Diffraction, Aberrations, and Image Quality

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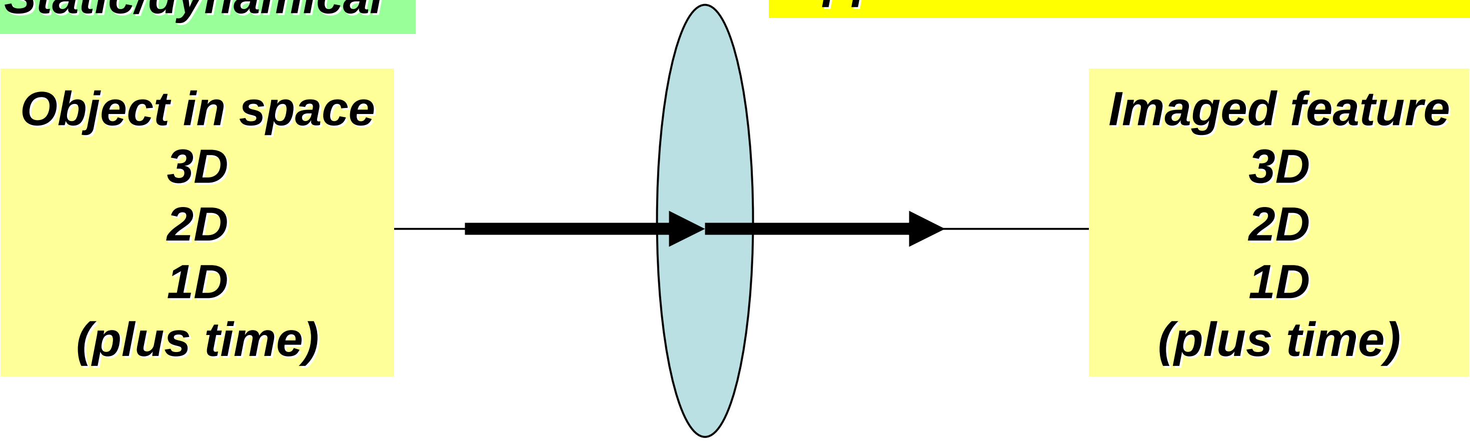
Textbook:
Robert E. Fischer, Biljana Tadic-Galeb, Paul R. Yoder
Optical System Design 2nd, ed. (SPIE Press, 2008)
Chapter 3

What image quality is all about

Static/dynamical

Application sets the criterion on quality

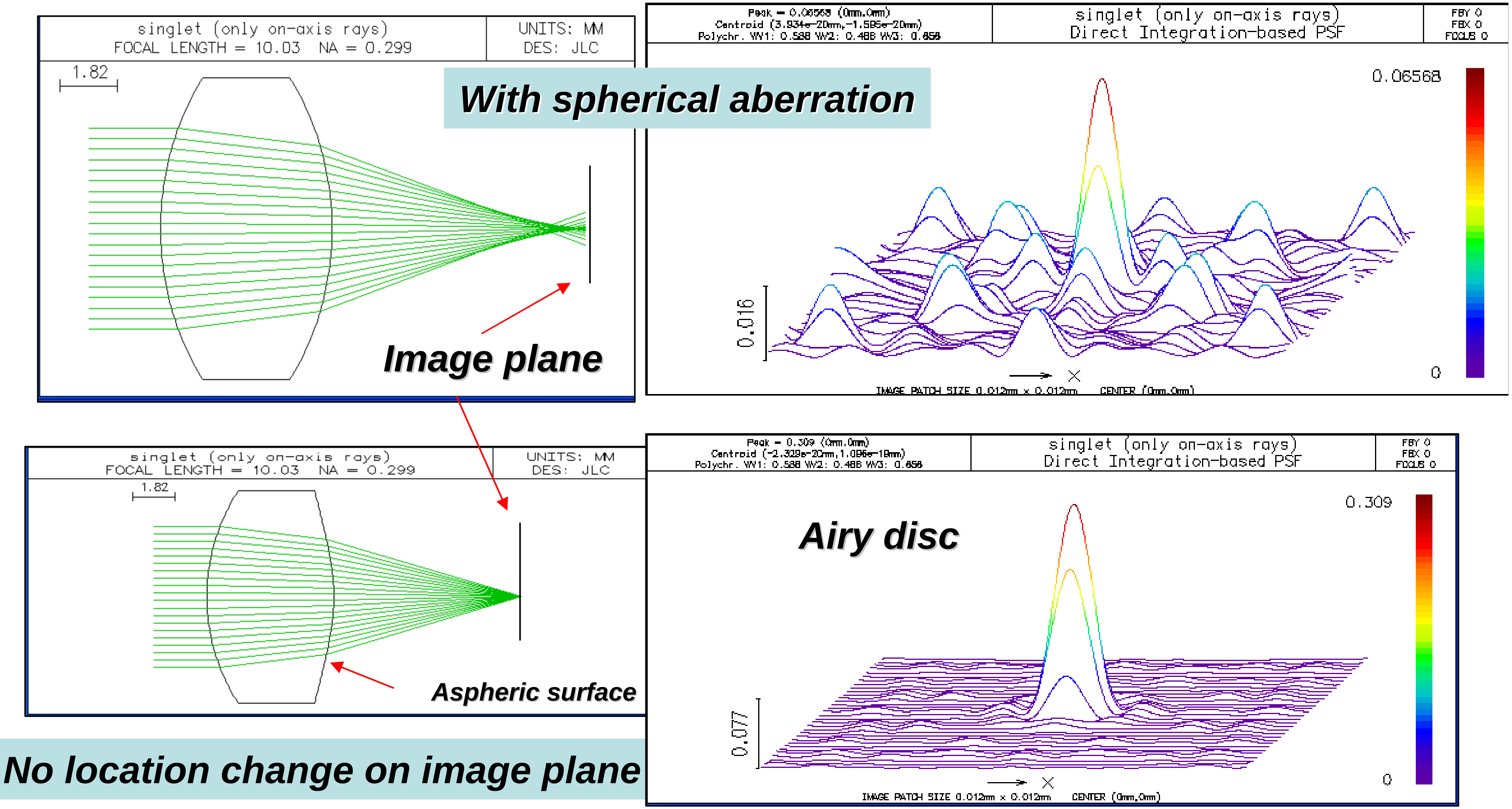


Object in space
3D
2D
1D
(plus time)

Imaged feature
3D
2D
1D
(plus time)

Object to image	**Concerns/ applications**	**Object to image**	**Concerns/ applications**	**Object to image**	**Concerns/ applications**
3D -> 3D	Virtual image Virtual reality	3D -> 2D	Extended depth of field (imaging)	3D-> 1D	Concentration
2D -> 3D	3D projection	**2D -> 2D**	**Imaging**	2D-> 1D	Image compression
1D -> 3D	Dynamical 3D projection Virtual reality	1D -> 2D	Dynamical projection Laser projection Scanning	**1D-> 1D**	**Point to point** Point to line Line to point **Line to line**

Numerical illustration (point to point)



