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INTRODUCTION TO OPTICAL WAVEGUIDE ANALYSIS

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Solving Maxwell's Equations and the Schrödinger Equation

KENJI KAWANO and TSUTOMU KITOH

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To our wives,
Mariko and Kumiko

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PREFACE

This book was originally published in Japanese in October 1998 with the intention of providing a straightforward presentation of the sophisticated techniques used in optical waveguide analyses. Apparently, we were successful because the Japanese version has been well accepted by students in undergraduate, postgraduate, and Ph.D. courses as well as by researchers at universities and colleges and by researchers and engineers in the private sector of the optoelectronics field. Since we did not want to change the fundamental presentation of the original, this English version is, except for the newly added optical fiber analyses and problems, essentially a direct translation of the Japanese version.

Optical waveguide devices already play important roles in telecommunications systems, and their importance will certainly grow in the future. People considering which computer programs to use when designing optical waveguide devices have two choices: develop their own or use those available on the market. A thorough understanding of optical waveguide analysis is, of course, indispensable if we are to develop our own programs. And computer-aided design (CAD) software for optical waveguides is available on the market. The CAD software can be used more effectively by designers who understand the features of each analysis method. Furthermore, an understanding of the wave equations and how they are solved helps us understand the optical waveguides themselves.

Since each analysis method has its own features, different methods are required for different targets. Thus, several kinds of analysis methods have to be mastered. Writing formal programs based on equations is risky unless one knows the approximations used in deriving those equations, the errors due to those approximations, and the stability of the solutions.

Mastering several kinds of analysis techniques in a short time is difficult not only for beginners but also for busy researchers and engineers. Indeed, it was when we found ourselves devoting substantial effort to mastering various analysis techniques while at the same time designing, fabricating, and measuring optical waveguide devices that we saw the need for an easy-to-understand presentation of analysis techniques.

This book is intended to guide the reader to a comprehensive understanding of optical waveguide analyses through self-study. It is important to note that the intermediate processes in the mathematical manipulations have not been omitted. The manipulations presented here are very detailed so that they can be easily understood by readers who are not familiar with them. Furthermore, the errors and stabilities of the solutions are discussed as clearly and concisely as possible. Someone using this book as a reference should be able to understand the papers in the field, develop programs, and even improve the conventional optical waveguide theories.

Which optical waveguide analyses should be mastered is also an important consideration. Methods touted as superior have sometimes proven to be inadequate with regard to their accuracy, the stability of their solutions, and central processing unit (CPU) time they require. The methods discussed in this book are ones widely accepted around the world. Using them, we have developed programs we use on a daily basis in our laboratories and confirmed their accuracy, stability, and effectiveness in terms of CPU time.

This book treats both analytical methods and numerical methods. Chapter 1 summarizes Maxwell's equations, vectorial wave equations, and the boundary conditions for electromagnetic fields. Chapter 2 discusses the analysis of a three-layer slab optical waveguide, the effective index method, Marcatili's method, and the analysis of an optical fiber. Chapter 3 explains the widely utilized scalar finite-element method. It first discusses its basic theory and then derives the matrix elements in the eigenvalue equation and explains how their calculation can be programmed. Chapter 4 discusses the semivectorial finite-difference method. It derives the fully vectorial and semivectorial wave equations, discusses their relations, and then derives explicit expressions for the quasi-TE and quasi-TM modes. It shows formulations of E_x and H_y expressions for the quasi-TE (transverse electric) mode and E_y and H_x expressions for the quasi-TM (transverse magnetic) mode. The nonequidistant discretization scheme used in this chapter is more versatile than the equidistant discretization reported by Stern. The discretization errors due to these formulations are also discussed. Chapter 5 discusses beam propagation methods for the design of two- and three-dimensional (2D, 3D) optical waveguides. Discussed here are the fast Fourier transform beam propagation method (FFT-BPM), the finite-difference beam propagation method (FD-BPM), the transparent boundary conditions, the wide-angle FD-BPM using the Padé approximant operators, the 3D semi-vectorial analysis based on the alternate-direction implicit method, and the fully vectorial analysis. The concepts of these methods are discussed in detail and their equations are derived. Also discussed are the error factors of the FFT-BPM, the physical meaning of the Fresnel equation, the problems with the wide-angle FFT-BPM, and the stability of the FD-BPM. Chapter 6 discusses the finite-difference time-domain method (FD-TDM). The FD-TDM is a little difficult to apply to 3D optical waveguides from the viewpoint of computer memory and CPU time, but it is an important analysis method and is applicable to 2D structures. Covered in this chapter are the Yee lattice, explicit 3D difference formulation, and absorbing boundary conditions. Quantum wells, which are indispensable in semiconductor optoelectronic devices, cannot be designed without solving the Schrödinger equation. Chapter 7 discusses how to solve the Schrödinger equation with the effective mass approximation. Since the structure of the Schrödinger equation is the same as that of the optical wave equation, the techniques to solve the optical wave equation can be used to solve the Schrödinger equation.

Space is saved by including only a few examples in this book. The quasi-TEM and hybrid-mode analyses for the electrodes of microwave integrated circuits and optical devices have also been omitted because of space limitations. Finally, we should mention that readers are able to get information on the vendors that provide CAD software for the numerical methods discussed in this book from the Internet.

We hope this book will help people who want to master optical waveguide analyses and will facilitate optoelectronics research and development.

KENJI KAWANO and TSUTOMU KITOH

Kanagawa, Japan
March 2001

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