MTF-Modulation Transfer Function, Geometrical and Diffraction

The Modulation Transfer Function (MTF) option computes the MTF of the lens system including or excluding diffraction effects, assuming either a sine wave or a square wave object.

When to Use the Modulation Transfer Function (MTF) Option

Use the MTF option when your performance specification is given in terms of MTF (or its equivalents - OTF or sine wave response) or square wave response; the geometrical approximation to both is also available.

MTF generates a table of system response at designated frequencies at one or more focal positions. Separate tables are generated for each field and zoom position. In each table, each row represents a given spatial frequency; each column represents a given focus position previously chosen in the LDM. Various MTF commands allow the user to choose the frequencies (in integral multiples of some increment) and to plot either the rows (MTF vs. focus) or the columns (MTF vs. frequency).

The default is to compute diffraction sine wave response. The response can be converted to square wave response (**SQW**) or to the geometrical approximation (**GEO**) or to both. Computations are made for targets oriented at 0° and 90°; additional tables of response for target orientations of 45° can be added (**DIA**). A table of the spatial phase can be added (**PHA**), as can a table of the wavefront map (**WFR**); this latter table is similar to output from PMA. Image motion can be simulated in any vector direction determined by the X and Y components of motion (**IMX**, **IMY**); if these components are different in each wavelength, then the effects of chromatic smear are simulated.

Default Operation

Diffraction sine wave MTF tables are generated for all field angles and zoom positions, using the depth-of-focus values defined in the LDM (FFO, IFO, NFO). The maximum frequency is the axial diffraction limit, and the increment in frequency is 1/60 of the axial diffraction limit. MTF uses Gaussian or user-defined (.INT files) apodization, and polarization, if present in the lens data.

Command Mnemonics
(alphabetical)

DIA	FOC	FRE	GEO	GRD	IFR
IMX	IMY	LAB	MFR	NRD	OVE
PHA	PLO FRE	PLO FOC	SPA	SQW	TIT
WFR					

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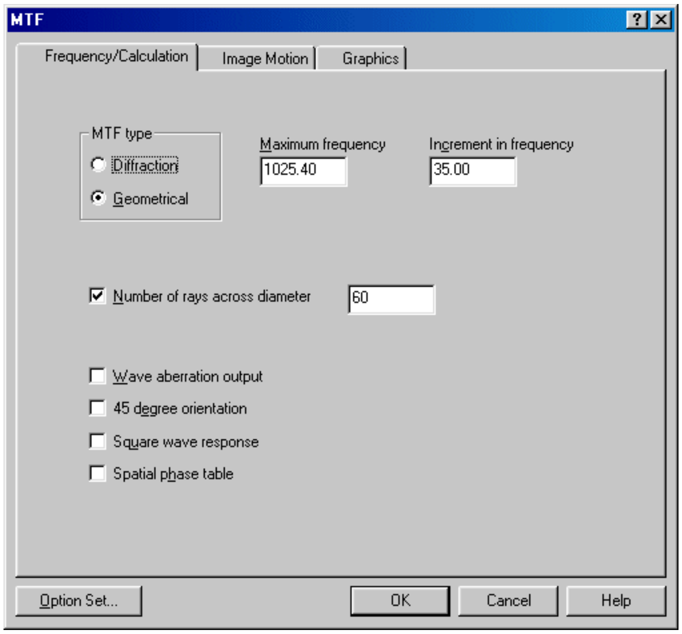
Defining Frequency/Calculation Settings

To define frequency/calculation settings for the MTF option, choose the **Analysis** > **Geometrical** > **MTF** menu, or the **Analysis** > **Diffraction** > **MTF** menu. The **MTF** dialog box displays, with the **Frequency/Calculation** tab automatically displayed. Define the controls on this tab as described in the table below.



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Command Syntax		
Screen Control	**Explanation**	**Default**
GEO Yes | No....z		
MTF type: **Diffraction** **Geometrical**	Selects whether you want geometrical response. **Note**: if polarization is present in the lens data, it is ignored for geometrical response calculations.	No. Includes the effects of diffraction.
MFR max_lines_per_mm....z		
Maximum frequency	Maximum frequency (line pairs/mm) for computation, measured on the image surface.	Axial diffraction limit at shortest wavelength.
IFR incr_lines_per_mm....z		
Increment in frequency	Increment in frequency (line pairs/mm) between each successive computation of response, measured on the image surface.	Axial diffraction limit at shortest wavelength/NRD
NRD num_rays_across_diameter | No....z		
Number of rays across diameter	If IFR is not entered, NRD determines the IFR frequency increment directly. If IFR is entered, NRD defines a range of valid grid sizes from 0.63 * NRD to 1.26 * NRD.	60.
WFR Yes | No....z		
Wave aberration output	Adds diagnostic display of wave aberration in exit pupil for each wavelength focus, field and zoom position, relative to a reference sphere centered on the chief ray intersection with the image surface.	No. No added display.
DIA Yes | No....z		
45 degree orientation	Adds output table for diagonal targets orientation of 45°, 135° to the normal 0° and 90° (radial and tangential, response).	No. 45°,135° not included.
SQW Yes | No....z		
Square wave response	Square wave response.	No. Sine wave response.
PHA Yes | No....z		
Spatial phase table	Adds spatial phase table to modulation tables - complete OTF.	No. Only modulation tables provided.