With spherical aberration

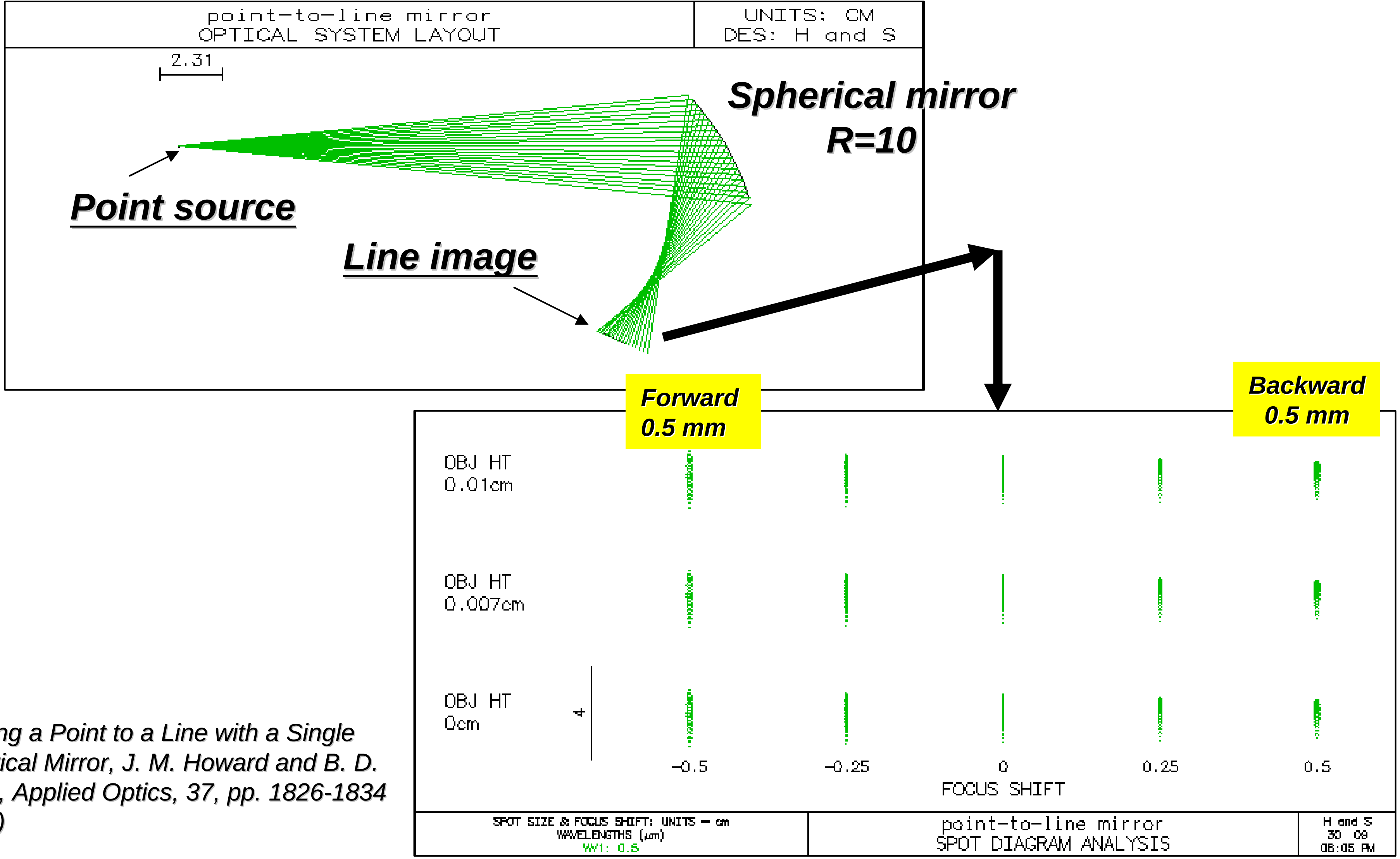
Image plane

Airy disc

Aspheric surface

No location change on image plane

Numerical illustration: point to line



Spherical mirror
R=10

Point source

Line image

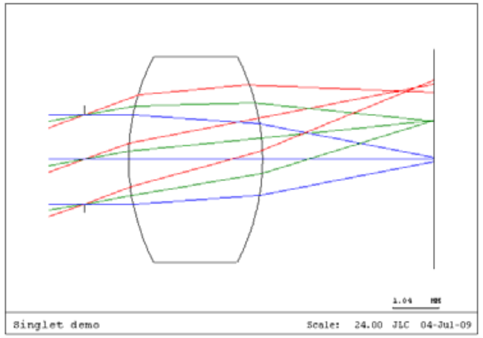
Forward 0.5 mm

Backward 0.5 mm

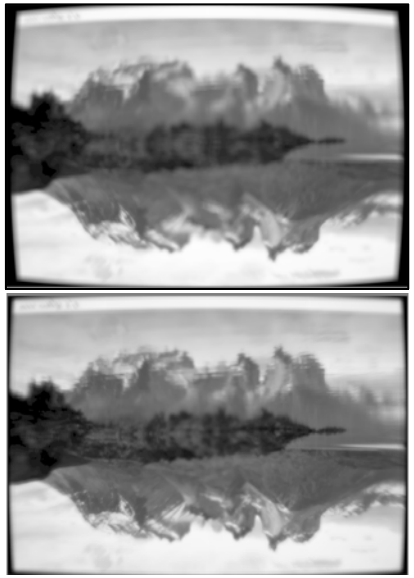
Imaging a Point to a Line with a Single Spherical Mirror, J. M. Howard and B. D. Stone, Applied Optics, 37, pp. 1826-1834 (1998)

Numerical illustration: plane to plane

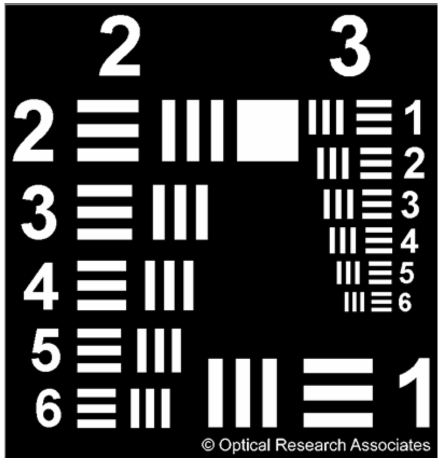
Object

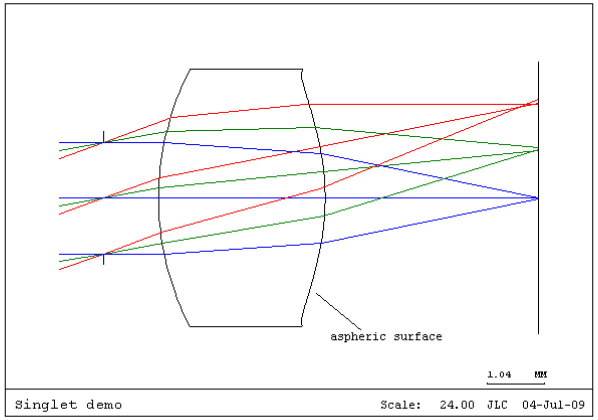


Image

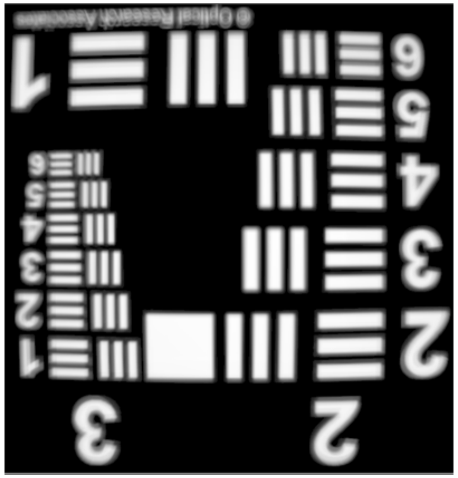


Object

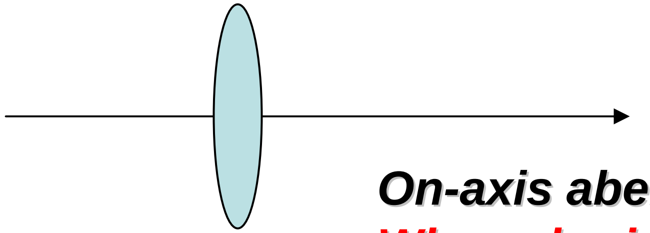




Image



What are geometrical aberrations and where do they come from?



