Pulse Propagation in Fibers

Dr. Kun Xu

Key Lab of Optical Communication and Lightwave Technologies (Beijing University of Posts and Telecommunications), Ministry of Education, P.R.China

Tel: +86-10-62282332 E-mail: xukun@bupt.edu.cn

Beijing University of Posts and Telecommunications

Course Outline

1. Introduction
2. Pulse Propagation in Fibers
3. Group-Velocity Dispersion
4. Self-Phase Modulation
5. Optical Solitons
6. Polarization Effects
7. Cross-Phase Modulation
8. Stimulated Raman Scattering
9. Stimulated Brillouin Scattering
10. Parametric Processes

Beijing University of Posts
and Telecommunications

Outline

- Maxwell's Equations
- Fiber Modes
- Pulse-Propagation Equation
- Numerical Methods

Maxwell's Equations

$$\begin{cases} \nabla\times\vec{H}=\vec{J}+\dfrac{\partial\vec{D}}{\partial t} \\ \nabla\times\vec{E}=-\dfrac{\partial\vec{B}}{\partial t} \\ \nabla\cdot\vec{B}=0 \\ \nabla\cdot\vec{D}=\rho \end{cases}$$

安培环路定律

法拉第电磁感应定律

磁通连续性定律

高斯定律

无源区域

$$\begin{cases} \nabla\times\vec{H}=\varepsilon\dfrac{\partial\vec{E}}{\partial t} \\ \nabla\times\vec{E}=-\mu\dfrac{\partial\vec{H}}{\partial t} \\ \nabla\cdot\vec{H}=0 \\ \nabla\cdot\vec{E}=0 \end{cases}$$

矢量形式的波动方程

$$\begin{cases} \nabla^2\vec{E}-\mu\varepsilon\dfrac{\partial^2\vec{E}}{\partial t^2}=0 \\ \nabla^2\vec{H}-\mu\varepsilon\dfrac{\partial^2\vec{H}}{\partial t^2}=0 \end{cases}$$

Constitutive Relations

$$\begin{cases} \vec{D}=\varepsilon_0\vec{E}+\vec{P}=\varepsilon\vec{E} \\ \vec{B}=\mu_0\vec{H}+\vec{M} \\ \vec{J}=\sigma\vec{E} \end{cases}$$

时谐场复数形式的
波动方程
（亥姆霍兹方程）

$$\vec{E}(r,t)=\vec{E}(r)e^{-i\omega t}$$

$$\begin{cases} \nabla^2\vec{E}+k^2\vec{E}=0 \\ \nabla^2\vec{H}+k^2\vec{H}=0 \end{cases}$$

媒质的传播常数

$$k=\omega\sqrt{\mu\varepsilon}$$

一定边界条件下
求解波动方程：

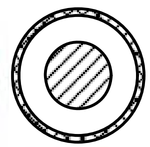
$$\vec{n}\cdot(\vec{D}_1-\vec{D}_2)=\sigma \quad \vec{n}\cdot(\vec{B}_1-\vec{B}_2)=0$$

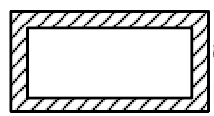
$$\vec{n}\times(\vec{E}_1-\vec{E}_2)=0 \quad \vec{n}\times(\vec{H}_1-\vec{H}_2)=\vec{i}$$

TEM波

- 正弦平面电磁波（无界或一维空间）
- 导行电磁波（二维或三维封闭空间）
- ……

TE波、TM波





How to get the solution?

$$\begin{cases}\nabla\times\vec{H}=\vec{J}+\frac{\partial\vec{D}}{\partial t}\\ \nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}\\ \nabla\cdot\vec{B}=0\\ \nabla\cdot\vec{D}=\rho\end{cases}$$

时谐场复数形式的波动方程
（亥姆霍兹方程）

$$\begin{cases}\nabla^2\vec{E}+k^2\vec{E}=0\\ \nabla^2\vec{H}+k^2\vec{H}=0\end{cases}$$

一定边界条件下求解

$$k=\omega\sqrt{\mu\varepsilon}$$

媒质的传播常数

$$\begin{cases}\nabla^2=\nabla_t^2+\nabla_z^2\\ \vec{E}=\vec{E}_{uv}(u,v)\cdot Z(z)\end{cases}$$

$$\begin{cases}\frac{d^2Z}{dz^2}-\gamma^2\cdot Z=0 \Rightarrow Z=A_+\cdot e^{-\gamma z}\\ \nabla_t^2\vec{E}(u,v)+k_c^2\cdot\vec{E}(u,v)=0 \Rightarrow \vec{E}(u,v)=\{E_u,E_v,E_z\}\\ \gamma=j\beta=jk_z\end{cases}$$

截至波数或横向传播常数

$$k_c^2=k^2+\gamma^2$$

$$\nabla_t^2E_z(u,v)+k_c^2\cdot E_z(u,v)=0$$

$$\nabla_t^2U(u,v)+k_c^2\cdot U(u,v)=0$$

$$U(x,y)=X(x)\cdot Y(y)$$

$$U(r,\varphi)=R(r)\cdot\Phi(\varphi)$$

$$X(x),Y(y)\quad R(r),\Phi(\varphi)\quad ?$$

$$E_z(x,y,z)=X(x)\cdot Y(y)\cdot e^{-j\beta z}$$

$$E_z(r,\varphi,z)=R(r)\cdot\Phi(\varphi)\cdot e^{-j\beta z}$$

$$\vec{E}(u,v,z)=\{E_u,E_v,E_z\}$$

?

Method of Longitudinal Field Components

应用电磁场纵向分量Ez和Hz来求解矢量亥氏方程

$$\begin{cases}\begin{cases}\vec{E}=\vec{E}_T(u_1,u_2,z)+\vec{z}E_z(u_1,u_2,z)\\ \vec{H}=\vec{H}_T(u_1,u_2,z)+\vec{z}H_z(u_1,u_2,z)\end{cases}\\ \nabla=\nabla_T+\vec{z}\dfrac{\partial}{\partial z}\end{cases}$$

$$\begin{cases}\nabla^2\vec{E}+k^2\vec{E}=0\\ \nabla^2\vec{H}+k^2\vec{H}=0\end{cases}$$

$$\begin{cases}\begin{cases}\nabla^2\vec{E}_T+k^2\vec{E}_T=0\\ \nabla^2\vec{H}_T+k^2\vec{H}_T=0\end{cases}\\ \begin{cases}\nabla^2 E_z+k^2E_z=0\\ \nabla^2 H_z+k^2H_z=0\end{cases}\end{cases}$$

标量亥氏方程

$$\begin{cases}\nabla\times\vec{E}=-j\omega\mu\vec{H}\\ \nabla\times\vec{H}=j\omega\varepsilon\vec{E}\end{cases}$$

带入展开令纵
横向分别相等

$$\begin{cases}\nabla_T\times\vec{E}_T=-j\omega\mu H_z\vec{z} & ①\\ \nabla_T\times\vec{z}E_z+\vec{z}\times\dfrac{\partial\vec{E}_T}{\partial z}=-j\omega\mu\vec{H}_T & ②\\ \nabla_T\times\vec{H}_T=-j\omega\varepsilon E_z\vec{z} & ③\\ \nabla_T\times\vec{z}H_z+\vec{z}\times\dfrac{\partial\vec{H}_T}{\partial z}=-j\omega\varepsilon\vec{E}_T & ④\end{cases}$$

联立2，4式得：

$$\begin{cases}\left(k^2+\dfrac{\partial^2}{\partial z^2}\right)\vec{E}_T=\dfrac{\partial}{\partial z}\nabla_T E_z+j\omega\mu\vec{z}\times\nabla_T H_z\\ \left(k^2+\dfrac{\partial^2}{\partial z^2}\right)\vec{H}_T=\dfrac{\partial}{\partial z}\nabla_T H_z-j\omega\varepsilon\vec{z}\times\nabla_T E_z\end{cases}$$

$$\begin{cases}\vec{E}_T=\dfrac{1}{K_c^2}\left(\dfrac{\partial}{\partial z}\nabla_T E_z+j\omega\mu\vec{z}\times\nabla_T H_z\right)\\ \vec{H}_T=\dfrac{1}{K_c^2}\left(\dfrac{\partial}{\partial z}\nabla_T H_z-j\omega\varepsilon\vec{z}\times\nabla_T E_z\right)\end{cases}$$