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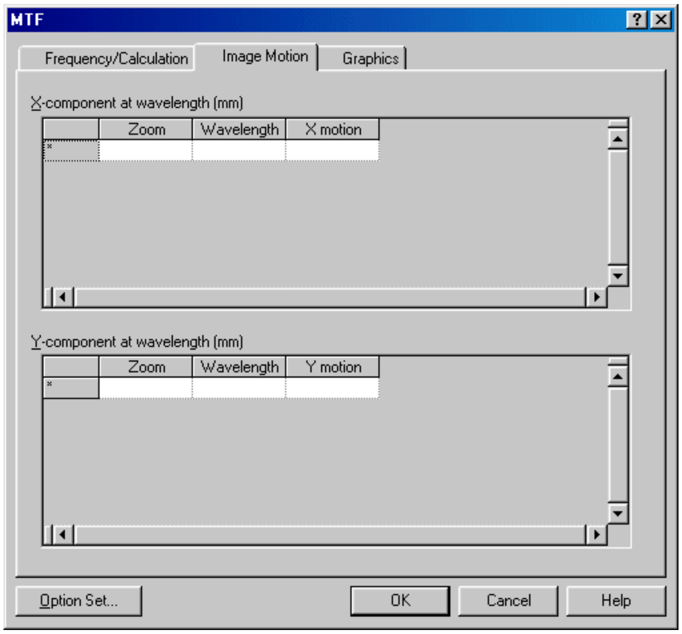
Defining Image Motion Settings

To define linear image motion/lateral color simulation settings for the MTF option, choose the **Analysis** > **Geometrical** > **MTF** menu, or the **Analysis** > **Diffraction** > **MTF** menu. The **MTF** dialog box displays. Click the **Image Motion** tab, and define the controls on this tab as described in the table below.



Command Syntax		
Screen Control	**Explanation**	**Default**
IMX x_mm_image_motion....w,z		
X-component at wavelength	X component of image motion (in millimeters) for all wavelengths; if desired, different values can be applied to each wavelength.	0.0
IMY y_mm_image_motion....w,z		
Y-component at wavelength	Y component of image motion (in millimeters) for all wavelengths; if desired, different values can be applied to each wavelength.	0.0

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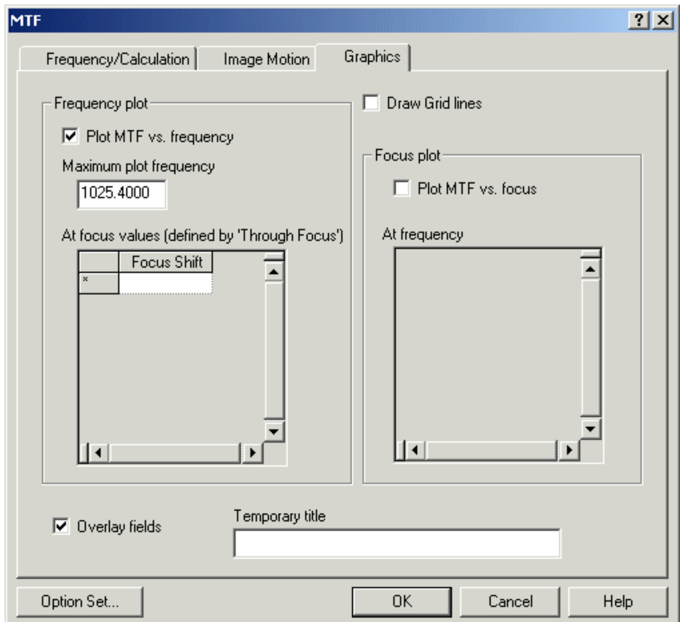
Defining Plotting Controls

To define plotting controls for the MTF option, choose the **Analysis** > **Geometrical** > **MTF** menu, or the **Analysis** > **Diffraction** > **MTF** menu. The **MTF** dialog box displays. Click the **Graphics** tab, and define the controls on this tab as described in the table below.



Command Syntax		
Screen Control	**Explanation**	**Default**
PLO FRE Yes | No....z		
Plot MTF vs. frequency	Plot response vs. frequency graphs at focus positions specified by FOC command(s). Uses all computed frequencies. Up to 10 PLO FRE commands may be included.	Yes.
FOC plotted_focus_shift....z		
Focus shift	Value of focus position to be included in PLO FRE operation. **Range**: One of the focus positions specified by FFO, IFO, NFO in the LDM. Up to 10 FOC commands may be included for each PLO FRE command.	Mid-focus (or left of middle, if even number were computed).
SPA max_frequency....z		
Maximum plot frequency	Maximum frequency to be plotted for the PLO FRE command. **Range**: Any value up to MFR value.	MFR value.

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Command Syntax		
Screen Control	**Explanation**	**Default**
PLO FOC Yes | No....z		
Plot MTF vs. focus	Plot through-focus curves at frequencies specified by FRE command(s). Uses all calculated focus positions. Up to 10 PLO FOC commands may be included.	No. No plotting done in any zoom position.
FRE plotted_frequency....z		
At frequency	Value of frequency to be included in the PLO FOC operation. **Range**: One of the frequencies specified by MFR, IFR for computation. Up to 10 FRE commands may be included for each PLO FOC command.	First computed frequency (=IFR).
LAB Yes | No....z		
Command line only	Label plots in standard manner. Use of No omits labeling for that zoom position - quick plot. To suppress the automatic time and date stamp on plots, use the TAD No command. See "Plot Labeling" on page 1-87 for details.	Yes. Labeling is included.
TIT 'temporary-title'(20+20)....z		
Temporary title	Up to 40 characters are plotted in two 20 character lines for the title of each zoom position for the PLO FRE or PLO FOC command; if only one is given, the title is applied to all zoom positions.	Uses the first 40 characters of the system title.
OVE Yes | No....z		
Overlay fields	Overlay all field angles of a zoom position in standard manner. Use of No draws each field position separately for that particular zoom position.	Yes. Fields are overlaid.
GRD Yes | No....z		
Draw grid lines	Add grid lines to the plot. Applies to both PLO FRE and PLO FOC.	No.

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Discussion of Input and Computations

What to Include in the LDM Data

No additional LDM data is needed to run the MTF option.

If you wish to perform through-focus analyses of MTF, enter the LDM data for depth of focus (NFO, FFO, IFO). Note that regardless of lens units (I, C, M), the MTF spatial frequencies are always expressed in line pairs per mm (or cycles/mm) at the image surface. If the system is afocal, the perfect lens focal length can be adjusted to change the scale of the spatial frequencies (for example, if the perfect lens focal length is one meter in lens units divided by the system magnification, then cycles/mm in image space numerically correspond to cycles/mrad in object space).