On-axis aberration → spherical aberration

Why spherical surface?

Manufacturing spherical lenses may be probably the simplest

Above two photos are from
http://www.fdtimes.com/articles/cooke/Cooke%20Book6_Web150dpi.pdf

Geometrical aberration

Violation of perfect imaging → aberration

Perfect imaging:

1. Point to point → On-axis: Spherical aberration; Off-axis: Coma, Astigmatism
2. Line to line — Field curvature
3. Surface to surface — Distortion

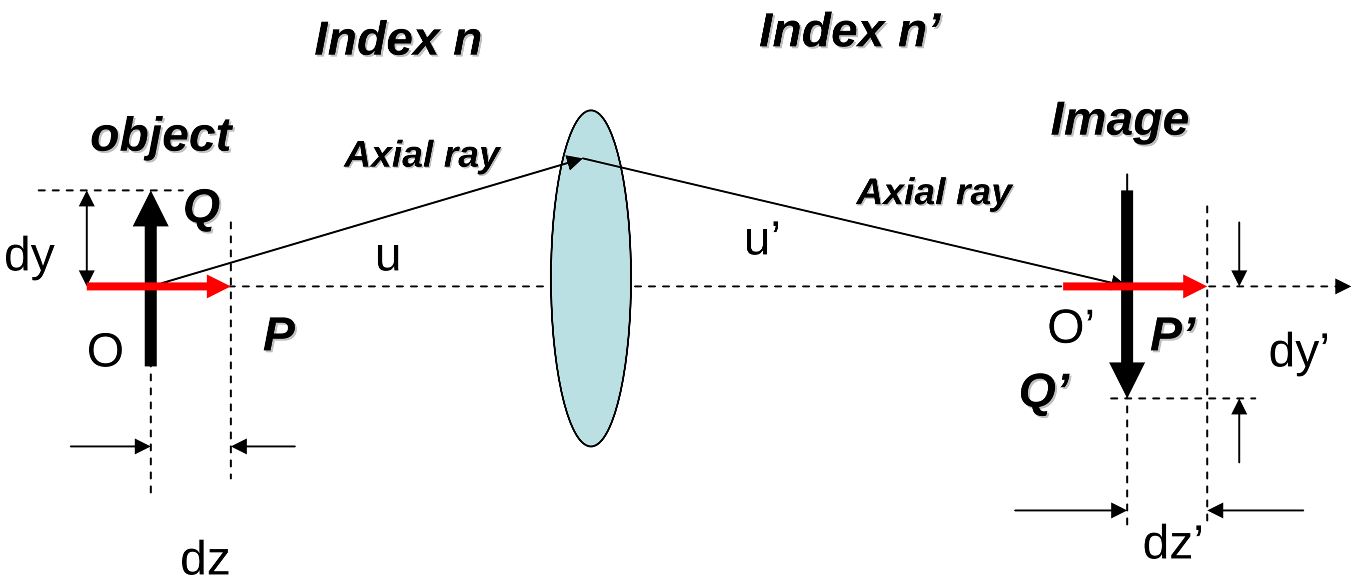
Maxwell's Perfect lens (imaging): Stigmatic

	Statement	Violation	Nature of imaging
1	All the rays from object point O after passing through the lens must go through the image point O'	Image degradation (image aberration)	Point to point Spherical aberration Coma Astigmatism
2	Every segment of the object plane normal to the optical axis that contains the point O must be imaged as a segment of a plane normal to the optical axis which contains O'	Image curvature (field curvature)	Line to line Field curvature
3	The image height h' must be a constant multiple of the object height h no matter where O is located in the object plane	Image distortion	Surface to surface Distortion

Condition of perfect imaging:

Herschel condition

$n*dz*\sin^2(u/2)=n'*dz*'\sin^2(u'/2)$



Abbe sine condition

$n*dy*\sin(u)=n'*dy'*\sin(u')$

Violation of the Abbe sine condition

n*dy*sin(u)=n'*dy'*sin(u')

We can set the criterion as the following D1 or D2 to judge the derivation

A1=n*dy*sin(u)-n'*dy*sin(u')

A2=n*dy*sin(u)/n'*dy*sin(u')-1

Magnification M=dy'/dy is fixed

Numerical illustration

OSC Operand

OSLO Premium/OSLO Standard

Offense against the sine condition (OSC) is a dimensionless measure of the total amount of linear coma present in a lens. OSC only takes linear coma into account, i.e., coma that is proportional to the first power of the field coordinate; it is an exact measure in the aperture coordinate. The following definition of OSC is valid for rotationally symmetric systems.

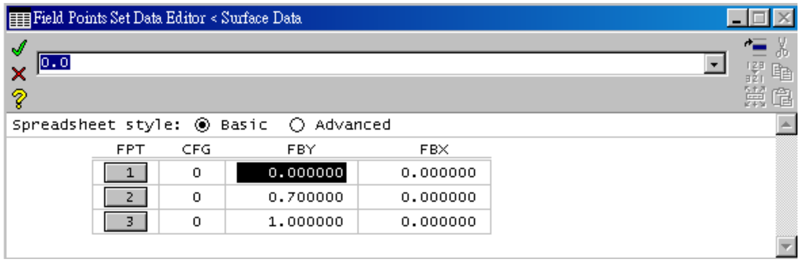
$$\mathrm{OSC} = \frac{\sin U}{u} \cdot \frac{u'}{\sin U'} \cdot \frac{l' - \bar{l}'}{L' - \bar{l}'} - 1$$

osc(fpt, ray, wvn, cfg) offense against the sine condition (use for meridional rays and on-axis field point for rotationally-symmetric systems only)

In the above equation, u and u?/i> are the initial and final slopes of the paraxial ray, U and U?/i> are the initial and final convergence angles of the real ray, l?/i> and L?/i> are the distances from the last lens surface to the intersections of the paraxial and real rays, respectively, with the optical axis, and $\bar{l}'$ *is the distance from the last lens surface to the exit pupil.*

In the absence of spherical aberration (l?/i> = L?/i>), zero OSC is seen to be equivalent to satisfying the Abbe sine condition. Also, when spherical aberration is present, OSC is a function of aperture stop position (through the $\bar{l}'$ *term), as would be expected from the Seidel aberration stop shift equations.*

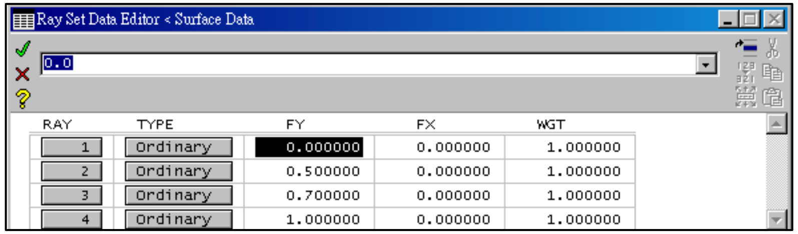
More details on OSC and derivations of the above equation may be found in Smith, Kingslake, and Welford.



