters (D^2).

Sensitivity

The lowest level of brightness which can be seen or detected is determined by the light level to which the eye has become accustomed. When the illumination level is reduced, the pupil of the eye expands, admitting more light, and the retina becomes more sensitive (by switching from cone vision to rod vision and also by an electrochemical mechanism involving rhodopsin, the visual purple pigment). This process is called dark adaptation. Figure 5.5 illustrates the adaptation

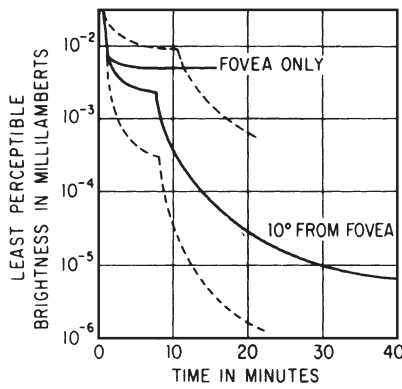


Figure 5.5 The threshold of vision. The minimum brightness perceptible drops sharply with time as the eye adapts itself to darkness. The upper and lower dashed curves show the effect of high and low illumination levels (respectively) before adaptation begins. For areas subtending more than 5° the threshold is almost constant, but rises rapidly as target size is reduced. Curves shown are for a target subtending about 2° .

process as a function of the length of time that the eye is in darkness. The “fovea only” curve indicates that after 5 or 10 minutes, the level of brightness detectable by the portion of the retina used for distinct vision is as low as it will ever get. At lower levels of illumination, only the outer portions of the retina are useful; the fovea becomes a blind spot. Figure 5.5 is for a target which subtends about 2° ; the threshold brightness is lower for larger targets and higher for smaller targets. As indicated by the dashed lines, the conditions of the test have a great bearing on the threshold of vision, and the data of Fig. 5.5 should be regarded as indicating only an order of magnitude for the threshold.

The eye is a poor photometer; it is very inaccurate at judging the absolute level of brightness. However, it is an excellent instrument for comparison purposes, and can be used to match the brightness or color of two adjacent areas with a high degree of precision. Figure 5.6 indicates the brightness difference that the eye can detect as a function of the absolute brightness of the test areas. At ordinary brightness levels, a brightness difference of about 1 or 2 percent is detectable. (Note that in comparison photometry, in which the eye is called upon to match two areas, the precision of setting is increased by making a series of readings. In half the readings, the brightness of the variable area is raised until an apparent match is obtained; in the other half of the readings, the brightness is lowered to obtain the apparent match. The average is then much more accurate than either set.) Contrast sensitivity is best when there is no visible dividing line between the two areas under comparison. When the areas are separated, or if the demarcation between areas is not distinct, contrast sensitivity drops markedly.

Figure 5.7 indicates the capability of the normal eye as a comparison colorimeter. Again, the eye is poor at determining the absolute wavelength of a color but quite good at determining a color match;

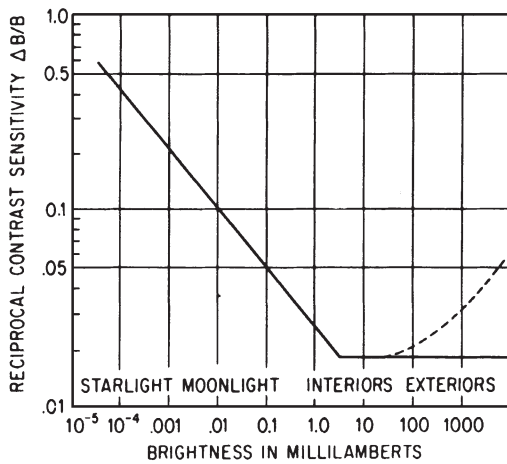


Figure 5.6 The contrast sensitivity of the eye as a function of field brightness. The smallest perceptible difference in brightness between two adjacent fields (ΔB) as a fraction of the field brightness B remains quite constant for brightnesses above 1 millilambert if the field is large. The dashed line indicates the contrast sensitivity for a dark surrounded field. (One millilambert is approximately the brightness of a perfect diffuser illuminated by one footcandle, i.e., one foot-lambert.)

