

A simple method to determine the thermal focal length of solid-state lasers with rate equation

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Abstract

A simple method to measure the thermal focal length of solid-state laser with rate equation was presented. By measuring the output beam power of lasers, the beam waist of oscillating wave can be deduced from rate equation under the stable oscillation condition. Based on the ABCD matrix theory, the thermal focus of the gain medium was easily obtained. The thermal focus of a LD end-pumped Nd:GdVO₄ laser was measured and the experimental results agreed with the theoretical predication.

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1. Introduction

LD pumped solid-state laser has become an important research area in laser field nowadays due to the advantages of high conversion efficiency and good stability and beam quality, etc. However, the end-pumping can lead to the uneven thermal dispersion in the gain media and thus cause thermal effect that can badly influence the performances of lasers. Now, the thermal effect has become the final limited factor to achieve high power [1]. So it is very important to study the thermal effect of laser crystals.

As a new host material for Nd³⁺ ion, the GdVO₄ crystal has been developed recently by Zagumennyi et al. [2]. Compared with Nd:YAG and Nd:YVO₄ crystal widely used in LD-pumped solid-state laser, Nd:GdVO₄ has a higher absorption cross section at 808 nm, a larger emission cross section at 1.06 μm and a larger thermal conductivity. A lot of research has been done on the performance properties of Nd:GdVO₄ lasers. However, as far as we know, there is no report on the thermal focal length of Nd:GdVO₄ crystal. So it is very important to study the thermal effect of Nd:GdVO₄.

The thermal effect in laser crystal can be treated in first order as a spherical lens expressed by the factor f_t [3]. In recent years, various ways have been reported to determine the induced factor f_t , such as stable cavity method [4] and CCD camera [3], etc. For stable cavity method or interference stripe method [5], there is no output beam when measuring the relative factors, which leads to a smaller value than the real one because a lot of energy used in simulating photon transition is conveyed into inner heat of crystal. For the method trying to detect the waist of output beam directly such as CCD camera, the experimental results are greatly limited by the properties of detector and thus are not so accurate. In this paper, a simple method to measure the thermal focal length of solid-state laser with rate equation is presented. Using the rate equation and ABCD matrix theory, we only need to detect the output beam power of the cavity to get the beam waist of resonate wave before calculating f_t . This method not only has the advantage of simply operation, but also can be directly used in analyzing the thermal effect in any cavity structure and output power range.

In this paper, we determined the thermal focus f_t of LD end-pumped Nd:GdVO₄ lasers with rate equation method. The experimental results agree well to the theoretical estimation.

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2. Theoretical analysis of measurement

For CW operating lasers, the resonate wave can undergo the gain from the amplification of laser crystal, while the absorption scattering, imperfect mirror coatings, the output power and so on lead to the loss of inner cavity. When the round-trip gain equals the loss, the laser reaches a stable oscillating state in the laser cavity.

For LD end-pumped solid-state laser, the pump laser and the oscillation laser are considered as coaxial Gaussian beams. Neglecting the divergence of the pump beam in a laser crystal of length l (i.e. the waist radius ω_p of the pump beam is considered to be a constant), and only considering the amplification of the gain medium, we can obtain the rate of change of the oscillation beam power P_l in a saturated gain medium [6]:

$$\frac{dP_l}{dz} = \frac{2\sigma\tau\lambda_p\alpha_p P_0 \exp(-\alpha_p z) P_l}{hc\pi(\omega_p^2 + \omega_c^2)(1 + 4P_l/\pi\omega_c^2 I_s)}, \quad (1)$$

where the coordinate z is measured from the input end of the gain medium, P_0 is the pump power at $z = 0$, σ is the emission cross section, τ is the emission lifetime, λ_p is the pump wavelength, α_p is the absorption coefficient, h is Planck's constant, c is the speed of light, ω_c is the waist radius of cavity mode, I_s is the saturation intensity ($I_s = h\nu/\sigma\tau$, where $h\nu$ is the energy of an oscillation photon).