0.0

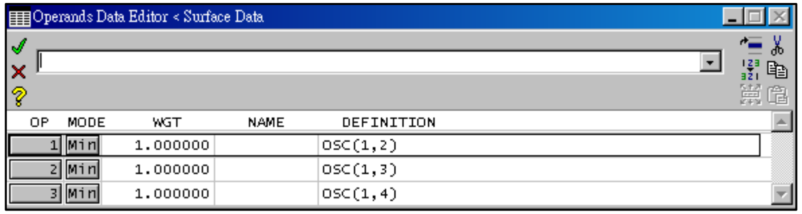
Spreadsheet style: ● Basic ○ Advanced

FPT	CFG	FBY	FBX
1	0	0.000000	0.000000
2	0	0.700000	0.000000
3	0	1.000000	0.000000



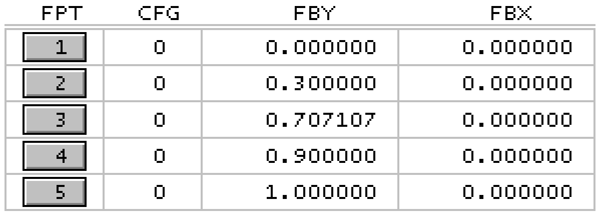
0.0

RAY	TYPE	FY	FX	WGT
1	Ordinary	0.000000	0.000000	1.000000
2	Ordinary	0.500000	0.000000	1.000000
3	Ordinary	0.700000	0.000000	1.000000
4	Ordinary	1.000000	0.000000	1.000000

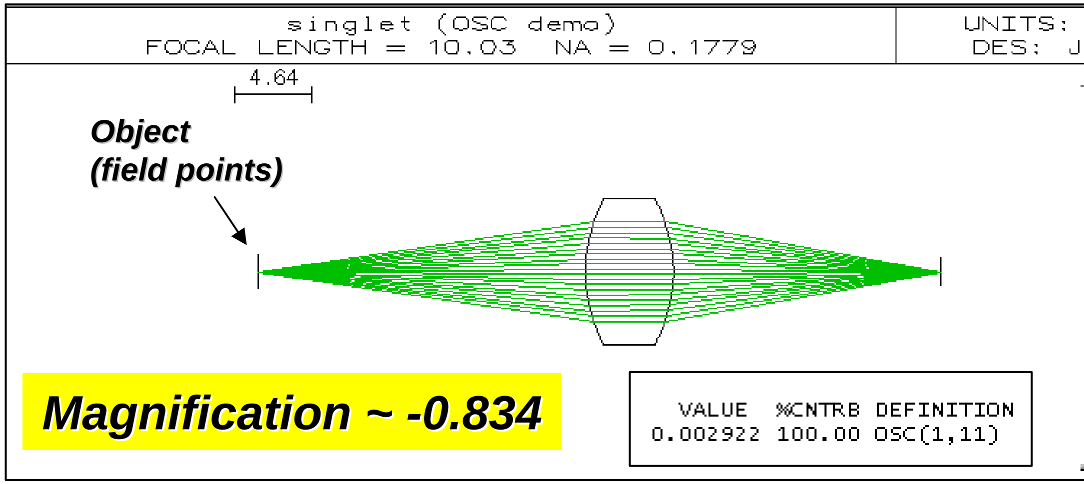


OP	MODE	WGT	NAME	DEFINITION
1	Min	1.000000		OSC(1,2)
2	Min	1.000000		OSC(1,3)
3	Min	1.000000		OSC(1,4)

Illustration



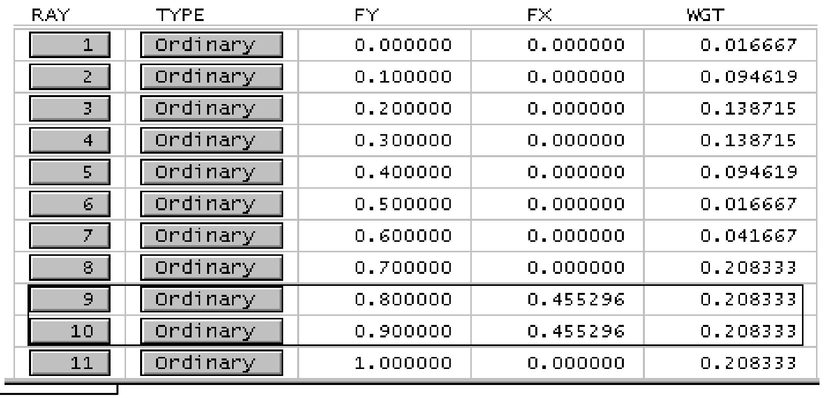
FPT	CFG	FBY	FBX
1	0	0.000000	0.000000
2	0	0.300000	0.000000
3	0	0.707107	0.000000
4	0	0.900000	0.000000
5	0	1.000000	0.000000



Object (field points)

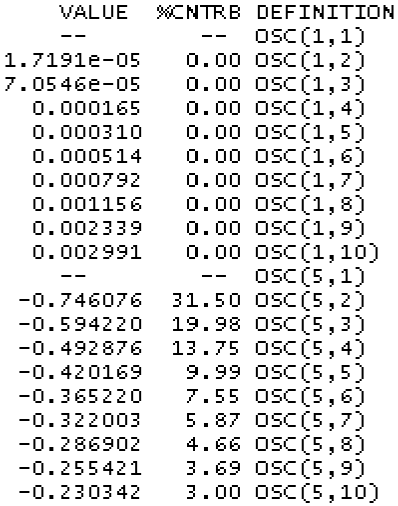
Magnification ~ -0.834

```
  VALUE    %CNTRB DEFINITION
0.002922  100.00 OSC(1,11)
```



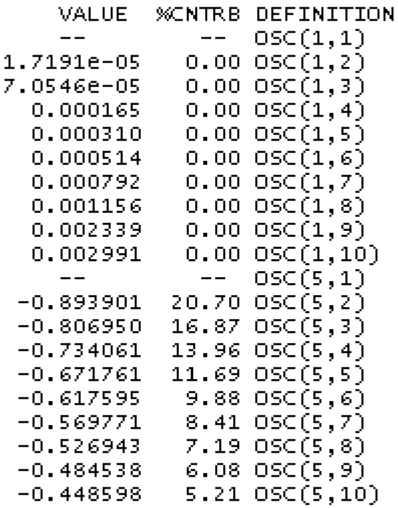
RAY	TYPE	FY	FX	WGT
1	Ordinary	0.000000	0.000000	0.016667
2	Ordinary	0.100000	0.000000	0.094619
3	Ordinary	0.200000	0.000000	0.138715
4	Ordinary	0.300000	0.000000	0.138715
5	Ordinary	0.400000	0.000000	0.094619
6	Ordinary	0.500000	0.000000	0.016667
7	Ordinary	0.600000	0.000000	0.041667
8	Ordinary	0.700000	0.000000	0.208333
9	Ordinary	0.800000	0.455296	0.208333
10	Ordinary	0.900000	0.455296	0.208333
11	Ordinary	1.000000	0.000000	0.208333