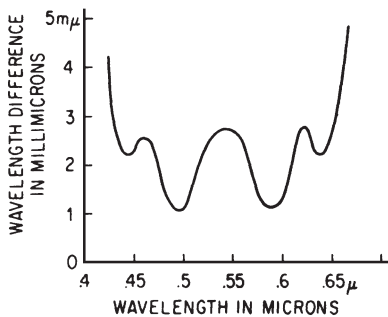


Figure 5.7 Sensitivity of the eye to color differences. The amount by which two colors must differ for the difference to be detectable in a side-by-side comparison is plotted as a function of the wavelength. Some data indicates a more uniform sensitivity of about twice that shown here.

wavelength differences of a few millimicrons are detectable under suitable conditions. The comments of the preceding paragraph regarding dividing lines between test areas apply to color sensitivity as well.

The sensitivity of the eye to light is a function of the wavelength of the light. Under normal conditions of illumination, the eye is most sensitive to yellow-green light at a wavelength of 0.55 μm , and its sensitivity drops off on either side of this peak. For most purposes the sensitivity of the eye may be considered to extend from 0.4 to 0.7 μm . Thus, in designing an optical instrument for visual use, the

monochromatic aberrations are corrected for a wavelength of 0.55 or 0.59 μm and chromatic aberration is corrected by bringing the red and blue wavelengths to a common focus. The wavelengths usually chosen are either *e* (0.5461 μm) or *d* (0.5876 μm) for the yellow, *C* (0.6563 μm) for the red, and *F* (0.4861 μm) for the blue.

Figure 5.8 shows the sensitivity of the eye as a function of wavelength for normal levels of illumination and also for the dark-adapted eye. The photopic curve applies for brightness levels of 3 cd/m^2 or more, and the scotopic curve applies for brightness levels of 3×10^{-5} cd/m^2 or less. Between these levels, the term “mesopic” is used. Notice that the peak sensitivity for the dark-adapted eye shifts toward the blue end of the spectrum, to a value near 0.51 μm . This “Purkinje shift” is due to the differing chromatic sensitivities of the rods and cones of the retina, as shown in Fig. 5.8. Figure 5.9 is a tabulation of the values used in plotting Fig. 5.8. Figure 5.10a is a standardized plot of ocular sensitivity which is used in colorimetry determinations. The long-wavelength portion of this curve (Fig. 5.10b) is useful in estimating the visibility of near-infrared searchlights (as used on sniper-scopes, etc.) under conditions where security is desired.

5.4 Defects of the Eye

Nearsightedness (*myopia*) is a defect of focus resulting from too much power in the lens and cornea and/or too long an eyeball. The result is that the image of a distant object falls ahead of the retina and cannot be focused sharply. Since myopia results from an excessive amount of positive power, it can be corrected by placing a negative lens before the eye. The power of the negative lens is chosen so that its image is formed at the most distant point on which the myopic eye can focus. For example, a person with 2 diopters of myopia cannot see clearly beyond $\frac{1}{2}$ m (20 in), and a -2 diopter lens (focal length = $-\frac{1}{2}$ m or

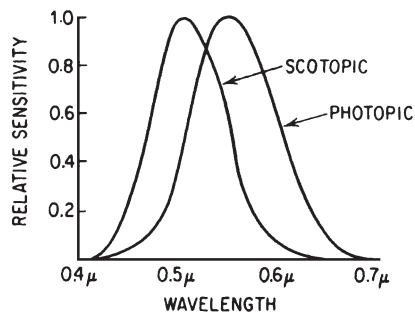


Figure 5.8 The relative sensitivity of the eye to different wavelengths for normal levels of illumination (photopic vision) and under conditions of dark adaptation (scotopic vision).