Fiber Modes

- 从数学上看，在确定边界条件下得到的Maxwell方程的一个本征解即为一种模式；从数学上看，模式是在确定边界条件下能存在的电磁场的一个本征态
- 任何实际存在的电磁场形式都可以用模式展开

例：矩形波导中传输的电磁场总可以表示为：

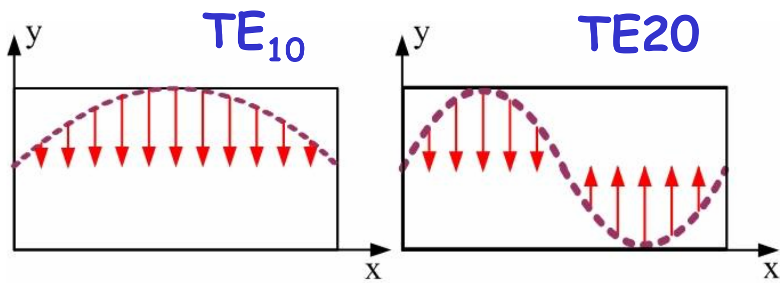
$$\vec{E}(\vec{r},t)=\sum B_{mn}\vec{E}_{TM_{mn}}+\sum C_{mn}\vec{E}_{TE_{mn}}$$

- 标征一个模式的参数

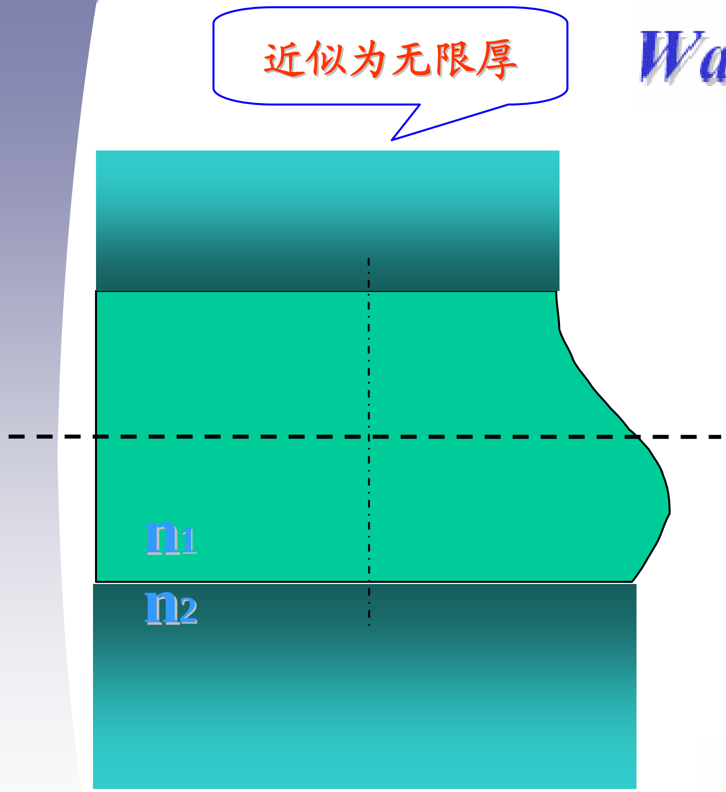
✓电磁场的空间分布，尤指横截面内场的分布

✓模式的谐振频率或传播常数

✓偏振方向



阶跃折射率光纤的波动理论求解—1



$$n(r)=\begin{cases} n_1 & r \le a \\ n_2 & r > a \end{cases}$$

Wave equations $\nabla^2\tilde{\vec{E}}+\omega^2\mu\cdot\varepsilon\tilde{\vec{E}}=0$

$$\tilde{\vec{E}}=\tilde{\vec{E}}_{r\varphi}(r,\varphi)\cdot\tilde{Z}(z)$$

$$\nabla_t^{\,2}\tilde{\vec{E}}_{r\varphi}+\left(\omega^2\mu\cdot\varepsilon+T^2\right)\tilde{\vec{E}}_{r\varphi}=0$$

$$K_c^{\,2}=\omega^2\mu\cdot\varepsilon+T^2=k^2+T^2$$

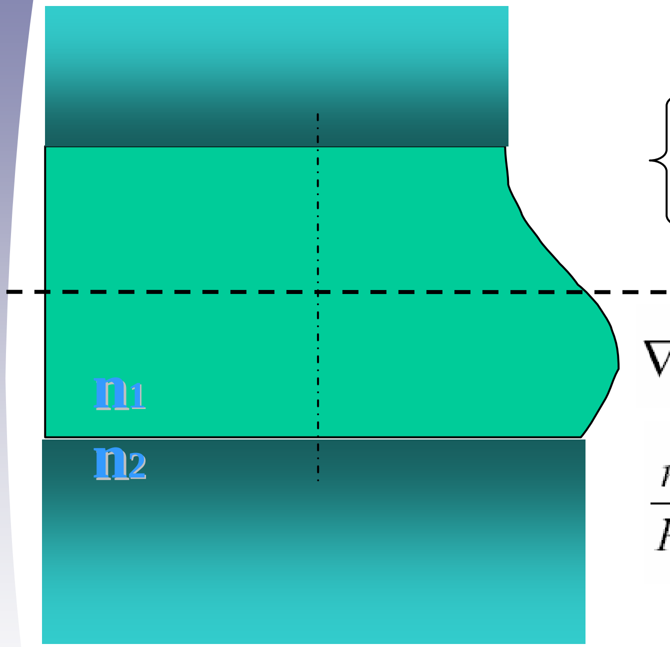
$$\tilde{\vec{E}}_{r\varphi}=\{E_r(r,\varphi),E_\varphi(r,\varphi),E_z(r,\varphi)\}\xrightarrow{\ *\ }\tilde{\vec{E}}$$

$$\nabla_t^{\,2}\tilde{\vec{E}}_{r\varphi}+K_c^{\,2}\cdot\tilde{\vec{E}}_{r\varphi}=0 \Rightarrow \begin{cases}\nabla_t^{\,2}\tilde{E}_r+K_c^{\,2}\cdot\tilde{E}_r=0\\ \cdots \\ \nabla_t^{\,2}\tilde{E}_z+K_c^{\,2}\cdot\tilde{E}_z=0\end{cases}$$

Beijing University of Posts and Telecommunications

阶跃折射率光纤的波动理论求解—2

First We want to get Ez or Hz:



$$\begin{cases} \nabla_t^{\,2}\tilde{E}_z(r,\varphi)+K_c^{\,2}\cdot\tilde{E}_z(r,\varphi)=0 \\ \nabla_t^{\,2}\tilde{H}_z(r,\varphi)+K_c^{\,2}\cdot\tilde{H}_z(r,\varphi)=0 \end{cases}$$

$$\nabla_t^{\,2}\tilde{U}(r,\varphi)+K_c^{\,2}\cdot\tilde{U}(r,\varphi)=0 \quad \tilde{U}(r,\varphi)=\tilde{R}(r)\cdot\tilde{\Phi}(\varphi)$$

$$\frac{r}{R}\frac{d}{dr}\left(r\cdot\frac{dR}{dr}\right)+K_c^{\,2}\cdot r^2=-\frac{1}{\Phi}\cdot\frac{d^2\Phi}{d\varphi^2}=Constant=i^2$$

$$\Phi_m(\varphi)=A_m\sin(m\varphi)+B_m\cos(m\varphi)=\begin{matrix}\cos(m\varphi)\\ \sin(m\varphi)\end{matrix}$$

$$\frac{d^2R}{dr^2}+\frac{1}{r}\frac{dR}{dr}+\left(K_c^2-\frac{m^2}{r^2}\right)R=0$$

$$K_c^2=\omega^2\mu\varepsilon+T^2=k^2-\beta^2>0?$$

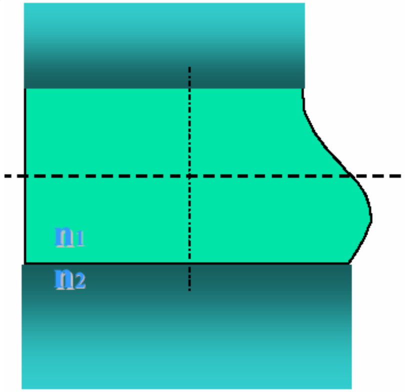
Beijing University of Posts and Telecommunications

阶跃折射率光纤的波动理论求解—3

$k_0 = \omega^2 \mu_0 \varepsilon_0$

$$\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr} + \left(K_c^2 - \frac{m^2}{r^2}\right)R = 0$$