Object height 1.0 mm



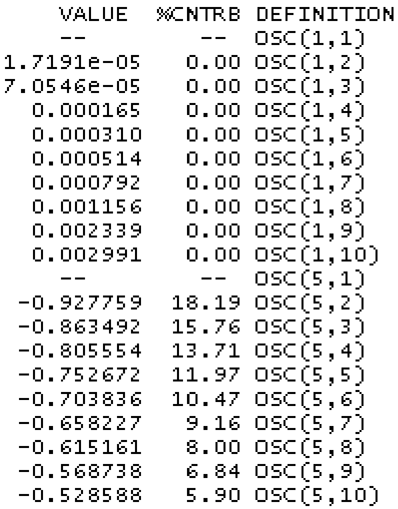
```
      VALUE    %CNTRB DEFINITION
         --        -- OSC(1,1)
 1.7191e-05      0.00 OSC(1,2)
 7.0546e-05      0.00 OSC(1,3)
   0.000165      0.00 OSC(1,4)
   0.000310      0.00 OSC(1,5)
   0.000514      0.00 OSC(1,6)
   0.000792      0.00 OSC(1,7)
   0.001156      0.00 OSC(1,8)
   0.002339      0.00 OSC(1,9)
   0.002991      0.00 OSC(1,10)
         --        -- OSC(5,1)
  -0.746076     31.50 OSC(5,2)
  -0.594220     19.98 OSC(5,3)
  -0.492876     13.75 OSC(5,4)
  -0.420169      9.99 OSC(5,5)
  -0.365220      7.55 OSC(5,6)
  -0.322003      5.87 OSC(5,7)
  -0.286902      4.66 OSC(5,8)
  -0.255421      3.69 OSC(5,9)
  -0.230342      3.00 OSC(5,10)
```

Object height 3.0 mm



```
      VALUE    %CNTRB DEFINITION
         --        -- OSC(1,1)
 1.7191e-05      0.00 OSC(1,2)
 7.0546e-05      0.00 OSC(1,3)
   0.000165      0.00 OSC(1,4)
   0.000310      0.00 OSC(1,5)
   0.000514      0.00 OSC(1,6)
   0.000792      0.00 OSC(1,7)
   0.001156      0.00 OSC(1,8)
   0.002339      0.00 OSC(1,9)
   0.002991      0.00 OSC(1,10)
         --        -- OSC(5,1)
  -0.893901     20.70 OSC(5,2)
  -0.806950     16.87 OSC(5,3)
  -0.734061     13.96 OSC(5,4)
  -0.671761     11.69 OSC(5,5)
  -0.617595      9.88 OSC(5,6)
  -0.569771      8.41 OSC(5,7)
  -0.526943      7.19 OSC(5,8)
  -0.484538      6.08 OSC(5,9)
  -0.448598      5.21 OSC(5,10)
```

Object height 5.0 mm



```
      VALUE    %CNTRB DEFINITION
         --        -- OSC(1,1)
 1.7191e-05      0.00 OSC(1,2)
 7.0546e-05      0.00 OSC(1,3)
   0.000165      0.00 OSC(1,4)
   0.000310      0.00 OSC(1,5)
   0.000514      0.00 OSC(1,6)
   0.000792      0.00 OSC(1,7)
   0.001156      0.00 OSC(1,8)
   0.002339      0.00 OSC(1,9)
   0.002991      0.00 OSC(1,10)
         --        -- OSC(5,1)
  -0.927759     18.19 OSC(5,2)
  -0.863492     15.76 OSC(5,3)
  -0.805554     13.71 OSC(5,4)
  -0.752672     11.97 OSC(5,5)
  -0.703836     10.47 OSC(5,6)
  -0.658227      9.16 OSC(5,7)
  -0.615161      8.00 OSC(5,8)
  -0.568738      6.84 OSC(5,9)
  -0.528588      5.90 OSC(5,10)
```

Violation of the Herchel condition

Herschel condition
$n*dz*\sin^2(u/2)=n'*dz*'\sin^2(u'/2)$

We can set the criterion as the following D1 or D2 to judge the derivation

$$H1= n*dz*\sin^2(u/2)-n'*dz'*\sin^2(u'/2)$$

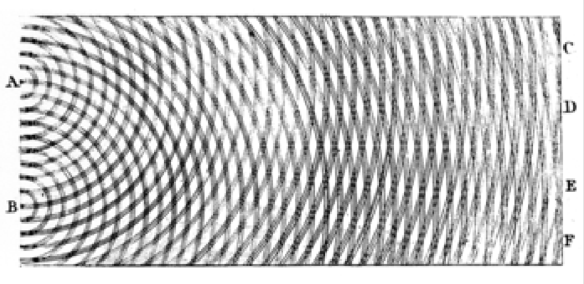
$$H2= n*dz*\sin^2(u/2)/n'*dz'*\sin^2(u'/2)-1$$
$$=(n/n')(1/M_z)(\sin^2(u/2)/\sin^2(u'/2))$$

Magnification $M_z=dz'/dz$ is fixed
n and n' are also fixed (given)

What is diffraction?

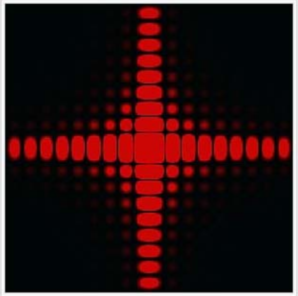
"Diffraction is a phenomenon or effect resulting from the interaction of light with the sharp limiting edge or aperture of an optical system."

Historical (Young's double slit)



Thomas Young's sketch of two-slit diffraction, which he presented to the Royal Society in 1803

Square aperture



The intensity pattern formed on a screen by diffraction from a square aperture

Star (Circular aperture) and Airy disc

The Airy disk around each of the stars from the 2.56m telescope aperture can be seen in this *lucky image* of the binary star zeta Boötis.

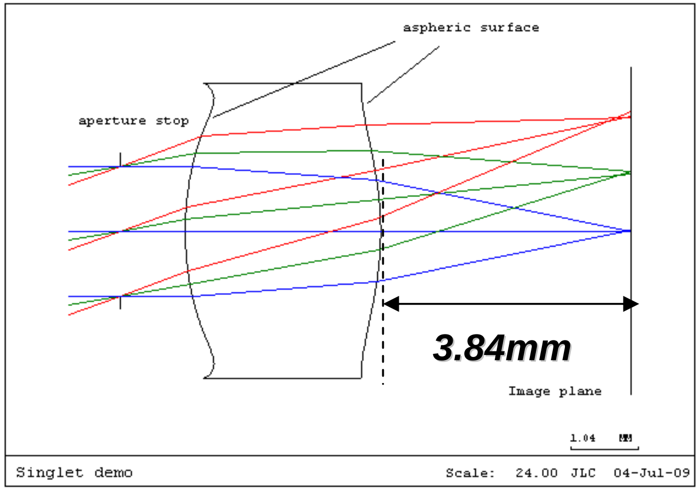
Solar glory at the steam from hot springs. A **glory** is an optical phenomenon produced by light backscattered (a combination of diffraction, reflection and refraction) towards its source by a cloud of uniformly-sized water droplets.