If we ignore the variance of ω_c in the gain medium, from Eq. (1), the double-pass power gain K in the cavity can be approximately given as

$$K = \frac{4\sigma\tau\lambda_p P_0 [1 - \exp(-\alpha_p l)]}{hc\pi(\omega_p^2 + \omega_c^2)(1 + 4P_l/\pi\omega_c^2 I_s)}. \quad (2)$$

Treating the output power as a nonlinear loss, for an output mirror with transmission of T , the nonlinear loss L_n can be given as

$$L_n = \frac{P_{out}}{P_l} = T. \quad (3)$$

When operating in stable oscillation, the double-pass power gain K must equal the inner cavity loss (including linear loss L_c and nonlinear loss L_n), that is

$$K = L_c + L_n \quad (4)$$

thus we can get

$$\omega_c^4 - B\omega_c^2 + C = 0, \quad (5)$$

where

$$B = \frac{4\sigma\tau\lambda_p P_0 [1 - \exp(-\alpha_p l)]}{hc\pi(L_c + L_n)} - \frac{4P_l}{\pi I_s} - \omega_p^2, \quad (6)$$

$$C = \frac{4P_l \omega_p^2}{\pi I_s}. \quad (7)$$

Solving Eq. (5) yields the solution for ω_c

$$\omega_c = \left[\frac{B - \sqrt{B^2 - 4C}}{2} \right]^{1/2}. \quad (8)$$

The waist radius ω_c of the cavity mode is the function of the beam waist radius ω_0 :

$$\omega_c = \left[\omega_0^2 + \frac{\lambda^2 d^2}{\pi^2 \omega_0^2} \right]^{1/2}, \quad (9)$$

where d is measured from the position of beam waist, λ is the wavelength of the resonate wave.

From Eq. (9) we can get the function of ω_0 with ω_c :

$$\omega_0 = \left[\frac{\omega_c^2 + \sqrt{\omega_c^4 - ((4\lambda^2 d^2)/\pi^2)}}{2} \right]^{1/2}. \quad (10)$$

To determine ω_0 , the oscillation beam power P_b , the linear loss L_c and the waist radius ω_p of the pump beam must be known. For an output power P_{out} , P_l in the gain medium is $P_l = P_{out}/T$, where T is the transmission of the output mirror. The linear loss L_c can be calculated from $-\ln(1-T) = 2kP_{in,th} - L_c$ [7], where $P_{in,th}$ is the pump power at threshold, k is a pump coefficient. Changing T and measuring $P_{in,th}$, k and L_c can be directly determined. If the experimental setup is carefully adjusted, the waist radius ω_p of the pump beam approximately equals to the ideal intra-cavity laser mode (TEM₀₀) size ω_{00} .

For a stable cavity structure shown in Fig. 1, M_1 is the output mirror with the radius of curvature R_1 and M_2 is the reflective mirror with the radius of curvature R_2 . Compared to M_1 , the single propagation matrix is $m = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, where a, b, c, d are related to the thermal focal length f_T . The corresponding parameters g of the stable cavity are [8]

$$g_1 = a - \frac{b}{R_1}, \quad (11)$$

$$g_2 = d - \frac{d}{R_2}. \quad (12)$$

The relationship between the beam waist and parameters g is

$$\omega_0^2 = \frac{\lambda b}{\pi} \frac{\sqrt{g_1 g_2 (1 - g_1 g_2)}}{|g_2 + d^2 g_1 - 2d g_1 g_2|}. \quad (13)$$

Because g is the function of f_T , using the beam waist value determined with (10), we can obtain the value of thermal focal length in laser media.

The abovementioned principle can be used in any cavity structure. In our experiment, we use plane-concave resonator structure. Fig. 2 shows our experimental setup.

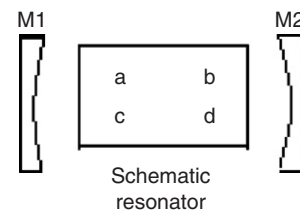


Fig. 1. Schematic of resonator.

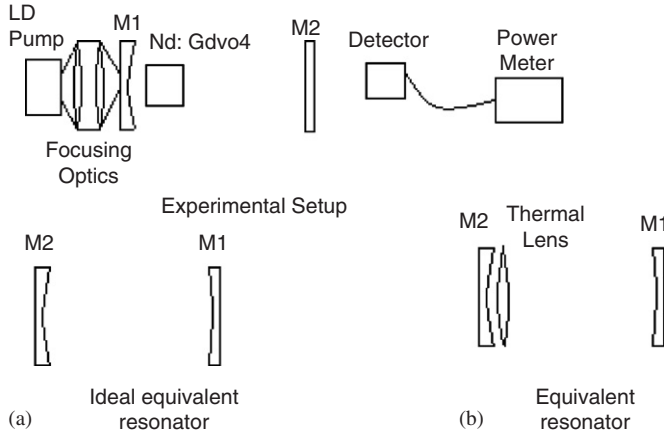


Fig. 2. Experimental setup; (a) ideal equivalent resonator, (b) equivalent resonator.