归一化相移常数



① $K_c^2 = \omega^2 \mu\varepsilon + T^2 = n_1^2 k_0^2 - \beta^2 > 0$ $u^2 = a^2 K_c^2$

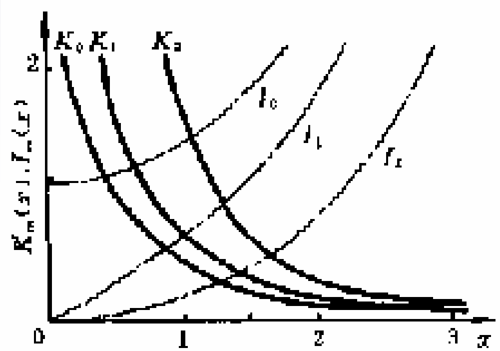
Bessel Function: $$\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr} + \left(\frac{u^2}{a^2} - \frac{m^2}{r^2}\right)R = 0$$

$$R(r) = C_1 J_m(ur/a) + C_2 N_m(ur/a)$$

归一化衰减常数

$$n(r) = \begin{cases} n_1 = \sqrt{\varepsilon_1/\varepsilon_0} & r \le a \\ n_2 = \sqrt{\varepsilon_2/\varepsilon_0} & r > a \end{cases}$$

② $K_c^2 = \omega^2 \mu\varepsilon + T^2 = n_2^2 k_0^2 - \beta^2 < 0$ $w^2 = -a^2 K_c^2$



变态Bessel Function: $$\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr} - \left(\frac{w^2}{a^2} + \frac{m^2}{r^2}\right)R = 0$$

$$R(r) = D_1 I_m(wr/a) + D_2 K_m(wr/a)$$

$r \to \infty$ $I_m(wr/a) \to \infty$ $K_m(wr) \to e^{-wr} \to 0$

阶跃折射率光纤的波动理论求解—4

$$E_Z(r,\varphi)=\begin{cases} A_1 J_m(ur/a)\begin{matrix}\cos m\varphi \\ \sin m\varphi\end{matrix} & r \le a \\ A_2 K_m(wr/a)\begin{matrix}\cos m\varphi \\ \sin m\varphi\end{matrix} & r > a \end{cases}$$

$$H_Z(r,\varphi)=\begin{cases} B_1 J_m(ur/a)\begin{matrix}\cos m\varphi \\ \sin m\varphi\end{matrix} & r \le a \\ B_2 K_m(wr/a)\begin{matrix}\cos m\varphi \\ \sin m\varphi\end{matrix} & r > a \end{cases}$$

$$\begin{cases} \vec{E}_T = \dfrac{1}{K_c^2}\left(\dfrac{\partial}{\partial z}\nabla_T E_z + j\omega\mu\vec{z}\times\nabla_T H_z\right) \\ \vec{H}_T = \dfrac{1}{K_c^2}\left(\dfrac{\partial}{\partial z}\nabla_T H_z - j\omega\varepsilon\vec{z}\times\nabla_T E_z\right) \end{cases}$$

光纤芯径和包层场结构

$$E_r = -\frac{j}{k_c^2}\left(\beta\frac{\partial E_z}{\partial r} + \frac{\omega\mu_0}{r}\frac{\partial H_z}{\partial \varphi}\right)$$

$$E_\varphi = -\frac{j}{k_c^2}\left(\frac{\beta}{r}\frac{\partial E_z}{\partial \varphi} - \omega\mu_0\frac{\partial H_z}{\partial r}\right)$$

$$H_r = -\frac{j}{k_c^2}\left(\beta\frac{\partial H_z}{\partial r} - \frac{\omega\varepsilon}{r}\frac{\partial E_z}{\partial \varphi}\right)$$

$$H_\varphi = -\frac{j}{k_c^2}\left(\frac{\beta}{r}\frac{\partial H_z}{\partial \varphi} + \omega\varepsilon\frac{\partial E_z}{\partial r}\right)$$

两种无损介质分界面没有自由电荷和面电流

$$\vec{n}\times\left(\vec{E}_1-\vec{E}_2\right)=0 \rightarrow E_{1t}=E_{2t}$$

$$\vec{n}\times\left(\vec{H}_1-\vec{H}_2\right)=\vec{J}_s=0 \rightarrow H_{1t}=H_{2t}$$

$$\begin{cases} E_{z1}=E_{z2} \\ E_{\varphi 1}=E_{\varphi 2} \end{cases} \qquad \begin{cases} H_{z1}=H_{z2} \\ H_{\varphi 1}=H_{\varphi 2} \end{cases}$$

$$\left[\frac{J'_m(u)}{uJ_m(u)}+\frac{K'_m(w)}{wK_m(w)}\right]\cdot\left[n_1^2\frac{J'_m(u)}{uJ_m(u)}+n_2^2\frac{K'_m(w)}{wK_m(w)}\right]=\left(\frac{\beta m}{k_0}\right)^2\left(\frac{1}{u^2}+\frac{1}{w^2}\right)$$

传输模特性方程

阶跃折射率光纤中传播条件

传输模特性方程

$$\left[\frac{J'_m(u)}{uJ_m(u)}+\frac{K'_m(w)}{wK_m(w)}\right]\cdot\left[n_1^2\frac{J'_m(u)}{uJ_m(u)}+n_2^2\frac{K'_m(w)}{wK_m(w)}\right]=\left(\frac{\beta m}{k_0}\right)^2\left(\frac{1}{u^2}+\frac{1}{w^2}\right)$$

$u^2=a^2\left(n_1^2k_0^2-\beta^2\right)$　　$w^2=a^2\left(\beta^2-n_2^2k_0^2\right)$　　$n_1,n_2,a,\omega \Rightarrow \beta_{mn}$

$$E_Z(r,\varphi,z)=\begin{cases}A_1J_m\left[\left(n_1^2k_0^2-\beta_{mn}^2\right)r\right]e^{im\varphi}e^{i(\beta_{mn}z-\omega t)} & r\le a\\ A_2K_m\left[\left(\beta_{mn}^2-n_2^2k_0^2\right)r\right]e^{im\varphi}e^{i(\beta_{mn}z-\omega t)} & r>a\end{cases}\quad\begin{Bmatrix}J_m\\K_m\end{Bmatrix}$$

纤芯中u为实数，否则场将衰减：　$\beta\le k_0n_1$

$wr\to\infty$ 时，$K_m(wr)\to e^{-wr}\to 0$　　$w>0,\beta>k_0n_2$

为虚数时，包层电磁场沿 r 方向为辐射状态

传输条件：　$u>0\to\ k_0n_2<\beta<k_0n_1$

截止条件：　$w=0\to\ k_0n_2=\beta$

$W^2<0$，辐射状态

阶跃折射率光纤中模式分析

传输模特性方程

$$\left[\frac{J'_m(u)}{uJ_m(u)}+\frac{K'_m(w)}{wK_m(w)}\right]\cdot\left[n_1^2\frac{J'_m(u)}{uJ_m(u)}+n_2^2\frac{K'_m(w)}{wK_m(w)}\right]=\left(\frac{\beta m}{k_0}\right)^2\left(\frac{1}{u^2}+\frac{1}{w^2}\right)$$

$$u^2=a^2\left(n_1^2k_0^2-\beta^2\right) \qquad w^2=a^2\left(\beta^2-n_2^2k_0^2\right)$$

$$n_1,n_2,a,\omega(k_0) \Rightarrow \beta_{mn}$$

- 模式：每一个传输常数对应着一种可能的光场分布
- 对每个m，都存在多个解一记为β_{mn}。每个β_{mn}对应于一个光场分布。
- 对应每组（mn），本征方程又包含两组解，两组解的Ez和Hz相对值不同：

混合模：场的六个分量都存在

- 当Hz >Ez，称为HEmn模
- 当Hz <Ez，称为EHmn模

$m=0 \Rightarrow$

$$\begin{cases}E_{z1}=0\\E_{z2}=0\end{cases} \qquad \frac{J_1(u)}{uJ_0(u)}+\frac{K_1(w)}{wK_0(w)}=0 \qquad \text{TE}_{0n}\text{模}$$

$$\begin{cases}H_{z1}=0\\H_{z2}=0\end{cases} \qquad n_1^2\frac{J_1(u)}{uJ_0(u)}+n_2^2\frac{K_1(w)}{wK_0(w)}=0 \qquad \text{TM}_{0n}\text{模}$$