All MTF computations are computed polychromatically using the wavelength weights in the LDM. Additional wavelengths (up to 21 total) may be specified to improve the polychromatic approximation. The SPE option can be used to compute the appropriate wavelengths and wavelength weights to best approximate a given spectral response.

Implications of Choice of Spatial Frequencies

MTF is usually desired at different frequencies than the default conditions provide. Enter either or both of the **MFR** and **IFR** requests to modify the range and increments to the desired values. MTFs will be computed for integral multiples of the **IFR** up to the **MFR**. For curved image surfaces, the spatial frequencies are measured on the actual image surface rather than on the tangent plane.

If **IFR** is not entered, the frequency increment is determined from the default or entered value for **NRD** (IFR = Axial diffraction limit at shortest wavelength/NRD).

If **IFR** is entered, direct convolution is used to compute MTF as long as the required grid spacing for the shortest wavelength (axial_diffraction_limit/**IFR**) is less than 1.26 ∗ **NRD**. Otherwise, if there are no obstructions in the lens, interpolative convolution is used to compute MTF where the interpolation is done on the pupil function and the grid size is determined by **NRD**. In the case where there are obscurations in the lens and **NRD** is not large enough to allow direct convolution, the run is aborted and a message is printed suggesting alternate values for **NRD** or **IFR**.

If the **GEO** command has been used to compute geometrical MTF, there are no restrictions on frequency increment choices; any appropriate **IFR** value may be used. The method used is an approximation of the Hopkins method as the wavelength goes to zero. See "Geometrical Optical Treatment of Frequency Response," H. H. Hopkins, *Proc. Phys. Soc.*, B, LXX (1957), page 1162.

NRD can be entered to modify the range of valid grid sizes. Some examples of when to do this are:

- You want to evaluate the MTF of obstructed systems at specific spatial frequencies (enter both **IFR** and **NRD**).
- You want a rough but quick approximation to the MTF using direct convolution (use **NRD** only, and accept the resulting frequency increment).
- You want the high accuracy of a fine grid (for modeling unusual wavefronts or aperture shapes) but want tables printed at frequencies unrelated to the grid (use both **NRD** and **IFR**).
- You have a system for which the NA (numerical aperture) varies from run to run or between zoom positions and you want to have a consistent grid pattern in each run or zoom position (use **NRD** without **IFR**).

The square wave response (**SQW**) is calculated from a series summation of sine wave response values. For diffraction MTF, the sine wave calculation is carried out to the cutoff frequency. For geometrical MTF, it is carried out to 500 terms.

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Occasionally there is a need for diagnosis of the ray grids to determine whether to change the **NRD**. The **WFR** command allows the user to display the ray grids actually used for each wavelength, focus, field, and zoom position; these are most useful in finding fields where excessive vignetting or obscuration leaves too few rays to provide accuracy. The **NRD** command can then be used to increase the number of rays appropriately. For example, **NRD** 50 sets up a 50 x 50 grid over the full pupil with direct convolution MTF computed at multiples of the resulting frequency increment (approximately 1/50 of diffraction cutoff). If **IFR** is also specified, then depending on the **IFR** value the grid spacing will change to be somewhere between 32 x 32 and 63 x 63 to compute direct convolution MTF, or interpolative MTF will be computed with a 50 x 50 grid.

Plotting

A **PLO FRE** command will generate a single graph of modulation vs. frequency for each zoom position, with all field angles for a given zoom position overlaid on that plot. If multiple focus positions have been calculated, this is done for the position in the middle or to the left of middle.

A maximum of five fields can be plotted per plot. If more than five fields have been specified, then multiple plots will be generated per zoom as necessary to plot all fields.

Usually, plots are desired only for one focus position which has been chosen from a prior run. A specific focus position is selected by using a **FOC** command, with the value of the focus position entered in the fields for each zoom position. Typically, then, unless only one focus position has been calculated, both the **PLO FRE** and **FOC** commands will be used.

Finally, if the range of frequencies to be plotted is smaller than the range which is calculated, a **SPA** command may be used to specify the maximum frequency desired on the plot.

Multiple (or modified) focus positions may be specified in the LDM to display the through-focus (or shifted) characteristics of the image. For plotted depth of focus studies, a **PLO FOC** command will generate a single graph of modulation vs. focus for each zoom position, with all field angles for a given zoom position overlaid on that plot. If no specific frequencies are requested, the first frequency at all field angles (both S and T) will be included. It is therefore desirable to limit the number of frequencies to just those on which quality judgments are to be made; this is done by the **FRE** command. Typically, then, both the **PLO FOC** and **FRE** commands will be used together. Note that:

- If multiple **FOC** commands are given for a single **PLO FRE** command, then all of the plots specified by the **FOC** commands appear in a single plot window (if supported by the plot driver).
- If multiple **PLO FRE** commands are given and a single **FOC** command is given for each **PLO FRE** command, then each focus position will be plotted in a separate window.

The colors for the fields will be set by CLS FLD. The first two line styles set by STL will be used for the R and T curves, respectively. If both line styles for R and T curves are solid lines, the line style for the R curve will default to line style # 3. The default line style for the axis curve is solid and style # 4 for the diffraction limit curve. (See “Defining Configuration - Graphics” on page 24-16.)

Description of Output

The default output of MTF is a header giving information on focal lengths or magnifications and f/numbers for each field position followed by a table of MTF vs. spatial frequency for each specified field position. If only one focal position has been specified in the LDM, a warning message is printed saying that at least three focal positions are recommended for finding the best focus. If more than one focal position has been specified in the LDM, then the MTF for each focal position is given in the table. A separate set of output is generated for each zoom position.

If the optical system is not isoplanatic at a given field point (i.e., the point spread function is not nearly uniform over an image area of several cycles of the **IFR**), then MTF may not be meaningful, and a warning message is printed out.