Fig. 2a is the equivalent ideal cavity without thermal effect and Fig. 2b is the equivalent resonator with thermal effect. According to Fig. 2a, the ideal intra-cavity laser mode (TEM_{00}) size ω_{00} can be expressed as

$$\omega_{00}^2 = \frac{\lambda}{\pi} [L(R - L)]^{1/2},$$

where R is the curvature radius of the concave reflector, L is length of resonator.

According to Fig. 2b, the cavity parameters g can be expressed by $g_1 = 1 - (L/f_T) - (L/R)$, $g_2 = 1$, $a = 1 - (L/f_T)$, $b = L$, $c = -1/f_T$, $d = 1$. Then according to (13)

$$\omega_0^2 = \frac{\lambda L}{\pi} \frac{\sqrt{(1 - (L/f_T) - (L/R))((L/f_T) + (L/R))}}{|1 + (1 - (L/f_T) - (L/R)) - 2(1 - (L/f_T) - (L/R))|}. \quad (14)$$

Thus, the thermal focus can be obtained by

$$f_T = \frac{1}{(\lambda^2 L / (\pi^2 \omega_0^4 + \lambda^2 L^2) - 1/R)}. \quad (15)$$

3. The experimental results and discussion

Fig. 2 shows our experimental setup. The pump source is a fiber-coupled laser-diode (made by Coherent Inc. USA, maximum output power 16 W), which works at the maximum absorption wavelength (808 nm) of the Nd:GdVO₄ crystal. The mirror M_1 with 150-mm curvature radius (R) is high-antireflection-coated at 808 nm and high-reflection-coated at 1064 nm. The flat mirror M_2 is an output mirror with transmission of 10%. The a-cut Nd:GdVO₄ crystal doped with 1.0 at% Nd³⁺ ions is $4 \times 4 \times 5 \text{ mm}^3$ and its absorption coefficient at 808 nm is 74 cm^{-1} . Its front surface is antireflection-coated at 808 nm and its rear surface is high-antireflection-coated at 1064 nm. It is placed near M_1 . The Nd:GdVO₄ crystal is wrapped in indium foil and held in a copper block, which is cooled by semiconductor coolers. The distance L between

M_1 and M_2 is about 82 mm. The pump laser was focused with an optical coupling system that was carefully adjusted so that the diode-laser emitting areas could be matched to the intracavity laser mode (TEM_{00}) size. A detector is connected with a power meter and is placed behind the output mirror to measure the power of output laser beam. Some relative parameters used in calculation for this setup are $L_c = 0.04$, $T = 10\%$, $\omega_{00} = 0.159 \text{ mm}$.

For a LD end-pumped Nd:GdVO₄ CW solid-state laser, the output beam power versus the diode-laser pump power is shown in Fig. 3. The stars are the experimental results and the real line is the fit curve. From the figure, we can see that the output beam power increase approximately linearly with the pump power.

Fig. 4 gives the beam waist according to (10) under different pump power, where the stars are the experimental results and the real line is its fit curve. It illustrates that the beam waist has a polynomial increase in second order with the pump power.

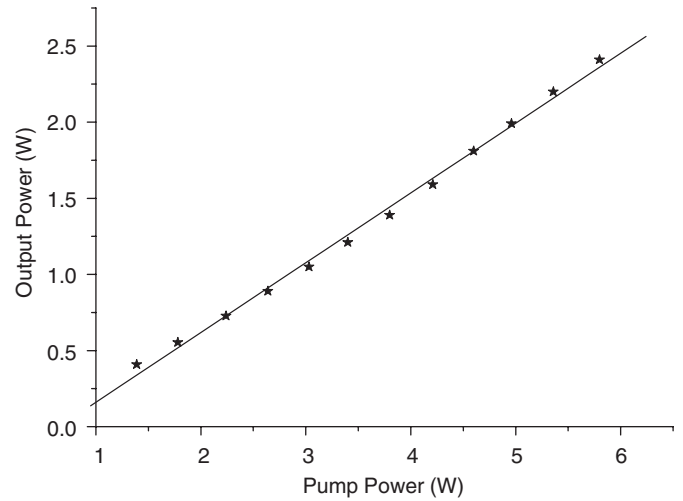


Fig. 3. Output power dependence on pump power.