弱导近似下光纤中各种模式的截止条件-1

弱导近似:

$$\Delta \approx (n_1 - n_2)/n_1 << 1, n_1 \approx n_2$$

截止波长无穷大，截止频率为0→主模

TE_{0n}模和TM_{0n}模: $J_0(u) = 0$ $u_{01} = 2.4048$ $u_{02} = 5.5201$ $\cdots$

EH_{mn}模: $J_m(u) = 0, u > 0$ $u_{11} = 3.8317$ $u_{21} = 5.1356$ $\cdots$

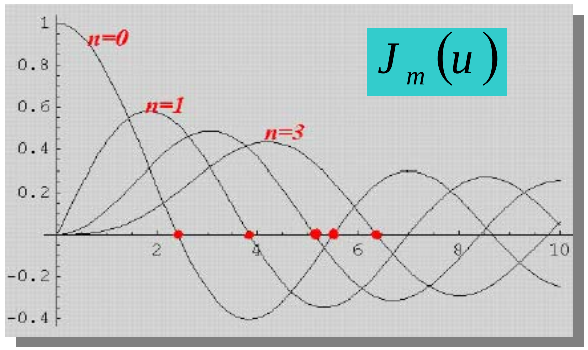
HE_{mn}模:

- HE_{1n}模(m=1) $J_1(u) = 0$ $u_{11} = 0$ $u_{12} = 3.8317$ $\cdots$
- HE_{mn}模(m>1) $J_{m-2}(u) = 0, u > 0$

$u_{01} = 2.4048$

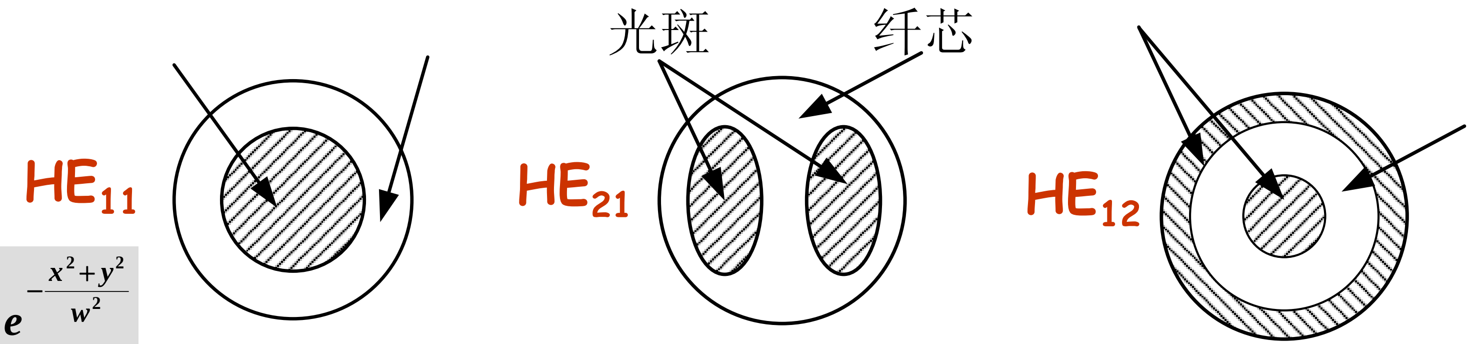
$u_{02} = 5.5201$

$\cdots$



光纤内模式的特点

$$F(x,y)=e^{-\frac{x^2+y^2}{w^2}}$$

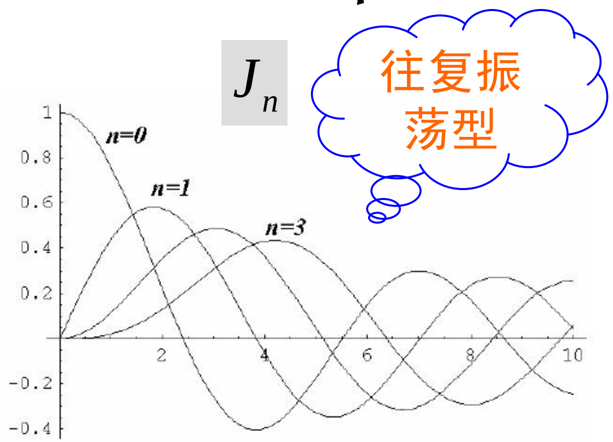


$E \sim J_0(r)$　　$E \sim J_1(r)\sin\varphi$　　$E \sim J_0(r)$

光斑　纤芯

- 模式截止波长或频率：当κ＝0时模式截止，光纤不支持此模式的传输。最低阶的模式为HE11模式，其截止波长为无穷大，截止频率为零（光纤基模）。基模场分布通常采用高斯分布近似;
- 模式传播常数β与频率的关系为色散关系，显然与折射率n1和模式指数m、n有关。

J_n



Beijing University of Posts and Telecommunications

弱导近似下光纤中各种模式的截止条件-2

$$\begin{cases} u^2 = a^2\left(n_1^2 k_0^2 - \beta^2\right) \\ w^2 = a^2\left(\beta^2 - n_2^2 k_0^2\right) \end{cases} \quad n_1, n_2, a, \omega(k_0) \Rightarrow \beta_{mn}$$

截止波长

截止条件： $w = 0 \rightarrow \quad k_0 n_2 = \beta \Rightarrow u_c^2 = a^2\left(n_1^2 k_c^2 - n_2^2 k_c^2\right) \Rightarrow \lambda_c = 2\pi / k_c$

确定传输模式和数量，由波动方程导出

归一化频率 $$V = \sqrt{u^2 + w^2} = \frac{2\pi a}{\lambda_0}\sqrt{n_1^2 - n_2^2} \approx \frac{2\pi a}{\lambda_0} n_1 \sqrt{2\Delta}$$

截止条件下 $$V_c = u_c = \frac{2\pi a}{\lambda_c}\sqrt{n_1^2 - n_2^2} \approx \frac{2\pi a}{\lambda_c} n_1 \sqrt{2\Delta}$$

传输条件 $$V > V_c = u_c \approx \frac{2\pi a}{\lambda_c} n_1 \sqrt{2\Delta}$$

HE11模

TE01、TM01和HE21模

单模传输条件： $$0 < V = \frac{2\pi a}{\lambda_0}\sqrt{n_1^2 - n_2^2} \approx \frac{2\pi a}{\lambda_0} n_1 \sqrt{2\Delta} < 2.4048$$

?

Beijing University of Posts and Telecommunications

Pulse-Propagation Equation

Pulse-Propagation Equation

非线性情况下的波动方程

$$\begin{cases}\nabla\cdot\vec{D}=0\\ \nabla\times\vec{E}=-\partial\vec{B}/\partial t\\ \nabla\cdot\vec{B}=0\\ \nabla\times\vec{H}=\partial\vec{D}/\partial t\end{cases}$$

$$\vec{D}=\varepsilon_0\vec{E}+\vec{P}$$
$$\vec{B}=\mu_0\vec{H}$$

利用E、P
来描述场

$$\nabla^2\vec{E}-\varepsilon_0\mu_0\frac{\partial^2\vec{E}}{\partial t^2}=\mu_0\frac{\partial^2\vec{P}}{\partial t^2}$$

$$\vec{P}=\varepsilon_0\chi\vec{E}$$

对光纤情况，只考虑到三阶非线性效应

$$\vec{P}=\varepsilon_0\chi^{(1)}\cdot\vec{E}+\varepsilon_0\vec{\vec{\chi}}^{(3)}\cdot\vec{E}\vec{E}\vec{E}$$
$$=\vec{P}_L+\vec{P}_{NL}$$

$$\nabla^2\vec{E}-\varepsilon_0\mu_0\frac{\partial^2\vec{E}}{\partial t^2}=\mu_0\frac{\partial^2\vec{P}_L}{\partial t^2}+\mu_0\frac{\partial^2\vec{P}_{NL}}{\partial t^2}$$

对比：

$$\begin{cases}\nabla^2\vec{E}-\varepsilon_0\mu_0\dfrac{\partial^2\vec{E}}{\partial t^2}=\mu_0\dfrac{\partial^2\vec{P}_L}{\partial t^2}\\ \nabla^2\vec{E}-\varepsilon\mu_0\dfrac{\partial^2\vec{E}}{\partial t^2}=0\end{cases}$$

Pulse-Propagation Equation

Several Simplifying Assumptions:

- P_{NL} is treated as a small perturbation to P_L
- Polarization Maintain & Scalar approach
- Quasi-monochromatic ($\Delta\omega/\omega \ll 1$)

$$\vec{E}(\vec{r},t) = \frac{1}{2}\hat{x}\left[\overline{E}(\vec{r},t)e^{-i\omega_0 t} + c.c\right]$$

时间的慢变函数

$$\vec{P}_L(\vec{r},t) = \frac{1}{2}\hat{x}\left[\overline{P}_L(\vec{r},t)e^{-i\omega_0 t} + c.c\right]$$

$$\vec{P}_{NL}(\vec{r},t) = \frac{1}{2}\hat{x}\left[\overline{P}_{NL}(\vec{r},t)e^{-i\omega_0 t} + c.c\right]$$

$$\overline{P}_L = \varepsilon_0 \chi^{(1)} \overline{E}$$

沿x方向偏振光的单位偏振矢量

Pulse-Propagation Equation

$$P_{NL} = \varepsilon_0 \chi^{(3)}_{xxxx} \cdot E \cdot E \cdot E$$

$$= \frac{1}{8}\varepsilon_0 \chi^{(3)}_{xxxx} \cdot \left[\bar{E}(x,y,z,t) e^{-i\omega_0 t} + c.c \right]^3$$

$$= \frac{1}{8}\varepsilon_0 \chi^{(3)}_{xxxx} \cdot \left[\bar{E}^3 e^{-i3\omega_0 t} + 3|\bar{E}|^2 \bar{E} e^{-i\omega_0 t} + 3|\bar{E}|^2 \bar{E}^* e^{i\omega_0 t} + (\bar{E}^*)^3 e^{i3\omega_0 t} \right]$$

$$= \frac{1}{8}\varepsilon_0 \chi^{(3)}_{xxxx} \left[3|\bar{E}|^2 \bar{E} e^{-i\omega_0 t} + \bar{E}^3 e^{-i3\omega_0 t} + c.c \right]$$

忽略3ω的频率项

$$P_{NL}(\vec{r},t) = \frac{3}{8}\varepsilon_0 |\bar{E}(\vec{r},t)|^2 \chi^3_{xxxx} \bar{E}(\vec{r},t) e^{-i\omega_0 t} + c.c = \varepsilon_0 \varepsilon_{NL} E(\vec{r},t)$$

$$= \frac{1}{2}\left[\bar{P}_{NL}(\vec{r},t) e^{-i\omega_0 t} + c.c \right]$$

$$\bar{P}_{NL} = \varepsilon_0 \varepsilon_{NL} \bar{E} \quad \Rightarrow \quad \varepsilon_{NL} = \frac{3}{4}|\bar{E}(x,y,z,t)|^2 \chi^{(3)}_{xxxx}$$

Nonlinear Pulse Propagation

非线性波动方程及各场量表达式：

$$\nabla^2 E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P_L}{\partial t^2} + \mu_0 \frac{\partial^2 P_{NL}}{\partial t^2}$$

$$E(x,y,z,t) = \overline{E}(x,y,z,t)e^{-i\omega_0 t}$$

$$P_L(x,y,z,t) = \overline{P}_L(x,y,z,t)e^{-i\omega_0 t} = \varepsilon_0 \chi^1 \overline{E}(x,y,z,t)e^{-i\omega_0 t}$$

$$P_{NL}(x,y,z,t) = \overline{P}_{NL}(x,y,z,t)e^{-i\omega_0 t} = \varepsilon_0 \varepsilon_{NL} \overline{E}(x,y,z,t)e^{-i\omega_0 t}$$

$$\varepsilon_{NL} = \frac{3}{4}\left|\overline{E}(x,y,z,t)\right|^2 \chi^{(3)}_{xxxx}$$

$$\frac{\partial^2 E}{\partial t^2} = \left(\frac{\partial^2 \overline{E}}{\partial t^2} - 2i\omega_0 \frac{\partial \overline{E}}{\partial t} - \overline{E}\omega_0^2\right)e^{-i\omega_0 t} \approx -\left(2i\omega_0 \frac{\partial \overline{E}}{\partial t} + \overline{E}\omega_0^2\right)e^{-i\omega_0 t}$$

$$\frac{\partial^2 P_L}{\partial t^2} \approx -\left(2i\omega_0 \frac{\partial \overline{P}_L}{\partial t} + \overline{P}_L\omega_0^2\right)e^{-i\omega_0 t} = -\varepsilon_0\chi^1\left(2i\omega_0 \frac{\partial \overline{E}}{\partial t} + \overline{E}\omega_0^2\right)e^{-i\omega_0 t}$$

$$\frac{\partial^2 P_{NL}}{\partial t^2} \approx -\left(2i\omega_0 \frac{\partial \overline{P}_{NL}}{\partial t} + \overline{P}_{NL}\omega_0^2\right)e^{-i\omega_0 t} = -\varepsilon_0\varepsilon_{NL}\left(2i\omega_0 \frac{\partial \overline{E}}{\partial t} + \overline{E}\omega_0^2\right)e^{-i\omega_0 t}$$

$$\nabla^2 E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P_L}{\partial t^2} + \mu_0 \frac{\partial^2 P_{NL}}{\partial t^2}$$

在频域内求解非线性波动方程，作傅氏变换：

$$\tilde{E}(r,\omega) = \int \overline{E}(r,t) e^{-i\omega_0 t} e^{i\omega t} dt = \int \overline{E}(r,t) e^{i(\omega-\omega_0)t} dt$$

$$\Rightarrow \tilde{E}(r,\omega-\omega_0) = \int \overline{E}(r,t) e^{i(\omega-\omega_0)t} dt$$

$$\nabla^2 \tilde{E} - \frac{(-i\omega)^2}{c^2} \tilde{E}\left[1 + \chi^{(1)}(\omega) + \varepsilon_{NL}\right] = 0$$

$$\left(\frac{\partial}{\partial t} \to -i\omega \quad \omega \to i\frac{\partial}{\partial t}\right)$$

$$\nabla^2 \tilde{E} + \varepsilon(\omega) k_0^2 \tilde{E} = 0$$

$$k_0 = \frac{\omega}{c} \quad \varepsilon(\omega) = 1 + \chi^{(1)}(\omega) + \varepsilon_{NL}$$

$$\varepsilon(\omega)=1+\chi^{(1)}(\omega)+\varepsilon_{NL}$$

- 线性极化率$\chi^{(1)}$可能是复数，其实部与介质折射率有关，虚部则与介质的吸收有关

$$\varepsilon(\omega)=1+\chi^{(1)}(\omega)=\left(n+i\alpha/2k_0\right)^2$$

$$n=1+\frac{1}{2}\mathrm{Re}\left[\chi^{(1)}(\omega)\right] \quad \alpha=\frac{\omega}{nc}\mathrm{Im}\left[\chi^{(1)}(\omega)\right]$$

- ε_{NL}是小量，其对折射率和吸收系数的修正为：

$$\tilde{n}+i\frac{\tilde{\alpha}}{2k_0}=\left(1+\chi^{(1)}+\varepsilon_{NL}\right)^{1/2}=\left(1+\mathrm{Re}\left(\chi^{(1)}+\varepsilon_{NL}\right)+i\,\mathrm{Im}\left(\chi^{(1)}+\varepsilon_{NL}\right)\right)^{1/2}$$

$$=\left[1+\mathrm{Re}\left(\chi^{(1)}+\varepsilon_{NL}\right)\right]^{1/2}+\frac{i\,\mathrm{Im}\left(\chi^{(1)}+\varepsilon_{NL}\right)}{2\left[1+\mathrm{Re}\left(\chi^{(1)}+\varepsilon_{NL}\right)\right]^{1/2}}=\tilde{n}+\frac{i\,\mathrm{Im}\left(\chi^{(1)}+\varepsilon_{NL}\right)}{2\tilde{n}}$$

$$=\left[1+\mathrm{Re}\left(\chi^{(1)}\right)\right]^{1/2}+\frac{\mathrm{Re}\left(\varepsilon_{NL}\right)}{2\left[1+\mathrm{Re}\left(\chi^{(1)}\right)\right]^{1/2}}+\frac{i\,\mathrm{Im}\left(\chi^{(1)}+\varepsilon_{NL}\right)}{2n}$$

$$=n+\frac{\mathrm{Re}\left(\varepsilon_{NL}\right)}{2n}+\frac{i\,\mathrm{Im}\left(\chi^{(1)}+\varepsilon_{NL}\right)}{2n}$$

$$\tilde{n} + i\frac{\tilde{\alpha}}{2k_0} = n + \frac{\mathrm{Re}(\varepsilon_{NL})}{2n} + \frac{i\,\mathrm{Im}\left(\chi^{(1)} + \varepsilon_{NL}\right)}{2n}$$