Wavelength, μm	Photopic	Scotopic	Wavelength, μm	Photopic	Scotopic
0.39	0.0001	0.0022	0.59	0.7570	0.0655
0.40	0.0004	0.0093	0.60	0.6310	0.0332
0.41	0.0012	0.0348	0.61	0.5030	0.0159
0.42	0.0040	0.0966	0.62	0.3810	0.0074
0.43	0.0116	0.1998	0.63	0.2650	0.0033
0.44	0.0230	0.3281	0.64	0.1750	0.0015
0.45	0.0380	0.4550	0.65	0.1070	0.0007
0.46	0.0600	0.5672	0.66	0.0610	0.0003
0.47	0.0910	0.6756	0.67	0.0320	0.0001
0.48	0.1390	0.7930	0.68	0.0170	0.0001
0.49	0.2080	0.9043	0.69	0.0082	0.0000
0.50	0.3230	0.9817	0.70	0.0041	
0.51	0.5030	0.9966	0.71	0.0021	
0.52	0.7100	0.9352	0.72	0.0010	
0.53	0.8620	0.8110	0.73	0.0005	
0.54	0.9540	0.6497	0.74	0.0003	
0.55	0.9950	0.4808	0.75	0.0001	
0.56	0.9950	0.3288	0.76	0.0001	
0.57	0.9520	0.2076	0.77	0.0000	
0.58	0.8700	0.1212			

Figure 5.9 The standard relative luminosity factors (relative sensitivity or response) for photopic and scotopic conditions.

−20 in) is used to correct for this amount of myopia. The onset of myopia frequently coincides with adolescence, when growth is most rapid.

Instrument myopia occurs when an observer (especially an untrained observer) focuses an optical instrument such as a microscope or telescope. There is a tendency to focus the instrument so that the image appears to be about 20 in (2 diopters) away. This may be due to the observer's perception that the image is inside the instrument and therefore should be nearby. Most experienced observers will focus an instrument much nearer to an infinity setting. They do this by moving the microscope toward the object to focus, so that the image is behind the viewer's eye (and thus well out of focus) until it is in focus. Instrument myopia may be related to *night myopia*, where, in the dark and with no stimulus, the eye apparently also focuses at a close distance (60 to 80 in).

Farsightedness (*hyperopia*) is the reverse of myopia and results from too short an eye and/or too little power in the refracting elements of the eye. The image of a distant object is formed (when the eye is relaxed) behind the retina. Hyperopia can be corrected by the use of a positive spectacle lens. Obviously farsighted individuals can, to the extent that their power of accommodation will allow, refocus their eyes

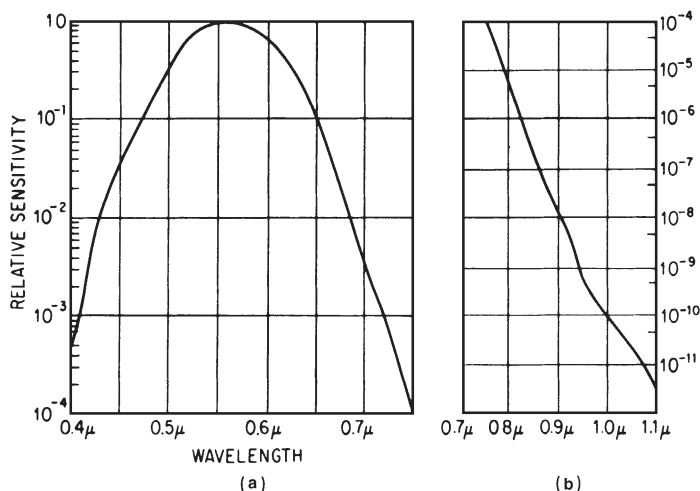


Figure 5.10 (a) Relative sensitivity of a standardized normal eye to light of varying wavelengths. (b) Sensitivity in the near-infrared.

to bring the image onto the retina. If prolonged, this may cause headaches.

Astigmatism is a difference in the power of the eye from meridian to meridian and usually results from an imperfectly formed cornea, which has a stronger radius in one direction than in the other. Astigmatism of the eye is corrected by the use of toroidal surfaces on the spectacle lenses.

A *contact lens*, placed in contact with the surface of the cornea, effectively changes the curvature of the outer surface of the eye (where most of the visual refractive power occurs). A rigid contact lens can easily correct astigmatism by replacing the toroidal surface of the cornea with its own spherical surface. Obviously, a soft (flexible) contact lens requires an orientation mechanism to align its toroidal power with that of the eye. Myopia and hyperopia can be corrected with contact lenses which flatten or strengthen the curvature of the outer surface of the visual optical system.