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Optional tabular output includes a wavefront map (similar to the output of PMA) with the **WFR** command, an additional MTF vs. spatial frequency table for diagonal targets with the **DIA** command, and a spatial phase table with the **PHA** command.

Optional plotted output can be obtained for MTF vs. spatial frequency for up to ten different focus positions, and MTF vs. focal position for up to ten different spatial frequencies.

Note that plotted MTF output (vs. frequency or vs. focus position) uses spline fitting for smooth curves. This fitting process may not produce accurate MTF data between computed points, especially if only a few points are plotted.

For an example, we shall use a Tessar lens, defined by the following:

```
LENS
RDM  Y
TITLE  'TESSAR'
WL   650  550  480
WTW  1  2  1
FNO  5.6
DIM  MM
NFO  5
FFO  -.1
IFO  .05
YAN  0  14  20
VUY  0  0  .2
VLY  0  0  .1
S    15.5  1.9  SK1
S   -58.5  1.8
S   -20.3  1.0  LLF4
S    14.0  2.7
     STO
S   -41.9  1.0  LLF4
S    12.2  3.0  SSK4
S   -15.6  0.0
     PIM
SI   0.0 -0.2
GO
```

This lens is shown in Figure 1.

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Command inputs:
VIE
GO

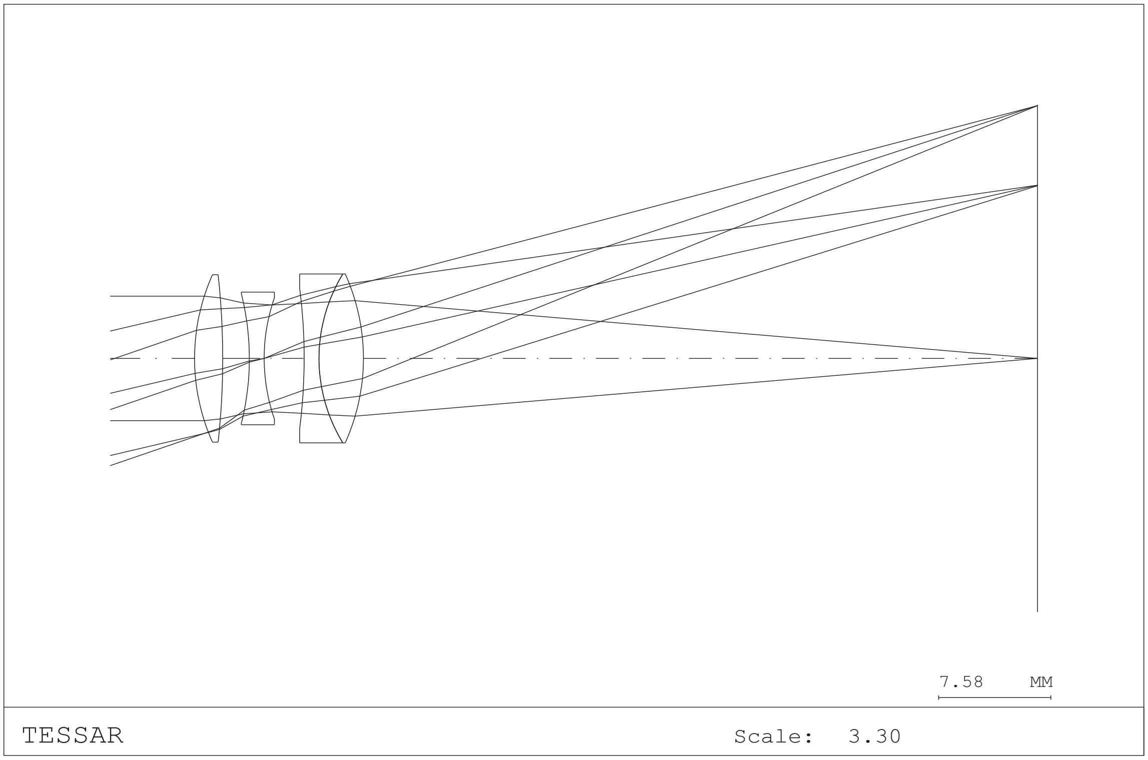

Figure 1. Layout of Tessar lens used for MTF examples

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Examples

Example 1. Compute and plot the MTF out to 50 lp/mm and the through-focus MTF at 20 lp/mm.

The commands to do this are:

```
Command inputs:
   MTF
   MFR  50
   IFR  10
   PLO  FRE
   PLO  FOC
   FRE  20
   GO
```

The tabular output for field 3 is given in Table 1. The other fields are similar.

Table 1. Tabular output for Tessar example

```
                                                                                MTF
                APERTURE STOP
                SEMI-DIAMETER =     3.835192
                  (Based on the maximum reference ray height at the stop.)

    X and Y focal lengths for each field angle

       X         0.498092E+02        0.513319E+02        0.530408E+02
       Y         0.498092E+02        0.545229E+02        0.603852E+02

    X and Y F-numbers for each field angle

       X             5.582152            5.752801            5.944319
       Y             5.582152            6.110414            7.961654

    Reference sphere radius for each field angle

                 0.519788E+02        0.535158E+02        0.552199E+02

(skipped output)

         POSITION  1         DIFFRACTION MTF

           TESSAR
                                                  WAVELENGTH   WEIGHT  NO. OF RAYS
    FIELD (X,Y)=( 0.00, 1.00)MAX, (  0.00, 20.00)DEG    650.0 NM     1      424
    RELATIVE ILLUMINATION =  70.7 PER CENT             550.0 NM     2      588
    ILLUMINATION (UNIT BRIGHTNESS) = 0.017866          480.0 NM     1      782
     DISTORTION =    0.13 PER CENT


    DIFFRACTION LIMIT                           FOCUS POSITION
    Formula  Actual     -0.10000    -0.05000    0.00000     0.05000     0.10000
 L/MM f/5.600 RAD  TAN    RAD  TAN    RAD  TAN    RAD  TAN    RAD  TAN    RAD  TAN
 ---- ----------------
---------------------------------------------------------
   0 .999   .999 .999   .999 .999   .999 .999   .999 .999   .999 .999   .999 .999
  10 .960   .960 .946   .901 .833   .925 .857   .932 .870   .921 .871   .893 .862
  20 .921   .920 .894   .787 .606   .843 .660   .847 .688   .795 .686   .691 .657
  30 .881   .880 .841   .699 .422   .779 .485   .762 .514   .647 .505   .457 .460
  40 .841   .840 .789   .629 .291   .725 .344   .679 .362   .498 .344   .248 .296
  50 .802   .800 .737   .567 .205   .678 .238   .601 .240   .365 .215   .095 .174
```

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MTF lists the aperture shapes on the aperture stop surface, as these will limit the ray bundles. If no aperture is specified for the aperture stop surface, then MTF uses the CODE V default aperture for that surface.