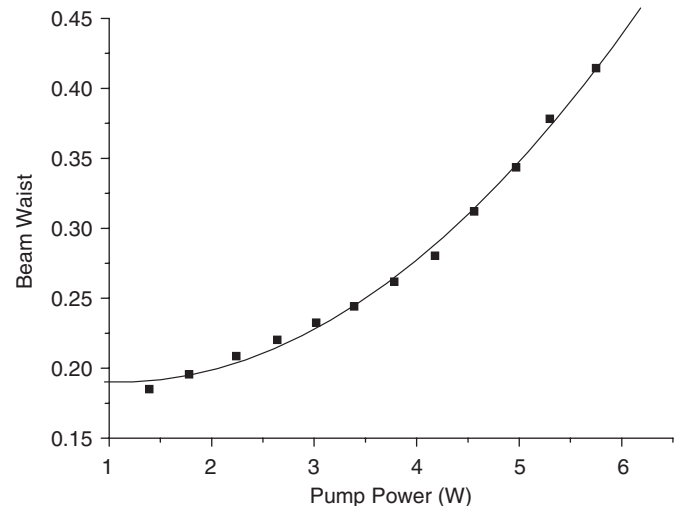


Fig. 4. Beam waist dependence on pump power.

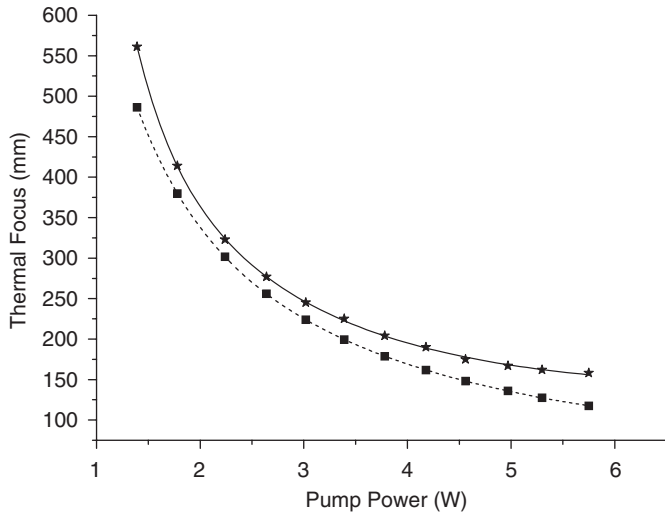


Fig. 5. Thermal focus dependence on pump power.

Thermal focus at different pump power in CW laser operation is shown in Fig. 5, where the stars are the experimental values with its fit curve of the solid line. From Fig. 5, we can see that the thermal focus decrease exponentially with the pump power. It means the higher the pump power is, the greater the thermal effect is.

For the laser crystal, if the thermal birefringent effect is neglected and the thermal-induced refractive index change and the thermal-induced stresses or strains are considered, the thermal focal length can be theoretically given as [9]

$$f_T = \frac{2\pi K_c}{\beta + n\alpha_T} \frac{\omega_p^2}{\xi P_{in} \exp(-\alpha l)}, \quad (16)$$

where K_c , $\beta = dn/dT$ and α_T are the thermal conductivity, thermal-optics coefficient and thermal expansion coefficient of the crystal, respectively, n is the refraction coefficient, ξ the thermal load ratio, ω_p is the efficient pump radius of the pump laser in crystal. The corresponding values of Nd:GdVO₄ is $K_c = 11.7 \text{ W/mK}$, $\beta = 4.7 \times 10^{-6} \text{ K}^{-1}$, $\alpha_T = 3.5 \times 10^{-6} \text{ K}^{-1}$, $n = 1.972$ and $\xi = 0.24$. The dots in Fig. 5 are the theoretical calculation results and the short dash line is its fit curve.

From Fig. 5 we can see, compared with the theoretical results, the experimental values are larger than the theoretical one. This may have resulted from the following factors:

- (1) The influence of the finite size of crystal end face has not been considered in the above theoretical calculation. It shows that the rigorous calculation results will be small in comparison with the above results.
- (2) The gain medium is rectangle, but the circle-symmetric condition is assumed in the above model. The measured results are relatively high due to the larger surrounding

area, which results in low-temperature difference distribution in the crystal.

If taking into account the above factors, we note that the experimental results are consistent with the theoretical one.

4. Conclusion

We have presented a simple method to determine the thermal focal length of the gain medium for end-pumped solid-state laser with stable operation. With rate equation of CW laser, the beam waist of oscillation wave can be accurately determined by knowing the output power and the thermal focal length of gain medium can be easily obtained with the standard matrix theory of stable cavity. This method has the advantages of simple setup and easy operation. The most important is that it can be used in any stable operation condition with accurate results. It is an effective and direct way to investigate the thermal effect of end-pumped solid-state lasers. Our experimental results of LD end-pumped Nd:GdVO₄ solid-state lasers are in agreement with the theoretical estimation.

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