Colors seen in a spider web are partially due to diffraction, according to some analyses.[1]

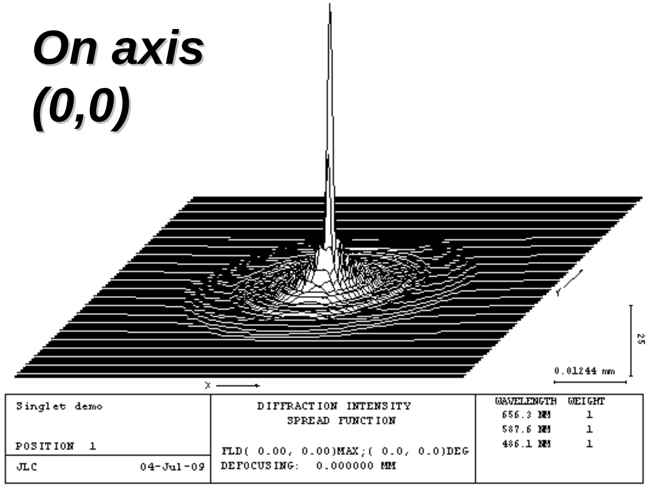
Photograph of single-slit diffraction in a circular ripple tank

More illustrations can be found at http://en.wikipedia.org/wiki/Diffraction
Above six photos are taken from there

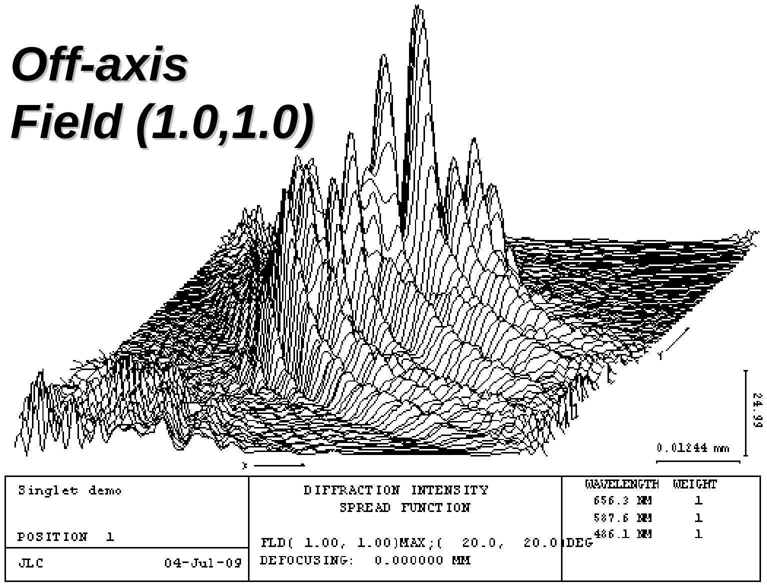
Numerical illustration – on image plane



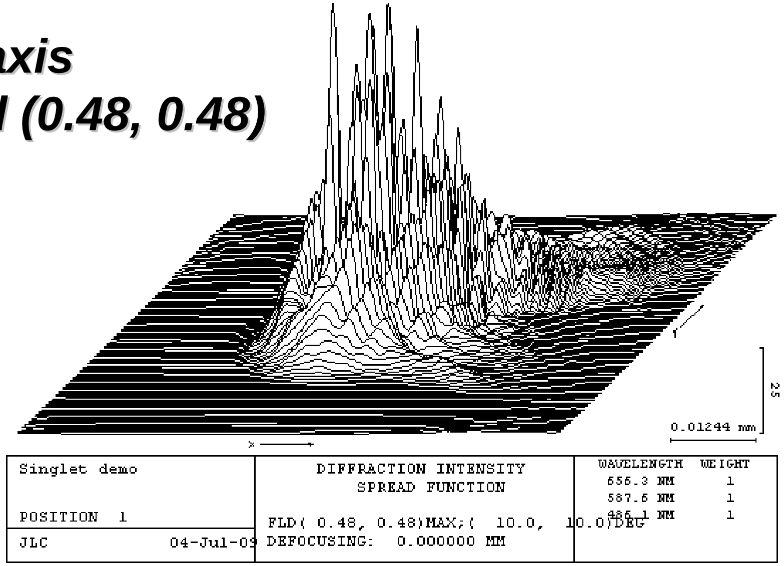
On axis
(0,0)



Off-axis
Field (1.0,1.0)