The X and Y focal lengths for each field are computed as the ratio of differential pupil position to differential numerical aperture at the chief ray for each field position. This computes the instantaneous magnification of the lens at that field position, and takes into account any image surface tilt or curvature. For finite conjugate systems, the magnification is listed in place of the focal lengths. The X and Y f/numbers are computed as the inverse of the difference in direction cosines of the plus and minus reference rays for that field position. Note that if you have not executed a SET VIG command, the reference rays may not correspond to the real edges of the pupil, and the f/number calculations may be in error. The MTF calculations will, however, be correct in any case.

For each field position specified, information is printed for the number of rays traced in the MTF grid for each wavelength, and the relative illumination and distortion at that field position. The relative illumination calculation assumes unit brightness at the exit pupil in units of W/cm^2-steradian. This calculation is done by counting rays traced and taking into account the image obliquity. The distortion calculation is the same as is output in ANA.

Next comes the MTF table. The first column is the spatial frequency in line pairs/mm at the image surface. The next column is the diffraction limit of the optical system based on the f/number in the specification data (or f/number computed from the specification data and first-order ray tracing). The third column, which may be a pair of columns, is the diffraction limit of the actual optical system in the radial and tangential directions, based on the actual vignetting, obscurations, and pupil aberration factors in the lens for that field position. After that comes the computed radial and tangential (or X and Y) MTFs for each focal position. Radial (or X) MTF is horizontal MTF (vertical bars) in local image surface coordinates; tangential (or Y) MTF is vertical MTF (horizontal bars) in local image surface coordinates.

The plotted output for this example is shown in Figure 2 and Figure 3. Figure 2 shows the MTF vs. spatial frequency at the nominal focus position, and Figure 3 shows the MTF vs. focus position for a spatial frequency of 20 lp/mm. In both plots, each field is plotted, with radial and tangential MTFs plotted separately. With the **OVE NO** command, separate plots are generated for each field position. For the MTF vs. frequency plot, if plotted output is desired over a frequency span different from what was computed, the **SPA** command may be used to limit the frequency span.