（1） $$\tilde{n} = n + \frac{\mathrm{Re}[\varepsilon_{NL}]}{2n} = n + \frac{3|E|^2}{8n}\mathrm{Re}\left[\chi^{3}_{xxxx}\right] = n + n_2|E|^2$$

$$n_2 = \frac{3}{8n}\mathrm{Re}\left[\chi^{(3)}_{xxxx}\right]$$

（2） $$\tilde{\alpha} = 2k_0 \cdot \frac{\mathrm{Im}\left[\chi^{(1)} + \varepsilon_{NL}\right]}{2n} = \frac{\omega}{nc}\mathrm{Im}\left[\chi^{(1)}\right] + \frac{\omega}{nc}\mathrm{Im}[\varepsilon_{NL}]$$

$$= \alpha + \frac{3\omega}{4nc}|E|^2\,\mathrm{Im}\left[\chi^{(3)}_{xxxx}\right] = \alpha + \alpha_2|E|^2$$

$$\alpha_2 = \frac{3\omega}{4nc}\mathrm{Im}\left[\chi^{(3)}_{xxxx}\right]$$

$$\nabla^2 E(r,\omega)+\varepsilon(\omega)k_0^2 E(r,\omega)=0$$

利用分离变量法求解
非线性波动方程：

$$\tilde{E}(r,\omega-\omega_0)=\tilde{A}(z,\omega-\omega_0)F(x,y)e^{i\beta_0 z}$$

$$\nabla^2\tilde{E}=\left(\nabla_t^2+\frac{\partial^2}{\partial z^2}\right)\left[\tilde{A}(z,\omega-\omega_0)\cdot F(x,y)\cdot e^{i\beta_0 z}\right]$$

$$=\tilde{A}\cdot e^{i\beta_0 z}\cdot\nabla_t^2 F(x,y)+F(x,y)\cdot e^{i\beta_0 z}\cdot\frac{\partial^2\tilde{A}}{\partial z^2}$$

$$+2i\beta_0\cdot F(x,y)\cdot e^{i\beta_0 z}\cdot\frac{\partial\tilde{A}}{\partial z}-\beta_0^2\cdot\tilde{A}\cdot F(x,y)e^{i\beta_0 z}$$

$$\approx\tilde{A}\cdot e^{i\beta_0 z}\cdot\nabla_t^2 F(x,y)+2i\beta_0\cdot F(x,y)\cdot e^{i\beta_0 z}\cdot\frac{\partial\tilde{A}}{\partial z}-\beta_0^2\cdot\tilde{A}\cdot F(x,y)e^{i\beta_0 z}$$

$$\frac{\nabla_t^2 F(x,y)}{F(x,y)}+\varepsilon(\omega)k_0^2=-\frac{2i\beta_0\frac{\partial\tilde{A}}{\partial z}}{\tilde{A}}+\beta_0^2$$

$$\frac{\nabla_t^2 F(x,y)}{F(x,y)} + \varepsilon(\omega)k_0^2 = -\frac{2i\beta_0 \frac{\partial \tilde{A}}{\partial z}}{\tilde{A}} + \beta_0^2$$

方程两边必须等于一常数，则此方程可分离成两个独立的方程：

$$\begin{cases} \nabla_t^2 F(x,y) + \left[\varepsilon(\omega)k_0^2 - \bar{\beta}^2\right] F(x,y) = 0 \\ 2i\beta_0 \frac{\partial \tilde{A}}{\partial z} + \left(\bar{\beta}^2 - \beta_0^2\right)\tilde{A} = 0 \end{cases}$$

特征值方程

方程（1）—横向场分布方程：

$$\nabla_t^2 F(x,y) + \left[\varepsilon(\omega) k_0^2 - \bar{\beta}^2\right] F(x,y) = 0$$

- 上述方程为线性情况下光纤的本征方程，仅折射率发生微小变化：

$$\varepsilon = \tilde{n}^2 = (n + \Delta n)^2$$

$$\approx n^2 + 2n\Delta n$$

微扰法：先求n²下方程的解，然后对方程考虑微扰量的影响；一阶微扰近似：微扰量不影响模F(x,y)的分布，只影响 β

$$\Delta n = n_2 |E|^2 + \frac{i\tilde{\alpha}}{2k_0}$$

考虑了非线性导致的折射率变化和光纤的本征损耗

$$\tilde{\beta}(\omega) = \beta(\omega) + \Delta\beta$$

- 折射率发生微小变化情况下，本征值也发生微小变化（其中包括非线性和损耗）：

$$\Delta\beta = \frac{k_0 \int_{-\infty}^{+\infty}\int \Delta n |F(x,y)|^2 dxdy}{\int_{-\infty}^{+\infty}\int |F(x,y)|^2 dxdy}$$

the modal distribution of the fundamental fiber mode HE11

Beijing University of Posts and Telecommunications

$$\Delta n = n_2 |E|^2 + \frac{i\tilde{\alpha}}{2k_0}$$

$$\tilde{E}(r, \omega - \omega_0) = \tilde{A}(z, \omega - \omega_0) F(x, y) e^{i\beta_0 z}$$

$$\Rightarrow \Delta\beta = \frac{k_0 \int_{-\infty}^{+\infty}\int \Delta n |F(x,y)|^2 dxdy}{\int_{-\infty}^{+\infty}\int |F(x,y)|^2 dxdy}$$

$$\Rightarrow \Delta\beta = \frac{k_0 \int_{-\infty}^{+\infty}\int \Delta n |F(x,y)|^2 dxdy}{\int_{-\infty}^{+\infty}\int |F(x,y)|^2 dxdy} = (\gamma |A|^2 + i\frac{\alpha}{2})$$

represents the optical power

the effective core area

$$\Rightarrow \gamma = \frac{k_0 n_2}{A_{eff}} = \frac{\omega_0 n_2}{c A_{eff}}$$

the nonlinear parameter

$$A_{eff} = \frac{\iint |F(x,y)|^2 dxdy}{\iint ||F(x,y)|^4 dxdy} \Rightarrow |A|^2$$

$$A_{eff} = \frac{[\iint |F(x,y)|^2 dxdy]^2}{\iint ||F(x,y)|^4 dxdy} \Rightarrow |A|^2 \iint_{\infty} |F(x,y)|^2 dxdy$$

方程（2）—复振幅方程：

$$2i\beta_0\frac{\partial\tilde{A}}{\partial z}+\left(\bar{\beta}^2-\beta_0^2\right)\tilde{A}=0$$

$$\frac{\partial\tilde{A}}{\partial z}=i\frac{\bar{\beta}^2-\beta_0^2}{2\beta_0}\tilde{A}=i\frac{\left(\bar{\beta}+\beta_0\right)\left(\bar{\beta}-\beta_0\right)}{2\beta_0}\tilde{A}\approx i\left[\beta(\omega)+\Delta\beta-\beta_0\right]\tilde{A}$$

$$\approx i\left[\beta_1(\omega-\omega_0)+\frac{1}{2}(\omega-\omega_0)^2\beta_2+\Delta\beta\right]\tilde{A}$$

作傅氏逆变换：

$$A(z,t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\tilde{A}(z,\omega-\omega_0)e^{-i(\omega-\omega_0)t}d\omega$$

$$\frac{\partial A(z,t)}{\partial z}=-\beta_1\frac{\partial A(z,t)}{\partial t}-\frac{i}{2}\beta_2\frac{\partial^2 A(z,t)}{\partial t^2}+i\Delta\beta\cdot A(z,t)$$

$$\frac{\partial A(z,t)}{\partial z}+\beta_1\frac{\partial A(z,t)}{\partial t}+\frac{i}{2}\beta_2\frac{\partial^2 A(z,t)}{\partial t^2}+\frac{\alpha}{2}A=i\gamma|A|^2A(z,t)$$

$$\gamma=\frac{\omega_0 n_2}{cA_{eff}}$$

最后光纤中的光场可表达为：

$$\vec{E}(x,y,z,t)=\frac{1}{2}\hat{x}\left[\overline{E}(x,y,z,t)e^{-i\omega_0 t}+c.c\right]$$

$$=\frac{1}{2}\hat{x}\left[A(z,t)F(x,y)e^{i(\beta_0 z-\omega_0 t)}+c.c\right]$$

纵向传播因之　横向分布因之

Nonlinear Schrodinger (NLS) Equation:

影响相位

$$\frac{\partial A(z,t)}{\partial z}+\beta_1\frac{\partial A(z,t)}{\partial t}+\frac{i}{2}\beta_2\frac{\partial^2 A(z,t)}{\partial t^2}+\frac{\alpha}{2}A=i\gamma|A|^2A(z,t)$$

- 参数意义：损耗、色散和Kerr非线性效应（非线性折射率效应）
- 色散和Kerr效应影响光场相位，其后传播中，相位变化通过色散转化为幅度变化（脉冲变形）
- Kerr效应产生的是附加的与功率相关的相位调制（SPM）
- 方程是一个单偏振、单波长光信号在单模光纤中的传输方程

Higher-Order Nonlinear Effects

在光信号非常窄的时候，即信号谱宽很宽，则必须考虑更高阶的色散效应和由喇曼效应导致的自频移等效应，此种情况下非线性薛定谔方程为：

$$\frac{\partial A}{\partial z}+\frac{\alpha}{2}A+\beta_1\frac{\partial A}{\partial t}+\frac{i}{2}\beta_2\frac{\partial^2 A}{\partial t^2}-\frac{1}{6}\beta_3\frac{\partial^3 A}{\partial t^3}=i\gamma\left[|A|^2A+\frac{i}{\omega_0}\frac{\partial}{\partial t}\left(|A|^2A\right)-T_RA\frac{\partial|A|^2}{\partial t}\right]$$

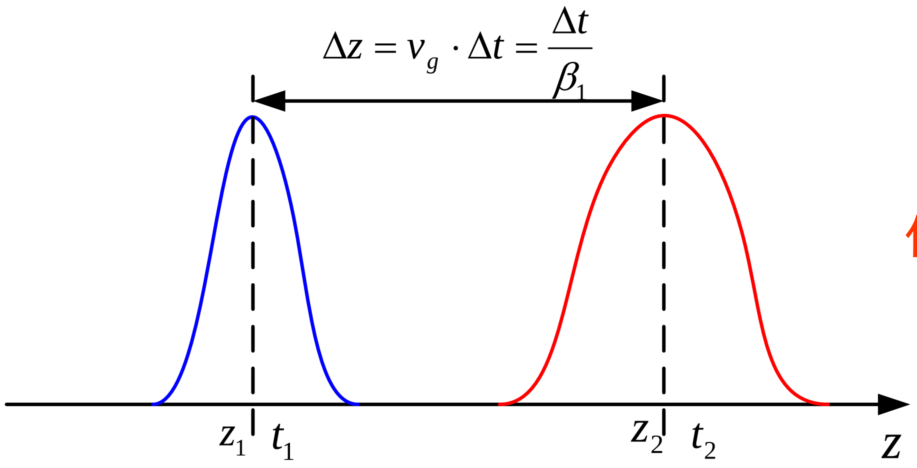
高阶色散效应，传输常数展开式的三阶项引起，因为超短脉冲的宽带宽，即使在波长与零色散波长相差较大时，高阶色散效应引起的超短脉冲也很重要。

非线性性极化强度的慢变部分PNL的一阶导数引起的，对应于脉冲沿的自陡峭。

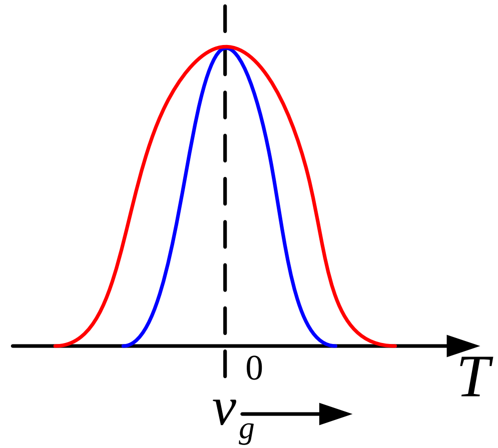
与延迟非线性响应有关，对应于自频移效应，在非线性极化PNL的表达式中，三阶电极化率的傅立叶变换是复数，且与频率有关，其虚部与喇曼增益有关，对其虚部有贡献，实部对其实部有影响，一般被忽略。

- 折射率变化导致传播常数的变化
- 非线性导致的折射率变化与时间相关
- 非线性极化的延迟响应表达

Retarded Frame



作变换：



上述方程都是在一个绝对坐标系中描述信号的传播

$$T = t - z/v_g = t - \beta_1 z$$

带入非线性薛定谔方程

$$\frac{\partial A}{\partial z} + \frac{\alpha}{2}A + \frac{i}{2}\beta_2\frac{\partial^2 A}{\partial T^2} - \frac{1}{6}\beta_3\frac{\partial^3 A}{\partial T^3} = i\gamma\left[|A|^2 A + \frac{2i}{\omega_0}\frac{\partial}{\partial T}\left(|A|^2 A\right) - T_R A\frac{\partial |A|^2}{\partial T}\right]$$

一般情况下，若脉冲宽度大于5ps，高阶非线性效应可不考虑，得简化方程

$$i\frac{\partial A}{\partial z} + \frac{i\alpha}{2}A - \frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2} + \gamma|A|^2 A = 0$$

Beijing University of Posts and Telecommunications

Numerical Methods

1. Split-Step Fourier Transform Method

$$\frac{\partial A}{\partial z}+\frac{\alpha}{2}A+\frac{i}{2}\beta_2\frac{\partial^2 A}{\partial T^2}-\frac{1}{6}\beta_3\frac{\partial^3 A}{\partial T^3}=$$

$$i\gamma\left[|A|^2 A+\frac{2i}{\omega_0}\frac{\partial}{\partial T}\left(|A|^2 A\right)-T_R A\frac{\partial |A|^2}{\partial T}\right]$$

$$\frac{\partial A}{\partial z}=(\hat{D}+\hat{N})A$$

微分算子D及非线性算子N分别为：

$$\hat{D}=-\frac{\alpha}{2}-\frac{i}{2}\beta_2\frac{\partial^2}{\partial T^2}+\frac{1}{6}\beta_3\frac{\partial^3}{\partial T^3}$$

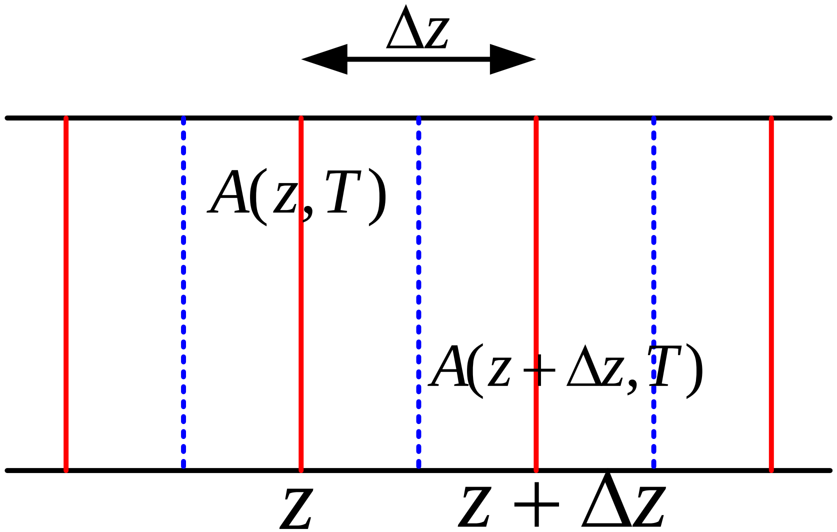
表示线性介质的色散和吸收

$$\hat{N}=i\gamma\left[|A|^2+\frac{i}{A\omega_0}\frac{\partial}{\partial T}\left(|A|^2 A\right)-T_R\frac{\partial |A|^2}{\partial T}\right]$$