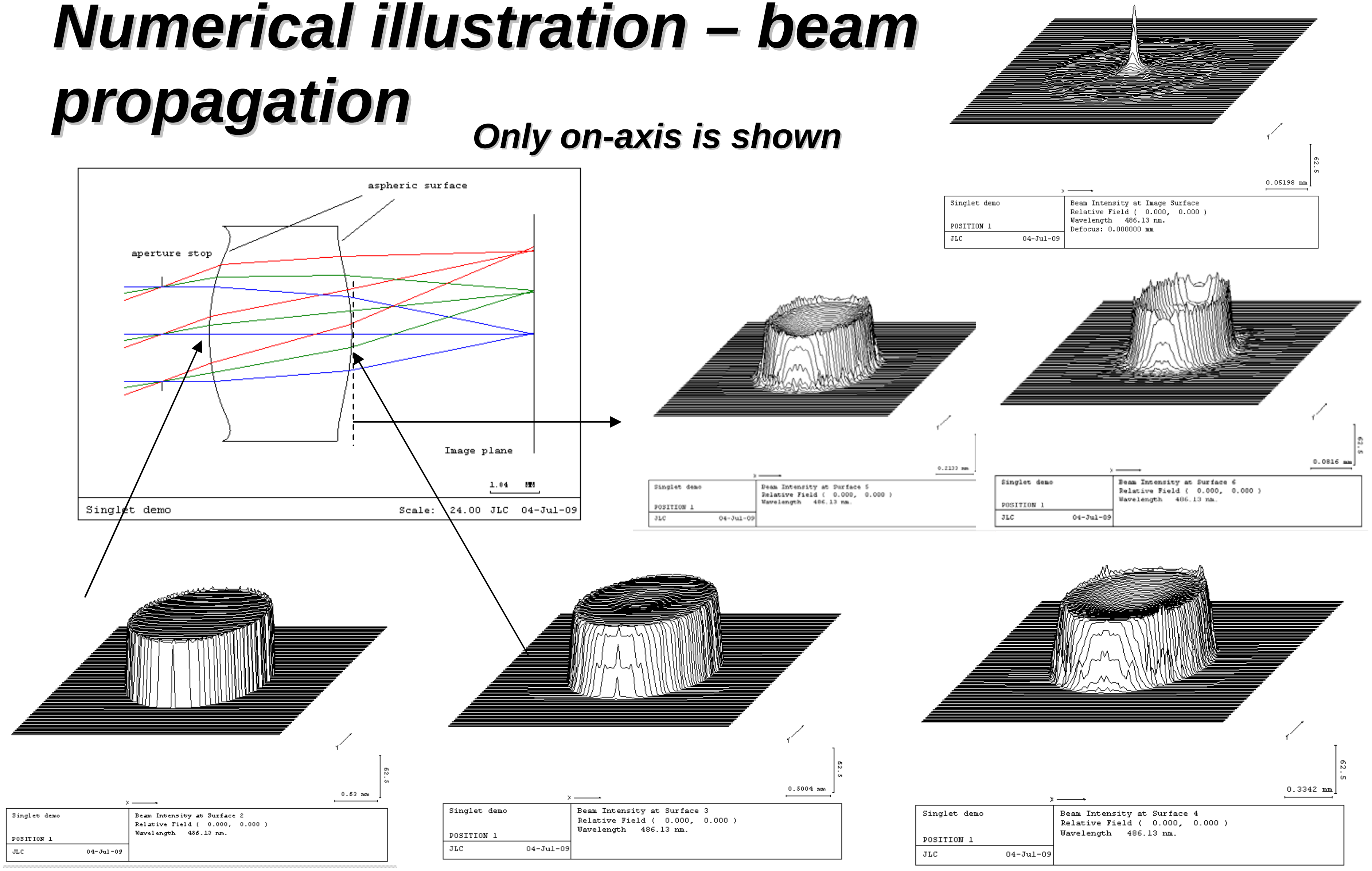


Off-axis
Field (0.48, 0.48)



Numerical illustration – beam propagation

Only on-axis is shown



Diffraction-limited performance

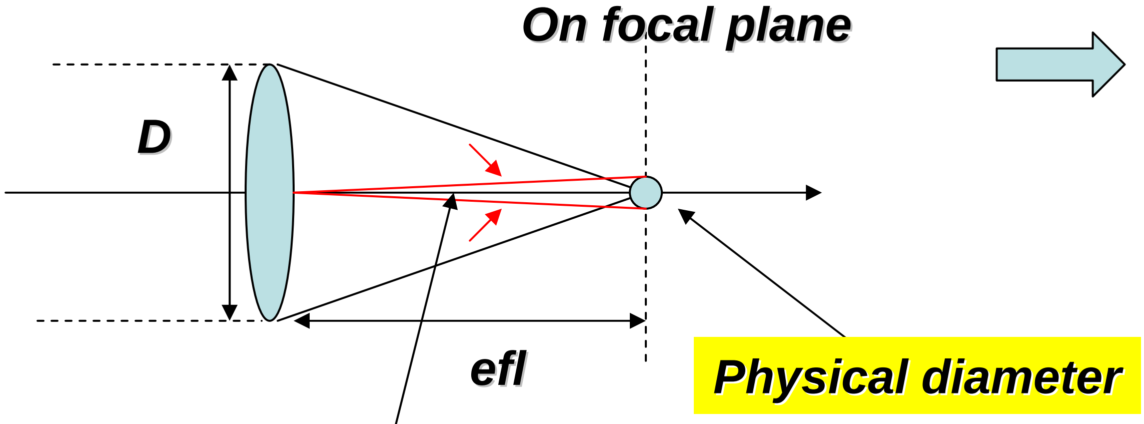
In the case of "point to point"

Physical diameter of the Airy disc=2.44* λ * f/#

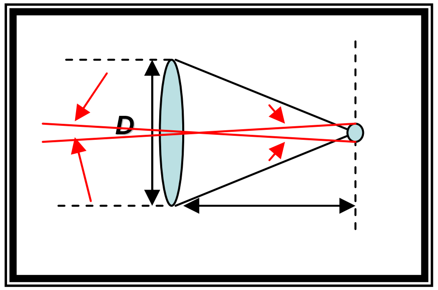
f/#=efl/D

Visible light λ =0.5 um

=1.22* f/# (um)
= 1.22 efl/D (um)



Angular diameter



Angular diameter of the Airy disk
=Physical diameter/efl~ θ
(in the unit of rad or mrad)
~ 2.44* λ /D ~ 1.22/D (for visible)

Derivation of system specifications

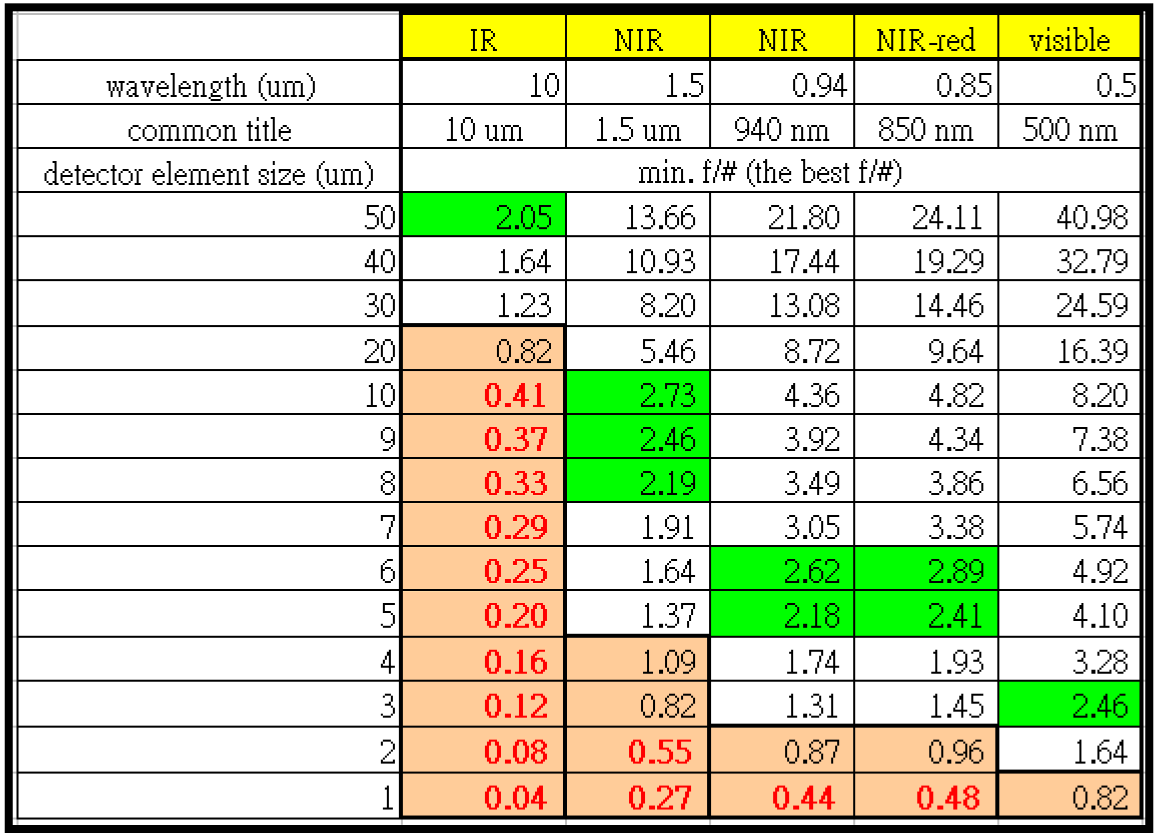
- *"Deriving the basic optical system parameter based on the functional system performance requirements" (p. 45 of textbook)*