Radial keratotomy is a surgical technique where radial cuts are made in the cornea (through most of its thickness). This weakens the cornea, and the internal pressure of the eye causes it to bulge in the region of the cuts, thus changing the shape and the power of the cornea. Two obvious drawbacks to this procedure are light scattering from the corneal scars left by the cuts, and the fact that the power of the eye tends to change as one ages, so that the correction may not be permanent. Another technique (PRK) involves a change in corneal shape by sculpting using laser ablation. LASIK slices a thin flap of the

cornea off and then ablates the cornea to change its shape; the flap is then replaced.

The chromatic aberration of the eye was discussed in Sec. 5.3; many eyes have some undercorrected spherical aberration as well. The lens of the eye has aspheric surfaces and a higher index of refraction in the central core of the lens than in the outer portions; both of these factors reduce the power of the system at the margin of the lens and tend to correct the heavy undercorrected spherical from the cornea. A few persons have overcorrected spherical. In most people, the spherical tends toward overcorrection with accommodation, since the lens bulges more at the center than at the edge when the eye focuses on a near point. As much as ± 2 diopters of spherical have been measured; however, like chromatic aberration, spherical seems to have little effect on the resolution of the eye.

Presbyopia is the inability to accommodate (focus) and results from the hardening of the material of the lens which comes with age. Figure 5.11 indicates the (typical) relationship between age and the power of accommodation. When the eye can no longer accommodate to reading distance (2 or 3 diopters), it is necessary to wear positive lenses to read comfortably.

Keratoconus is a conically shaped cornea and can be corrected by contact lenses which effectively overlay a new spherical surface on the cornea.

An opaque or cloudy lens (*cataract*) is frequently removed surgically to restore vision. The resultant loss of power can be made up by an extremely strong positive spectacle lens. But better solutions are a contact lens or by surgically implanting a plastic intraocular lens near

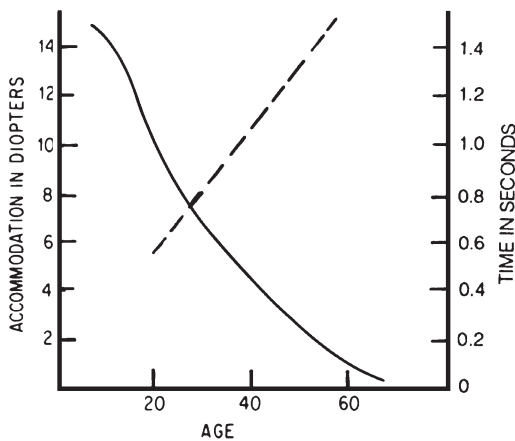


Figure 5.11 The variation of accommodation power with age (solid line). The dashed line indicates the time in seconds to accommodate to 1.3 diopters.

the iris. Such an aphakic eye, lacking a lens, cannot accommodate. Also, the change in retinal image size due to the shift in refractive power from inside to outside the eye (if due to the strong spectacle lens) will preclude binocular vision if only one eye is lensless.

Aniseikonia is the name given to a disparity in retinal image size from one eye to the other, occurring in otherwise normal eyes, and results in lack of binocular vision if the disparity is larger than a few percent. Aniseikonia can be corrected by special thick meniscus lenses which are effectively low-power telescopes whose magnifications balance out the difference in retinal image size.

In instrument design, a number of additional factors should be taken into consideration, especially for binocular instruments. An adjustment must be provided for the variation in interpupillary distance, so that both sides of the instrument can be aligned with the pupils of the eyes. This distance is typically about $2\frac{1}{2}$ in, but it ranges from 2 to 3 in. Both halves of a binocular instrument must have the same magnification (within $\frac{1}{2}$ to 2 percent, depending on the individual's tolerance) and both halves must have their axes parallel (to within $\frac{1}{4}$ prism diopter vertically, $\frac{1}{2}$ diopter divergence, and 1 diopter convergence). Each side must be independently focusable to allow for variations in focus between the two eyes. A focus adjustment of ± 4 diopters will take care of the requirements of all but a few percent of the population; ± 2 diopters will satisfy about 85 percent. The depth of field of the eye (the distance on either side of the point of best focus through which vision is distinct) is about $\pm \frac{1}{4}$ diopter. The Rayleigh quarter wave (see Chap. 11) depth of focus is $\pm 1.1/(\text{pupil diameter})^2$ diopters, which for a 3-mm pupil works out to $\pm \frac{1}{8}$ diopter. For biocular devices, such as head-up displays (HUDs), the angular disparity between the eyes should be less than 0.001 radians.