表示非线性作用

- 在一小段距离内，色散、损耗与非线性独立（分别）作用；
- 所取距离愈小，计算结果愈准确；
- 数值计算分两步进行：色散作用于一段距离，非线性作用于其后。

$$\frac{\partial A}{\partial z} = (\hat{D} + \hat{N})A$$



仅考虑色散和损耗：

$$\frac{\partial A}{\partial z} = \hat{D}A \Rightarrow A(z+\Delta z, T) = \exp\left[\Delta z \cdot \hat{D}\right]A(z,T)$$

$$\frac{\partial A}{\partial z} = -\frac{\alpha}{2}A - \frac{i}{2}\beta_2\frac{\partial^2 A}{\partial T^2} + \frac{1}{6}\beta_3\frac{\partial^3 A}{\partial T^3} \Rightarrow$$

$$\frac{\partial A(\omega,z)}{\partial z} = -\frac{\alpha}{2}A(\omega,z) - \frac{i}{2}\beta_2\cdot(-i\omega)^2\cdot A(\omega,z) + \frac{1}{6}\beta_3\cdot(-i\omega)^3\cdot A(\omega,z)$$

$$\Rightarrow \frac{\partial A(\omega,z)}{\partial z} = \left[-\frac{\alpha}{2} + \frac{i}{2}\beta_2\omega^2 + \frac{i}{6}\beta_3\omega^3\right]A(\omega,z)$$

$$\Rightarrow A(\omega, z+\Delta z) = \exp\left[\Delta z \cdot \hat{D}(-i\omega)\right]A(\omega,z)$$

$$\exp\left(\Delta z \cdot \hat{D}\right)A(z,T) = FFT^{-1}\left\{\exp\left[\Delta z \cdot \hat{D}(-i\omega)\right]\cdot FFT\left(A(z,T)\right)\right\}$$

仅考虑非线性：

$$\frac{\partial A}{\partial z} = \hat{N}A \Rightarrow A(z+\Delta z, T) = \exp\left[\Delta z \cdot \hat{N}\right]A(z,T)$$

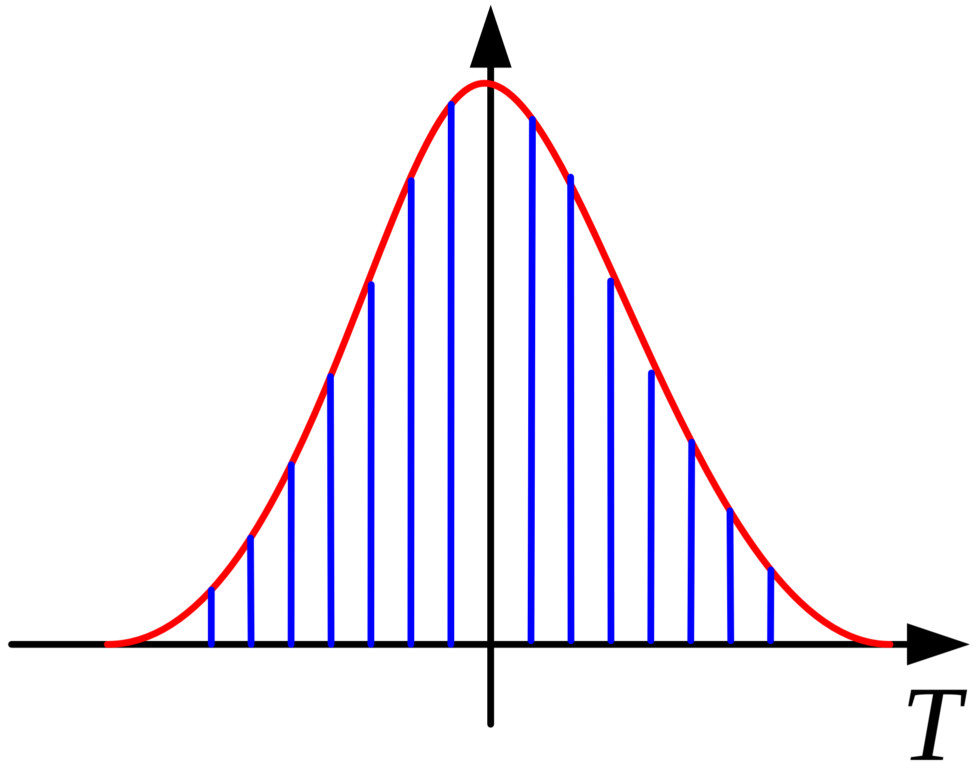
$$\hat{N} = i\gamma\left[|A|^2 + \frac{i}{A\omega_0}\frac{\partial}{\partial T}\left(|A|^2 A\right) - T_R \frac{\partial |A|^2}{\partial T}\right]$$

方程的最终近似数值解为：

$$A(z+\Delta z, T) \approx \exp\left[\Delta z \cdot \hat{D}\right]\exp\left[\Delta z \cdot \hat{N}\right]A(z,T)$$

- 数值解的精度取决于所取计算步长的大小
- 可以采用迭代的方法提高在增大计算步长的情况下保持计算精度

脉冲信号包络采样：



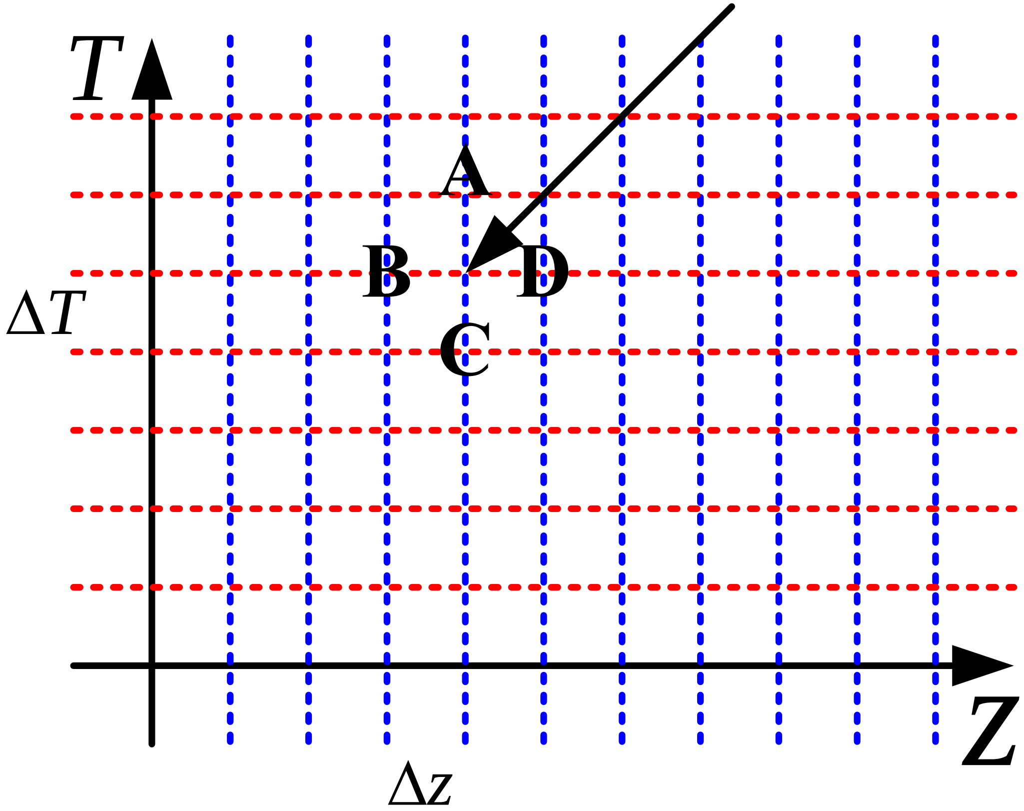
$$A(T) = [a_1 \quad a_2 \quad \cdots \quad a_{n-1} \quad a_n]$$

Numerical Methods

2. Finite-Difference Method

$$\frac{\partial A}{\partial z}+\frac{\alpha}{2}A+\frac{i}{2}\beta_2\frac{\partial^2 A}{\partial T^2}-\frac{1}{6}\beta_3\frac{\partial^3 A}{\partial T^3}=$$
$$i\gamma\left[|A|^2A+\frac{2i}{\omega_0}\frac{\partial}{\partial T}\left(|A|^2A\right)-T_R A\frac{\partial |A|^2}{\partial T}\right]$$

将时间坐标简单看作一个与空间坐标z等同的坐标变量，在二维的时空坐标中划分计算网格，在网格点处将微分方程线性化，利用迭代技术进行计算



$$A = A(z, T + \Delta T) \quad B = A(z - \Delta z, T)$$

$$C = A(z, T - \Delta T) \quad D = A(z + \Delta z, T)$$

Email: xukun@bupt.edu.cn
Tel: 86 -10 - 62282332
Web: oclt.sto.bupt.cn

Thank You All for Your Attention!

Any Questions and Suggestions are Welcome!

Beijing University of Posts and Telecommunications