Example: LWIR (long-wave infrared: 8-12 um spectral band)
Task: what is the best f/# and clear aperture diameter

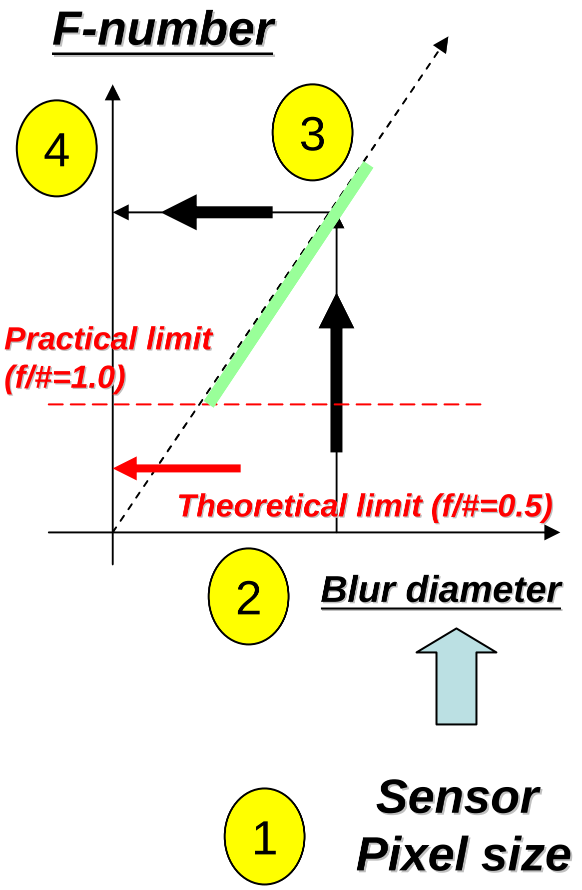
1. *To determine the best f/# (min. f/#)*
   Physical diameter of the Airy disc=2.44 λ * f/#= limited by sensor pixel*
   Wavelength is given at 10 um, hence
   *50 um=2.44*10 um*f/# -> f/#=5/2.44~2.05~2.0*

2. *To resolve an object with 0.25 mrad, we have*
   0.25 mrad= 2.44 λ /D=2.44*10 um/D*
   Hence, we can determine the clear aperture D
   → D=24.4 um/0.25 mrd ~97.6 mm ~ 100 mm

Numerical illustration



	IR	NIR	NIR	NIR-red	visible
wavelength (um)	10	1.5	0.94	0.85	0.5
common title	10 um	1.5 um	940 nm	850 nm	500 nm
detector element size (um)	min. f/# (the best f/#)				
50	2.05	13.66	21.80	24.11	40.98
40	1.64	10.93	17.44	19.29	32.79
30	1.23	8.20	13.08	14.46	24.59
20	0.82	5.46	8.72	9.64	16.39
10	0.41	2.73	4.36	4.82	8.20
9	0.37	2.46	3.92	4.34	7.38
8	0.33	2.19	3.49	3.86	6.56
7	0.29	1.91	3.05	3.38	5.74
6	0.25	1.64	2.62	2.89	4.92
5	0.20	1.37	2.18	2.41	4.10
4	0.16	1.09	1.74	1.93	3.28
3	0.12	0.82	1.31	1.45	2.46
2	0.08	0.55	0.87	0.96	1.64
1	0.04	0.27	0.44	0.48	0.82



F-number

4

3

Practical limit (f/#=1.0)

Theoretical limit (f/#=0.5)

2

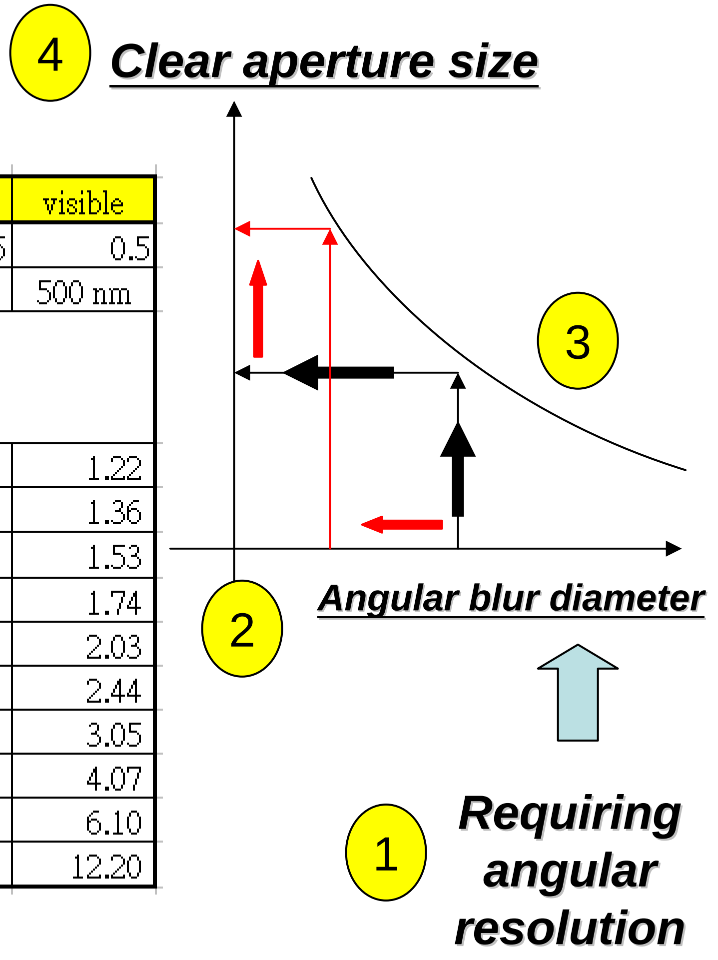
Blur diameter

1

Sensor Pixel size

Numerical illustration

suitable for astronomy application		IR	NIR	NIR	NIR-red	visible
wavelength (um)		10	1.5	0.94	0.85	0.5
common title		10 um	1.5 um	940 nm	850 nm	500 nm
observing object size in mrad	Field of view (degree)	Clear aperure size (mm)				
1	0.057	24.40	3.66	2.29	2.07	1.22
0.9	0.052	27.11	4.07	2.55	2.30	1.36
0.8	0.046	30.50	4.58	2.87	2.59	1.53
0.7	0.040	34.86	5.23	3.28	2.96	1.74
0.6	0.034	40.67	6.10	3.82	3.46	2.03
0.5	0.029	48.80	7.32	4.59	4.15	2.44
0.4	0.023	61.00	9.15	5.73	5.19	3.05
0.3	0.017	81.33	12.20	7.65	6.91	4.07
0.2	0.011	122.00	18.30	11.47	10.37	6.10
0.1	0.006	244.00	36.60	22.94	20.74	12.20



④ ***Clear aperture size***

③

② ***Angular blur diameter***

① ***Requiring angular resolution***

More information about infrared astronomy can be found in http://en.wikipedia.org/wiki/Infrared_astronomy

Homework 1

- Design a mirror system to project a line source to a rectangular image

Homework 2

- How the degree of violation of the Abbe sine condition could be identified?
  - This is so called OSC (offense sine condition) in the operands of optimization for perfect imaging of an 2D object
- How the degree of violation of the Herschel condition could be identified?