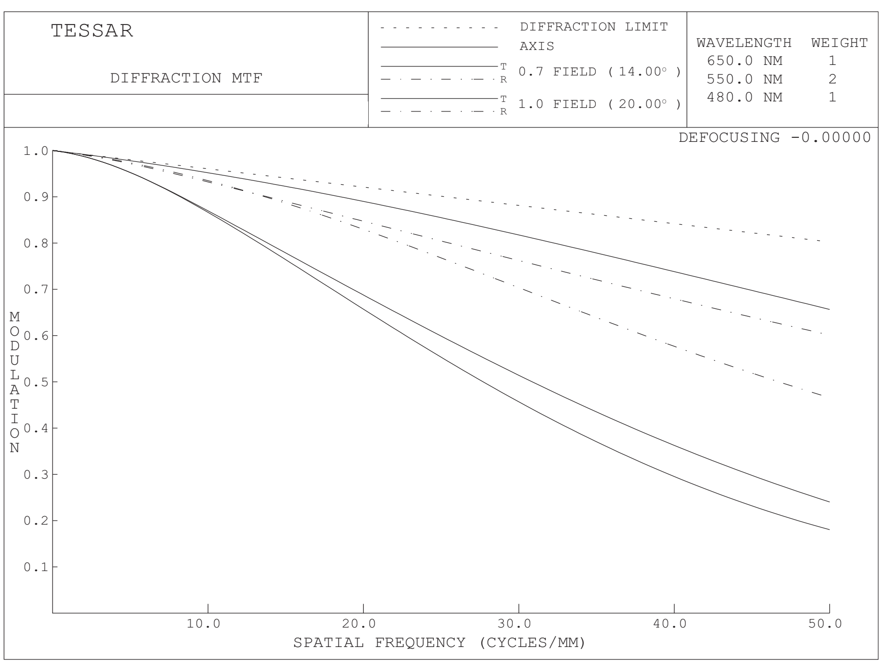

Figure 2. MTF vs. spatial frequency at the nominal focus position

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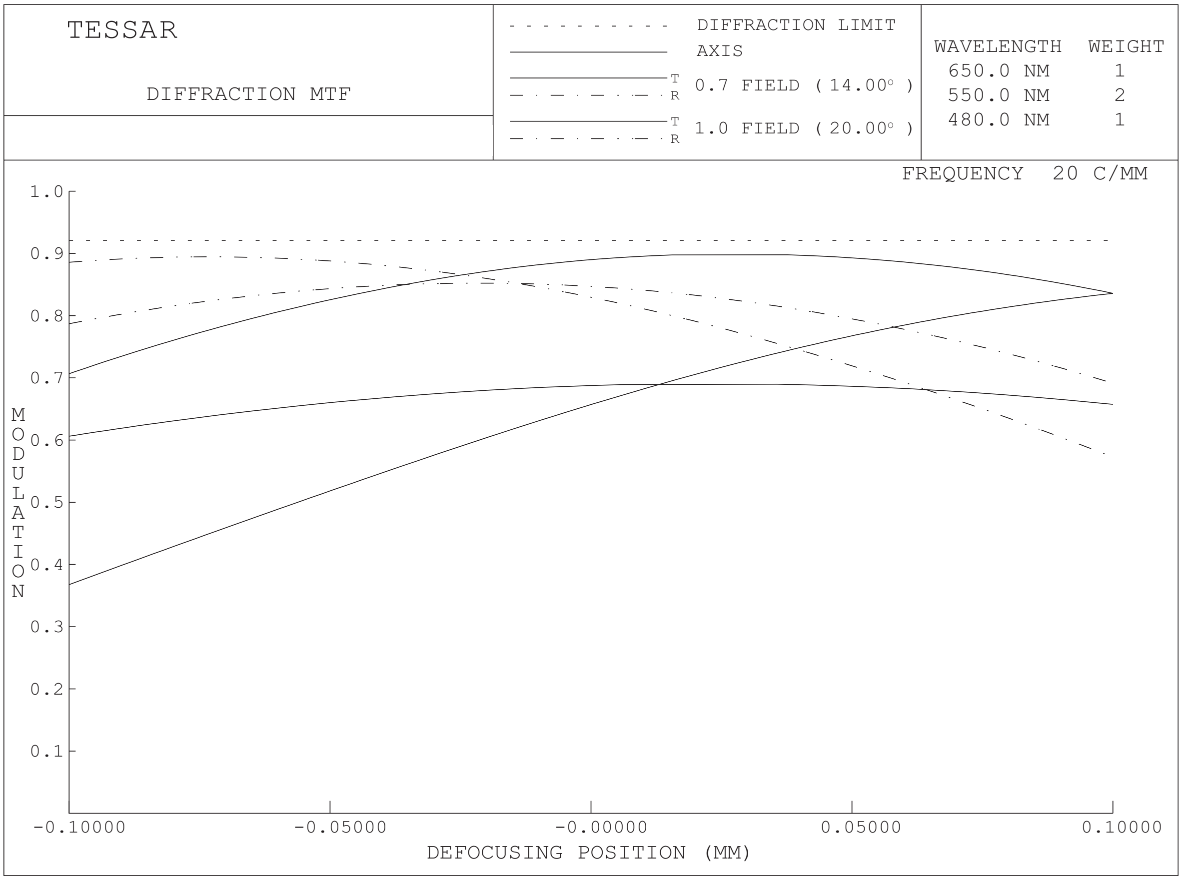

Figure 3. MTF vs. focus position for a spatial frequency of 20 lp/mm

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Example 2. Geometrical MTF calculation of the same Tessar lens

The input commands and plotted output are shown in Figure 4. Note that for the on axis field point, the geometrical MTF is above the diffraction limit. This is an obvious indicator that diffraction MTF should be used for this lens, or misleading MTF values will result. Geometrical MTF should only be used when the RMS spot size is significantly larger than the diffraction limited spot size.

```
Command inputs:
   MTF
   MFR  50
   IFR 10
   GEO
   PLO  FRE
   GO
```

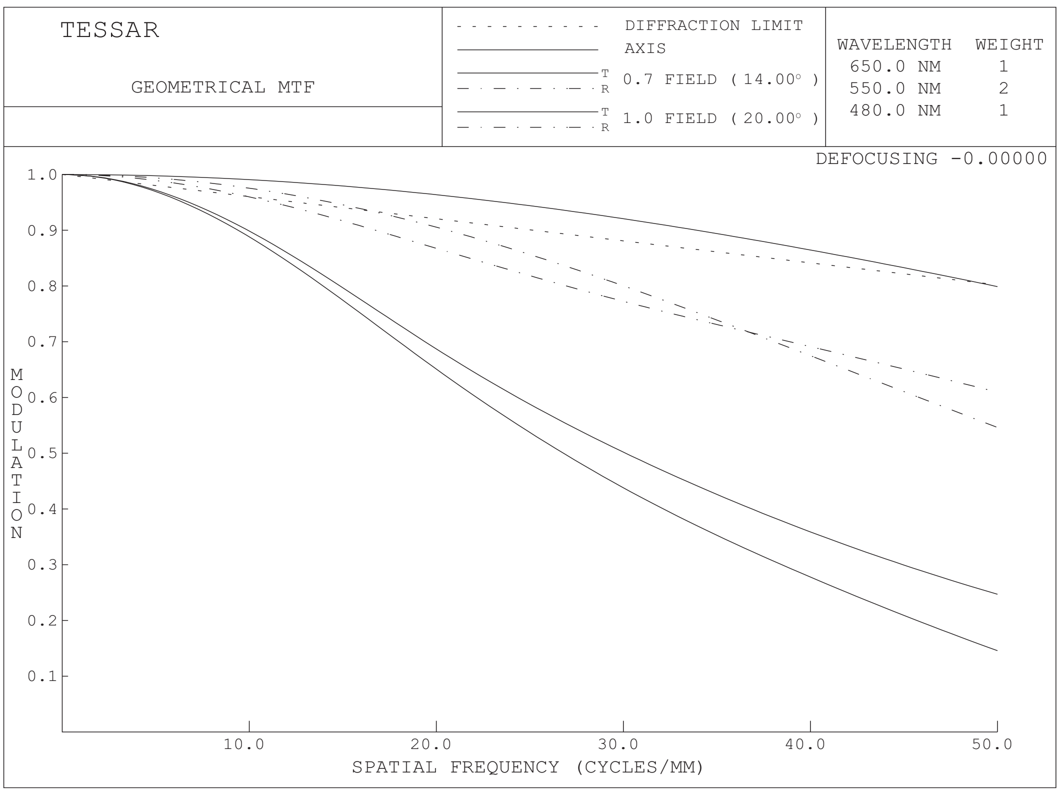

Figure 4. Geometrical MTF plotted output

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Example 3. Effects of obscuration on MTF

Central obscurations in a lens will affect the MTF, generally reducing the mid-frequency MTF and increasing the high frequency MTF. The crossover point is dependent on the fraction obscured, with higher obscuration ratios having greater effects and higher crossover frequencies. The most common example of obscured systems is in telescopes, such as a cassegrain, which have a primary and secondary mirror with the secondary blocking part of the energy reaching the system.