Bibliography

Note: Titles preceded by an asterisk are out of print.

*Adler, F., *Physiology of the Eye—Clinical Applications*, St. Louis, Mosby, 1959.

Alpern, M., "The Eyes and Vision," in W. Driscoll (ed.), *Handbook of Optics*, New York, McGraw-Hill, 1978.

Blaker, W., "Ophthalmic Optics," in Shannon and Wyant (eds.), *Applied Optics and Optical Design*, vol. 9, New York, Academic, 1983.

Charman, W. N., "Optics of the Eye," *Handbook of Optics*, vol. 1, New York, McGraw-Hill, 1995, Chap. 24.

*Davson, H., *The Physiology of the Eye*, London, Blakiston, 1950.

Dudley, L., "Stereoscopy," in Kingslake (ed.), *Applied Optics and Optical Engineering*, vol. 2, New York, Academic, 1965.

- Fry, G., "The Eye and Vision," in Kingslake (ed.), *Applied Optics and Optical Design*, vol. 2, New York, Academic, 1965.
- Geisler, W. S., and M. S. Banks, "Visual Performance," *Handbook of Optics*, vol. 1, New York, McGraw-Hill, 1995, Chap. 25.
- *Hartridge, H., *Recent Advances in the Physiology of Vision*, London, Blakiston, 1950.
- Kingslake, R., *Optical System Design*, New York, Academic, 1983.
- Lueck, I., "Spectacle Lenses," in Kingslake (ed.), *Applied Optics and Optical Engineering*, vol. 3, New York, Academic, 1966.
- Mouroulis, P., *Visual Instrumentation*, New York, McGraw-Hill, 1999.
- Richards, "Visual Optics," in MIL-HDBK-141, *Optical Design*, Washington, Defense Supply Agency, 1962.
- Zoethout, W., *Physiological Optics*, Professional Press, 1939.

Exercises

1 What power telescope is necessary to enable a person with "normal" visual acuity to read letters 1 mm high at a distance of 300 ft? (tangent of 1 minute of arc is 0.0003)

ANSWER: $135\times$

2 What power corrective lens would be prescribed for a nearsighted person who could not focus clearly on an object more than 5 in away?

ANSWER: -8 diopters

3 Assuming a depth of focus of $\pm 1/4$ diopter, over what range of distance is vision perfectly clear when the eye is focused at 10 in?

ANSWER: $1\frac{1}{4}$ in

4 It is desired to set an optical vernier to a precision of 0.0001 in. Assuming that the vernier projects the image of a ruled scale onto a screen which is viewed from a distance of 10 in and that the setting is made by aligning a scale line with a cross hair on the screen, what magnification must the projection lens of the optical vernier have? Use 10 seconds of arc for the vernier acuity. (Tangent of 1 second is 0.000005)

ANSWER: 5 power

5 A convex reflector of radius of curvature = 10 in is mounted on a spindle and rotated. (a) What is the largest amount that its center of curvature can be displaced from the axis of rotation without the motion of the reflected image of a distant object being detected by the naked eye? Assume the reflected image is viewed from 10 in. (b) What are the fastest and slowest speeds of rotation at which the motion caused by a decentration of 0.02 in can be detected?

ANSWER: (a) 0.00025 in, (b) 3 to 5 r/min, 300 to 500 r/s

6 (a) If a plane parallel plate is specified to have zero, ± 10 millidiopters, power, what is the shortest tolerable focal length it may have? (b) Assuming one surface is truly flat, what is the strongest (shortest) acceptable radius for the other surface if the index of refraction is 1.6? (c) If the piece has a diameter of 20 mm, how many Newton's rings will be visible when this surface is tested against a true flat? (Use $\lambda = 0.55 \mu\text{m}$. One fringe occurs for each $\lambda/2$ change in thickness of the air space.)

ANSWER: (a) $\pm 100 \text{ m}$, (b) $\pm 60 \text{ m}$ (c) 3 rings