For an example of the effect of obscurations, we will compute the MTF for a Ritchey-Chretien cassegrain telescope.

The sequence to run the MTF for this example and the plotted output are shown in Figure 5. Note that the shape of the diffraction limit curve is different in this case. This is due to the central obscuration of the lens which causes the response at lower frequencies to be reduced, while enhancing response at high frequencies; the amount of this affect is a direct function of the obscuration ratio. The printed output (not shown) also shows the reduced diffraction limit under the column “DIFFRACTION LIMIT - ACTUAL.”

Command inputs:

```
RES  CV_LENS:CASSRC

MTF
MFR  120
IFR   5
PLO  FRE
GO
```

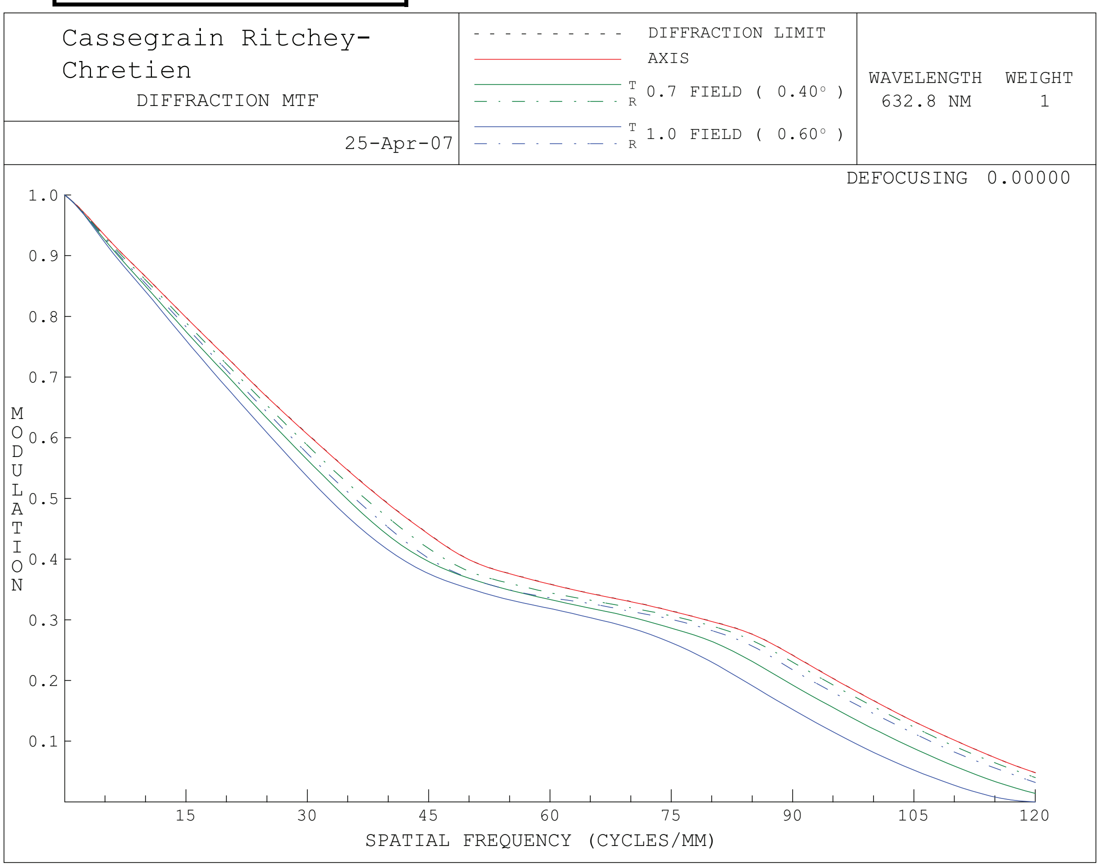

Figure 5. Plotted MTF output for obscured Cassegrain

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Example 4. Square wave MTF for a perfect optical system

We will compute the square wave MTF for a perfect optical system, to show the difference between sine wave and square wave MTF. The square wave MTF is computed from the following equation, using S for square wave MTF and M for sine wave MTF:

$$S(f) = \frac{4}{\pi}\left[M(f) - \frac{M(3f)}{3} + \frac{M(5f)}{5} - \frac{M(7f)}{7} + \ldots\right]$$

Because of the higher frequency terms required to compute square wave MTF, the MTF is calculated out to a frequency of 100 times the **IFR** or to cutoff, whichever is smaller, and thus will take more computer time than a corresponding sine wave MTF.

The sequence to define the perfect lens and run the square wave MTF and the plotted output for this example are shown in Figure 6. Note that the shape of the diffraction limit curve is different in this case. This is due to the effects of square wave vs. sine wave. Often, square wave response is higher than sine wave because of the $4/\pi$ factor in the function; it is always true of the last 2/3 of the frequencies to cutoff because no contribution can come from the higher order terms. This is shown by the "bump" in the curve at 1/3 of cutoff.

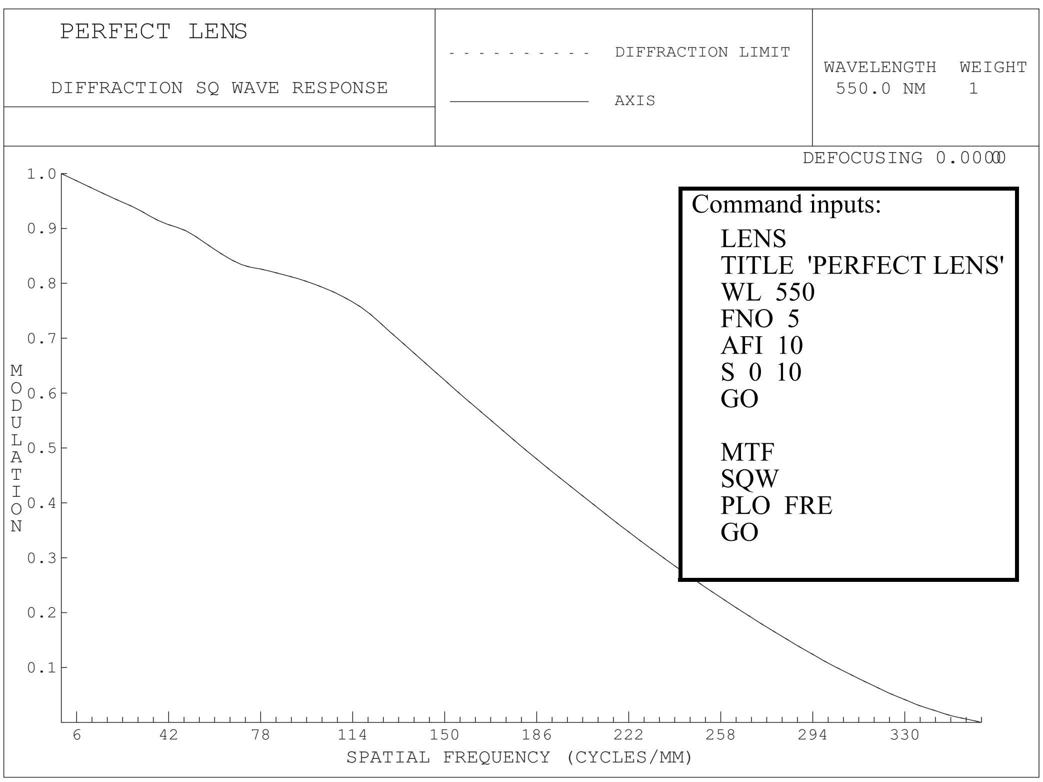

Figure 6. Square wave diffraction MTF

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Technical Notes

Calculation Method

Diffraction MTF is calculated utilizing H. H. Hopkins' method for the numerical evaluation of the auto correlation integral of the pupil function (H. H. Hopkins, *Proc. Phys. Soc.*, 70, B 1002 [1957]). The canonical pupil coordinates, suggested by Hopkins, are also employed. An adjustment in the calculation is made so that the frequencies are measured on the image surface itself.

The square wave response is calculated from a series summation of sine wave response values (see W. Smith, *Modern Optical Engineering*, McGraw-Hill, N.Y., 1966, p. 318).

The relative illumination of each field point is computed including all effects of vignetting, pupil expansion and cosine^4 (but not variations of transmission or angular sensitivity of coatings).

Isoplanatism and OSC

Any calculation that deals with extended objects depends upon isoplanatism constancy of the point image patch as the field changes over the width of the object, e.g. over enough cycles of the MTF target to integrate properly. Isoplanatism is connected with Offense Against Sine Condition (OSC) and is destroyed by significant OSC residuals. OSC can also lead to invalid distribution of rays over the exit pupil, if not properly compensated.

In all CODE V options that deal with diffraction or accurate intensity calculations (ALI, CEF, LSF, MTF, PAR, PMA, PSF, TOR, and WAV), special programming has been incorporated to represent the local intensity and magnification changes that occur with OSC, so that any wavefront and point image is correctly apodized and calculated, even in the presence of large OSC; for example, an f/0.3 parabola would have massive OSC. This special process is valid in all forms of calculation (convolution, FFT, etc.) and has little effect on compute time.

Even though the calculation is accurate, the lack of isoplanatism will still cast doubt on the usefulness of any option results dealing with extended objects, when the image is changing so rapidly across the target. In these options (CEF, LSF, MTF, PAR, PMA, PSF), a test is made to see if OSC is sufficient to affect results. If so, a warning is issued to alert you that the system is not isoplanatic at the field in question.

F-Numbers, Focal Lengths and Reduction Ratios

In some of the CODE V options that deal with diffraction calculations (CEF, LSF, MTF, PAR, PMA, and PSF), the listed output includes focal lengths (or reduction ratios if the object distance is finite) and f-numbers based on the references rays at each field. A value for both the X and Y directions is given. These numbers vary with field and are useful for relating image dimensions to object dimensions and for simple relative illumination calculations. Also included are the reference sphere radii for each field. Diffraction is assumed to take place at the reference sphere.

The f-numbers (fno) are based on a calculation of the image intensity. Thus, if the pupils are approximately elliptical, an estimate of the relative illumination can be obtained from the product of the X and Y numerical apertures. (The numerical aperture is given by NA = 1/(2*fno).) This calculation assumes a Lambertian object and a uniformly illuminated exit pupil and ignores transmission losses in the lens. It includes the effect of a non-flat image surface. Note that, for a perfect thin lens with stop in contact and the object at infinity, the X numerical aperture is proportional to the cosine of the field angle while the Y numerical aperture is proportional to the 3rd power of the cosine, thus giving the cosine 4th law.

The reduction ratio, which is listed for a system with a finite object distance, is the ratio of an incremental distance on the image to the corresponding distance on the object, measured in a coordinate system normal to the surface. In general, a square patch on the image can represent a parallelogram shaped patch on the object, with the X and Y reduction ratios used to calculate the sides. The angle of the parallelogram is not listed although it is included in the internal calculation.

The focal length, which is listed for a system with an infinite object distance, relates the image patch size to an angle in object space. It can be thought of as the product of the f-number and pupil diameter, similar to the paraxial definition. Consistent with the other calculations, it assumes a measurement normal to the object surface which is typically flat for an object at infinity. Thus, dividing the image patch size by the focal length gives the change in the sine of the field angle. To get the change in angle, divide this number by the cosine of the field angle.

The reference sphere radius is also listed for each field. It is used to calculate the wave aberrations and is also used in the calculation of the focal lengths and f-numbers although it normally has very little influence on these numbers. It can be used to identify potential problems when the radius is so short that the reference sphere is within the caustic of the image.